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Note

The Yarkovsky effect as a heat engine

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Abstract

We show how the Yarkovsky effect can be understood as a heat engine. The output of the engine, manifested in the rate of change in semimajor axis of the body, has a maximum at an intermediate heat capacity, depending on the rotation rate of the body. This maximum arises because the work output depends on the product of the solar heat absorbed by the body and transported from its morning to evening side (this am–pm heat flux increases with heat capacity) and the Carnot efficiency (which declines with heat capacity).

Keywords: Asteroid dynamics; Thermodynamics; Entropy

1. Introduction: dynamic and thermodynamic perspectives

The Yarkovsky effect is the radiation thrust due to the anisotropic radiation of heat from an illuminated rotating body in space. Once a rather obscure effect of interest in precision spacecraft dynamics, it is now seen to be of considerable importance in Solar System studies. The effect can significantly modify the orbital energy of meteorites and small asteroids and may be instrumental in the delivery of meteorites from the asteroid belt to the Earth (e.g., Bottke et al., 2001, 2003; Farinella et al., 1998; Vokrouhlický, 1998). The effect has recently been directly observed by its effects on the orbit of Asteroid 6489 Golevka (Chesley et al., 2003). The effect might even be exploited in asteroid hazard mitigation (Spitale, 2002). The unknown thermal properties of the Earth-crossing asteroids, and the consequent uncertainty in the Yarkovsky force can be the dominant uncertainty in their orbital evolution, and in whether they may collide with Earth, e.g., Giorgini et al. (2002).

Usually, the Yarkovsky effect is computed from a dynamic perspective—in other words by calculating the temperature distribution by analytic (e.g., Vokrouhlický, 1998, 1999) or numerical (e.g., Spitale and Greenberg, 2001) methods and then evaluating the radiative forces on each surface element of the body. This approach explicitly requires consideration of the momentum of the thermal photon streams leaving the object, and their direction (e.g., see Fig. 1).

In this paper, we explore the Yarkovsky effect from a new perspective, that of thermodynamics. Since work is being performed on the object, it follows that available heat must be degraded in order for some to be converted into work. The thermodynamic laws that govern the performance of heat engines must therefore apply.

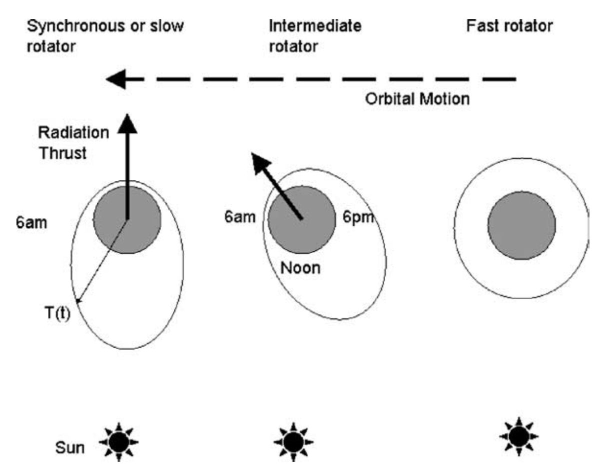

Fig. 1. Schematic of the Yarkovsky effect on an anticlockwise-rotating body (grey circle) orbiting the Sun right to left. The temperature distribution is illustrated as a polar plot (thin line). A slow rotator (or equivalently, one with a low thermal inertia) is everywhere near instantaneous radiative equilibrium and thus has a strong temperature bulge, but one that is symmetric about noon. The thermal photon thrust is therefore orthogonal to the direction of motion and thus performs no work on the object. A fast rotator has an even temperature distribution and thus no net photon thrust. An intermediate rotation/thermal inertia still has a substantial bulge, but slewed into the afternoon: the net photon thrust therefore has a component along the direction of motion and thus accelerates the body, increasing its orbital energy.

2. Yarkovsky effect as a heat engine

For simplicity of exposition, we consider only the diurnal Yarkovsky effect, where the solar flux on a point is modulated only by the rotation of

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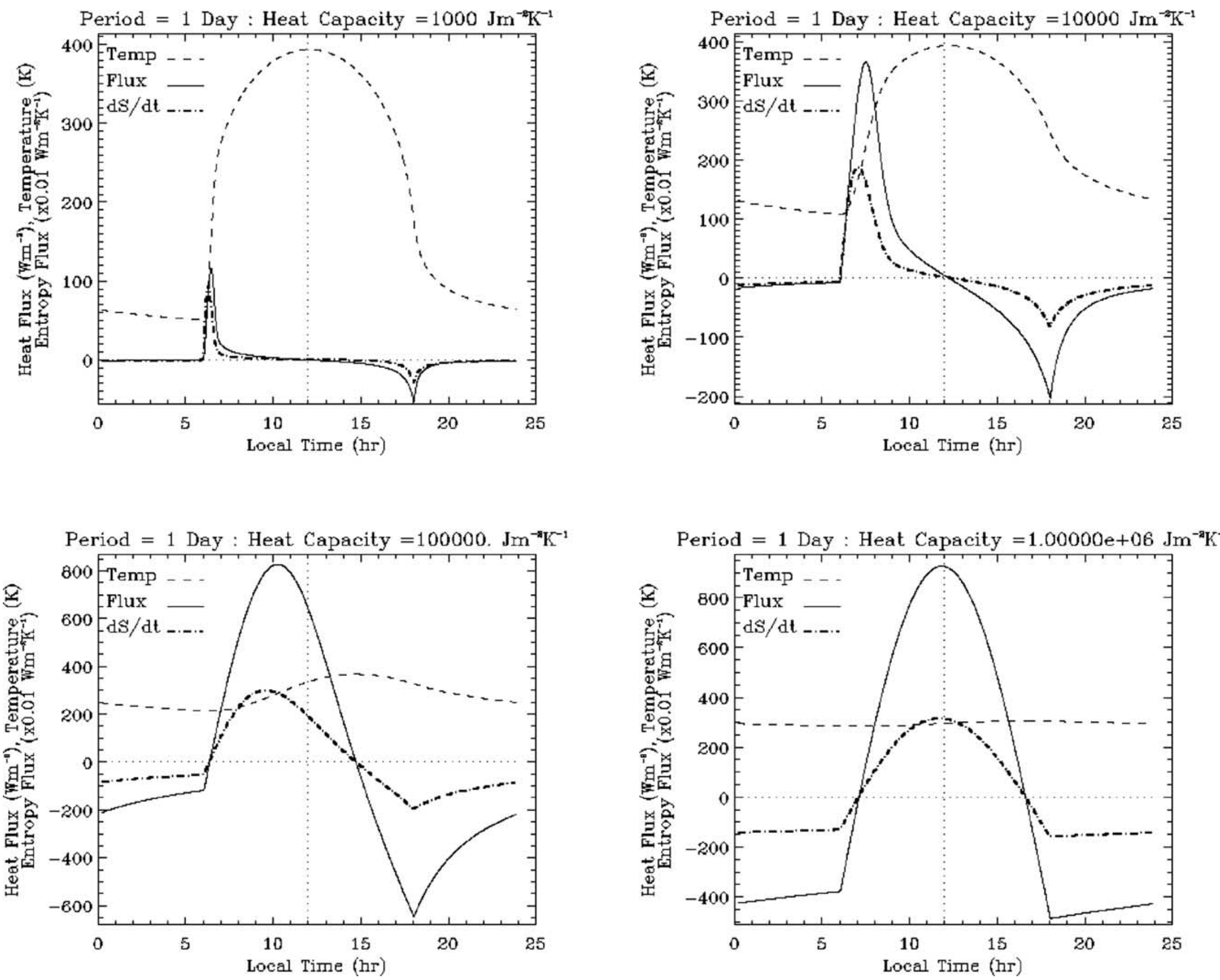

Fig. 2. Temperature and energy balance on a rotating cylindrical black body at 1 AU with a surface ('slab') heat capacity of 1000, 10000, 10^5, and 10^6 $\mathrm{J\,m^{-2}\,K^{-1}}$ and a period of 1 day. For the smallest capacity the absorbed heat flux (solid line) is small, and only deviates from zero at sunrise and sunset—the temperature excursion (dashed line) is large. Combining the two curves yields the entropy production (dashed-dot line). Note that the diurnally-integrated area between the solid line and the abscissa is always zero, since the body as a whole is at a thermal steady state; the integrated area between 0 and 12 hours represents the am–pm heat transport. Moving to larger heat capacities causes the absorbed flux to become more symmetric about noon: the entropy production by absorbed heat is initially small in amplitude and highly asymmetric about zero, and becomes progressively larger and more symmetric.

the body (and for simplicity we assume a cylindrical body with its rotation axis is normal to the orbital plane, following Peterson (1976)). The weaker, seasonal Yarkovsky effect, due to the modulation of flux by the obliquity and/or eccentricity of the body, could be considered in an analogous way.

Classical thermodynamics tends to consider only systems at equilibrium. However, such systems are rarely interesting. More recently, nonequilibrium thermodynamics (sometimes called 'finite-time thermodynamics'), which considers the heat flows explicitly and their limitations on engine performance—be the engine an artificial one or a natural one—has attracted interest. This is the perspective that must be applied to the problem here.

Considering the insolation and surface albedo, and thus the available energy supply, to be fixed, the Yarkovsky engine's performance is determined by two efficiencies multiplied together. The first of these may be considered an 'extractive' efficiency—for the engine to work on sunlight, it must first absorb some of that sunlight. This means that the local thermal balance of absorption and re-radiation must be non-zero. The object therefore absorbs energy in the morning and re-radiates it in the evening.

This process requires that the surface have a non-zero heat capacity or thermal inertia. An infinitesimally thin plate, or equivalently the surface of a perfect insulator, would instantaneously warm to its radiative equilibrium temperature, such that the outgoing thermal radiation equals the absorbed sunlight: there would be no heat available for the engine to transport to the pm side and thus convert into work. We will note at this point that the removal of heat from the radiative surface by conduction is intrinsically an entropy-generating process—heat is degraded by diffusion.

As thermal inertia increases, the morning temperature rises with time less steeply. The lower morning surface temperatures therefore allow less morning re-radiation, with the effect that more heat is absorbed by the body during the morning. This must be re-radiated in the afternoon. The local heat budgets as a function of local solar time for several values of heat capacity are shown in Fig. 2.

The smaller temperature peaks mean that the (greater) heat absorbed is transported down a shallower temperature gradient before being re-radiated. This weaker temperature difference therefore leads to a lower conversion efficiency (i.e., Carnot efficiency) of the heat transported in the body.

Thus, as the heat capacity of the surface increases, the am–pm heat transport in the body monotonically increases, while the Carnot efficiency monotonically decreases (see Fig. 3). The product of these two quantities yields the maximum mechanical work that the system is capable of performing. It is seen that for the case considered here, the available work peaks for

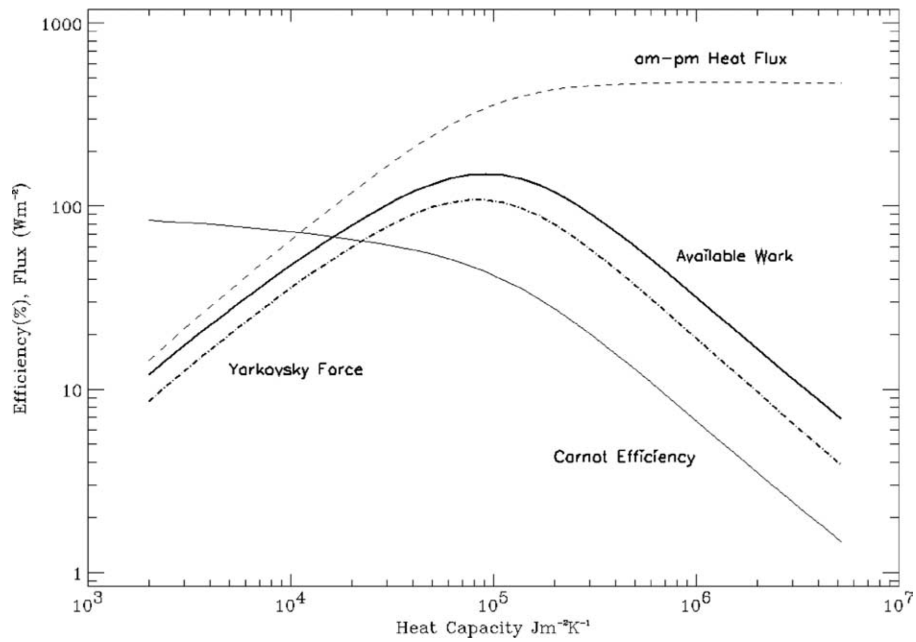

Fig. 3. The am–pm heat flux (dashed line) for a cylindrical body rotating with a period of 1 day at 1 AU as a function of the surface heat capacity. The corresponding Carnot efficiency is shown (thin solid line)—the product of these two curves is the available work from the system (thick solid line.) The Yarkovsky force in the direction of motion is also shown (dash-dot line).

a heat capacity of $\sim 10^5\ \mathrm{J\,m^{-2}\,K^{-1}}$ (given our imposed rotation period, this corresponds approximately to a thermal diffusivity of $\sim 5\times 10^{-7}\ \mathrm{m^2\,s^{-1}}$). The component of the Yarkovsky force along the direction of motion can be calculated in the usual way, and peaks at the same value of heat capacity.

Note that for ease of exposition and calculation, we have considered a cylinder with a 'slab' heat capacity C per unit area and a fixed rotation rate. Slab heat capacity is a simple approach widely used in climate models where there exists a single dominant timescale for temperature variations, and simply represents the amount of heat required to be absorbed per unit area to raise the temperature by 1 K. The slab heat capacity may be related to material properties as $C = \rho c_p L$, where ρ is the material density and c_p is the specific heat capacity of the material. L is a slab thickness, equivalent to the penetration depth of the thermal wave. In the classic thermal conduction problem with periodic forcing with period τ (ignoring factors of order unity), the penetration depth $L = (\kappa\tau)^{0.5}$, where is the thermal diffusivity ($= k/\rho c_p$) with k the thermal conductivity. For $\tau \sim 24$ hrs and a $\kappa \sim 10^{-6}\ \mathrm{m^2\,s^{-1}}$ typical for rocky materials (very porous, fine-grained material might have $\kappa \sim 10^{-7}\ \mathrm{m^2\,s^{-1}}$) $L \sim 0.1$–0.3 m. These parameters may be related to the thermal inertia $\Gamma = (k\rho c_p)^{0.5}$ and the thermal parameters used by other workers (e.g., Peterson's 'P' and the thermal parameter Θ used by Vokrouhlický). For example, $C = \Gamma\tau^{0.5}$.

All these parameters measure the same thing, the ability of a material in a certain setting to wick away heat from the surface. Only by absorbing heat can the asteroid transport it by its rotation from the morning to the evening side. If one imagines the rotating asteroid as a waterwheel, the slab heat capacity is analogous to the depth of the buckets on the wheel, determining how much water (= heat) can be held in each sector of the rotating body before it leaks out too quickly to accumulate.

In principle, our analysis could be extended to the more classical 3-D time-varying thermal conduction problem on a cylinder, sphere or other shape.

3. Some considerations on engine efficiency

We note that a simple linearized expression for the Yarkovsky effect (e.g., Burns et al., 1979; De Pater and Lissauer, 2001) includes a $(\Delta T/T)$ term. We may recognize $\Delta T/T$ as the Carnot efficiency—the maximum heat-to-work conversion efficiency that a system can attain. Specifically, they write the Yarkovsky force (assuming zero obliquity) as $F_y = (8/3)\pi R^2 \times (\sigma T^4/c)(\Delta T/T)$. Since the input power is $\sim 4\pi R^2 \times (\sigma T^4)$ and the work being done on the body (moving at orbital speed v) is vF_y it follows that the conventional engineering efficiency is $\sim (2/3) \times (v/c)(\Delta T/T)$. The prefactor (2/3) arises from the assumed Lambertian emission of the thermal photons: note that the ΔT term in this expression is the morning/evening hemispheric temperature difference—somewhat less than that shown in Fig. 3 which is the total day:night temperature difference.

The term (v/c) is familar as the propulsive efficiency in aerospace design where the exhaust velocity of an engine should be matched to the flight speed in order to maximize (in an inertial reference frame) the addition of kinetic energy to the vehicle, rather than to the exhaust. At nonrelativistic orbital speeds, it is clear that the use of photons as a working fluid leads to low efficiencies.

In comparison, the conversion of solar heating into kinetic energy by a fluid moving across a spherical planet's surface has a total efficiency at maximum power output of about 10% (de Vos and van der Wel, 1992). The Carnot efficiency for this maximum power condition is 30.7%. We may note in passing that at least the long-term average circulations of planetary atmospheres (Lorenz et al., 2001) can appear to organize themselves to maximize mechanical power output, or equivalently at steady-state dissipation. The situation is analogous to the situation considered in this paper—a vigorous circulation transports more heat, but reduces the temperature gradient and thus the Carnot efficiency; the power output is maximized at an intermediate heat transport, which is apparently the state at which the zonal climates of Earth, Titan and Mars find themselves. These states are indistinguishable in terms of heat flow from states of maximum entropy production, although mixing and phase-change processes in the latter case reduce the available mechanical power output (e.g., Ozawa et al., 2003).

The reduction of the Yarkovsky force for small bodies can be considered from a heat engine perspective. Vokrouhlický (1998) presents a detailed discussion of the diurnal Yarkovsky effect and notes a weakening of the effect in the 'small body' regime, where the rotating body has a radius that is smaller than the thermal skin depth. In this situation, some of the heat supplied to the day side of the object is conducted through the center of the body to the other side, rather than being transported around to the night side by the object's rotation. In effect the rotational heat engine is being 'shorted out'—heat is allowed to leak across the engine, rather than running through it.

4. Entropy budget

The entropy dS associated with a quantity of heat dQ is simply dQ/T, where T is the temperature. A flux of heat I from the Sun (with a black body temperature of $\sim$ 6000 K) therefore brings an entropy flux $\sim I/6000$ $\mathrm{Wm^{-2}\,K^{-1}}$ to a surface. That same flux may be re-radiated from a surface in radiative equilibrium with an effective temperature of (say) 300 K, so the entropy flux exported ($I/300$) is much higher than that imported ($I/6000$). Usually, the latter term can be ignored and the $\sim I/300$ flux exported is assumed to be essentially all generated on the surface of the asteroid. Making that approximation more generally, the entropy production dS/dt for a rotating body not in instantaneous radiative equilibrium everywhere is simply the integral of $I/T - \sigma T^3$ over the object, where T is the local temperature, σ the Stefan–Boltzmann constant and I the absorbed sunlight per unit area. Pollarolo and Sertorio (1979) discuss entropy production on rotating bodies, although without specific application to astrodynamics. Lesins (1990) considers the radiant entropy flux from a planet as a function of the heat transport by its atmosphere. He noted that the maximal radiant entropy production occurs when the body is isothermal—equivalently that the heat transports are at a maximum such that all temperature differences due to radiative imbalance are eliminated.

The total entropy production (of which the production by absorption shown by the dashed curve in Fig. 2 is only a small part) increases monotonically with rotation rate—its lowest value occurs for slow rotation (or, equivalently, a low heat capacity), where the peak temperature is obtained; the radiation pressure of the thermal emission is maximized here, although in the antisun direction. For fast rotation and maximum entropy production, the temperature distribution with hour angle is flattest and there is no thermal radiation thrust. The peak Yarkovsky effect, where rotation is slow enough to have a high temperature contrast, but quick enough to swing the temperature bulge away from the noon position, occurs for an intermediate rotation rate (e.g., Peterson, 1976). A plot of entropy production against the logarithm of rotation rate happens to show a maximum (logarithmic) *slope* at this rate, although the significance of that result is unclear.

Note that we have considered the entropy budget so far on a purely thermal basis. Entropy also has a probabilistic (or information-theoretic) basis, manifested in the number of states (photon directions) available to the photon streams. Sunlight is a low-entropy source in the sense that there are few photons, and they arrive from a narrow range of directions. The thermal photons re-emitted from the asteroid are more numerous (since longwave radiation has less energy per photon, and energy is conserved) and therefore have a larger possible number of states, and are emitted over a much wider range of directions.

For low rotation rates, the emission temperatures are highest and therefore there are few thermal photons. Furthermore, as virtually all the emitted radiation comes from the dayside of the object, the photons are concentrated in only 2π steradians of solid angle. For very fast rotation, the peak temperature is lowered and the wavelength of emitted radiation (and thus the number of emitted photons) is maximized. Finally, since the emitted radiation is most nearly isotropic, the emitted photons occupy the direction state space to the fullest extent possible. Fort et al. (1999) have shown how an information-theoretic approach can yield useful results, for example, in predicting solar limb-darkening.

Out-of-equilibrium systems with many degrees of freedom, such as the circulations of the atmospheres of Titan, Mars, and Earth, and of the Earth's mantle, appear to organize themselves so that their entropy production is maximized (Lorenz, 2002; Ozawa et al., 2003), a result which follows from information-theoretic grounds (Dewar, 2003; Lorenz, 2003). An isolated, symmetric, rotating body in space has no such freedom, but this paper has shown that thermodynamic analysis still offers useful insights. In future work, we intend to explore the heat engine and entropy aspects of the evolution of the spin rate and spin axis of bodies acting under the Yarkovsky or more particularly the Yarkovsky–O'Keefe–Radzievskii–Paddack (YORP) effect (see, e.g., Rubincam, 2000). We also note the conjecture of Lorenz (2002) that the collectively dynamics of many particles (e.g., in ring systems or protoplanetary disks) may seek extremal states.

5. Conclusions

We have shown that the entropy budget of a rotating illuminated body can be interpreted in the context of the work done on the object by the Yarkovsky force: a Carnot efficiency applies, modified by a relativistic propulsive efficiency. It is likely that thermodynamic analyses may be fruitful in other astrodynamic studies involving radiative forces.

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