JBIS VOLUME 72 2019 PAGES 146–152

A REACTION DRIVE

Powered by External Dynamic Pressure

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A new class of reaction drive is discussed, in which reaction mass is expelled from a vehicle using power extracted from the relative motion of the vehicle and the surrounding medium, such as the solar wind. The physics of this type of drive are reviewed and shown to permit high velocity changes with modest mass ratio while conserving energy and momentum according to well-established physical principles. A comparison to past propulsion methods and propulsion classification studies suggests new mission possibilities for this type of drive. An example of how this principle might be embodied in hardware suggests accelerations sufficient for outer solar system missions, with shorter trip times and lower mass ratios than chemical rockets.

Keywords: Propulsion, Reaction drive, Solar wind, External power, Dynamic pressure

1 INTRODUCTION

In the sixty years since the first interplanetary spacecraft (Luna 1), scientific probes have been flown to all the large bodies in the solar system, and, after decades of flight time, the twin Voyager 1 and 2 spacecraft are entering the boundary between the solar system and interstellar space. However, missions to the outer solar system are still very difficult, with long trip times, even with use of gravity assist maneuvers.

Substantial reductions in trip times to the outer solar system or for interstellar precursor missions are difficult for fundamental physical reasons. Fast trips imply high velocities: a constant speed of 100 km/s is only ~20 AU/year, beyond any demonstrated capability (though achievable with a close-solar flyby Oberth maneuver). Fast trips also imply that acceleration cannot be too small: a 29 AU trip (Neptune from Earth) of 100km/s peak velocity requires a constant acceleration of at least 0.005 m/s^2 to achieve a two-year flight time (ignoring Solar gravity), otherwise too much time is spent in acceleration and braking to take advantage of high speed.

With rocket propulsion, high velocity implies either high mass ratio (expense) or high exhaust velocity (high specific energy of the propellant). High acceleration implies high specific power, which is why electric rockets have not been able to overcome these limitations. Nuclear propulsion systems offer high specific energy, but whether they can combine high specific energy with high specific power remains to be demonstrated.

These well-known challenges have led to exploration of various types of 'sail' which use either the photons or the solar wind particles as an external source of momentum to harvest [1,2]. Most of these approaches offer low accelerations because of the large collection areas required, but at least one, the "plasma magnet" [3], offers useful accelerations by using large-scale magnetic fields from small generators. This work was motivated by the realization that while such sails offer near-term prospects for acceleration to high heliocentric velocities away from the sun (hundreds of km/s), that there is no current propulsion system which permits braking from those velocities or sunward acceleration.

NOMENCLATURE

$\vec{D}$	drag on the ship, in the ship frame (negative if it decreases a positive velocity), N
I_{SP}	specific impulse, m/s
P	power, (positive if supplied from ship, negative if supplied to the ship), W
$\vec{T}$	thrust applied to ship (positive if it increases a positive velocity), N
$\overrightarrow{T_{net}}$	net thrust of the ship, surplus of thrust over drag, N
m_e	reaction mass expelled, kg
m_{ship}	mass of the ship, kg
m_{wind}	mass of the surrounding medium which interacts with the ship, kg
q	dynamic pressure of the medium on the ship, Pa
v_{ship}	velocity of the ship in the rest frame, m/s
v_{wind}	velocity of the surrounding medium in the rest frame, m/s
$v_{wind(before)}$	v_{wind} before interaction with the propulsion system, in the rest frame, m/s
$v_{wind(after)}$	v_{wind} after interaction with the propulsion system, in the rest frame, m/s
$\overrightarrow{v_e}$	exhaust velocity of the reaction mass, relative to the ship, in the ship frame, m/s
$\overrightarrow{v_\infty}$	freestream velocity; velocity of the surrounding medium in the ship frame, m/s
ρ	density of the surrounding medium, kg/m^3

The widely known methods of accelerating and decelerating in a surrounding medium, including propellers, ramjets, turbojets, rockets, parachutes, and sails, form distinct classes of propulsion. Energy can be provided by the vehicle or by the surrounding medium, while reaction mass can be carried aboard or harvested from the surrounding medium. By classifying propulsion systems in this way (a "morphological analysis", following the methods of Zwicky [4]), a promising form of propulsion is identified, in which the reaction mass is carried aboard the vehicle, but the energy to expel that reaction mass is provided by the passage of the vehicle through the medium. This was anticipated by Alan Bond [5] in the limit of high-speed operation of ram-augmented interstellar rockets in which inert, rather than energetic, reaction mass could be used. The principle however is useful in contexts beyond the original application. We review the physics of the classical systems, and then explore the physics of this alternative form of propulsion. Finally, some examples of how this might be realized in an implementable device for fast transportation in the interplanetary medium are given.

2 REVIEW OF CLASSICAL APPROACHES AND THEIR PHYSICS

A review of the fundamental physics of existing propulsion is needed to understand how this method differs. The methods are grouped depending on whether propulsive energy is internally carried or externally harvested, and whether reaction mass is internally carried or externally harvested. Beginning with the equations of those well-known systems also provides the basis for deriving the physics of the new approach.

2.1 Propeller Systems (internal energy, external reaction mass)

The earliest known forms of propulsion (rowing, paddlewheels, propellers, turbojets, ramjets) involve pushing against the medium surrounding the vehicle, using energy carried aboard the vehicle. These forms of propulsion use the same basic physics: they are reaction drives, accelerating the medium around the vehicle [6]. They all depend on the surrounding medium, and the energy requirements to produce thrust increase with speed relative to the medium, governed by the propeller equations [7], so maximum velocities are limited. Where m_{wind} is the streamtube of the surrounding medium captured by the propulsion device, and to which mechanical work is done, and Δv_{wind} is the change in velocity of the wind caused by the propulsion system:

$$\vec{T} = -\frac{dm_{wind}}{dt}\Delta v_{wind} \tag{1}$$

Where $$\Delta v_{wind} = v_{wind(after)} - v_{wind(before)} \tag{2}$$

$$P = \frac{1}{2}\frac{dm_{wind}}{dt}\left[(\overrightarrow{v_\infty} + \Delta v_{wind})^2 - \overrightarrow{v_\infty}^2\right] \tag{3}$$

With $\overrightarrow{v_\infty}$ defined by: $$\overrightarrow{v_\infty} = \operatorname{sgn}(\vec{T})(v_{ship} - v_{wind}) \tag{4}$$

Note that care is required in observing the sign of these quantities, because the ship, medium, and reaction mass are all moving relative to each other. In the case of high freestream velocities ($\Delta v_{wind} << \overrightarrow{v_\infty}$), Equation 3 becomes:

$$P \cong \frac{dm_{wind}}{dt}\overrightarrow{v_\infty}\Delta v_{wind} \cong |\vec{T}|\overrightarrow{v_\infty} \tag{5}$$

This simplified form illustrates that the higher the freestream velocity, the more power is required for a given thrust, which is why rockets tend to dominate at higher speeds even when used within the atmosphere [8]. Recently, systems extending the propeller principle to the interplanetary plasma as a medium have been suggested, with the same general physical principles [9,10].

2.2 Rockets (internal energy, internal reaction mass)

As the limitations of propeller systems in reaching high velocities became apparent, the application of the rocket principle became attractive. All rocket-type systems, regardless of power source, have broadly similar behavior. They are governed by the rocket equations [11].

$$\frac{m_{ship(final)}+m_e}{m_{ship(final)}} = e^{\frac{|\Delta v_{ship}|}{|\overrightarrow{v_e}|}} \tag{6}$$

$$\vec{T} = -\frac{dm_e}{dt}\overrightarrow{v_e} \tag{7}$$

$$P = \frac{1}{2}\frac{dm_e}{dt}\left(\overrightarrow{v_e}^2\right) \tag{8}$$

In rocket systems, the reaction mass that is ejected to conserve momentum is carried aboard the ship, as is the energy that is converted into the kinetic energy of both the ship and the exhaust. For example, the chemical energy of fuels and oxidizers are converted to reaction mass. The rocket equation is derived from the conservation of energy and momentum. While such a form of propulsion works in a vacuum, the amount of velocity gain is limited by the onboard energy and mass storage. In the case of chemical rockets, with practical exhaust velocities of ≤4500 m/s, maximum vehicle propulsive velocity gains of ~20000 m/s are the greatest achieved to date, though missions with higher heliocentric velocities have been achieved by gravity assist maneuvers.

2.3 Drag Devices (external energy, external reaction mass)

Where the surrounding medium is moving relative to the ship, the application of drag can be useful, either to accelerate downwind (simple sails) or to brake a preexisting velocity (parachutes and aerobrakes). In these cases, any energy required is provided by (or carried away by) the surrounding medium, and the reaction mass is also formed by the surrounding medium. Drag devices are usually considered a distinct class of device from propellers and rockets.

One motivation for the current work is the recent proliferation of proposals for using the interplanetary or interstellar plasma as a medium for drag devices, which show that in spite of the low density (~10^{-20} kg/m^3 for the interplanetary medium at 1 AU, as low as ~10^{-22} kg/m^3 in hot plasma interstellar regions), useful accelerations can be achieved through electromagnetic interactions. This was first conceived as a magnetic sail or magsail [1], and more recently as an electric sail [2]. A particularly high drag to mass configuration is the "plasma magnet" magnetic sail, which offers a streamtube capture area far larger than the physical dimension of the coils involved in the device [3]. Fundamentally these are all drag devices, although the capture area, and hence the value of $\frac{dm_{wind}}{dt}$ involved at a given phase of flight, differ significantly. As drag devices, they provide thrust as in Equation 1 above, although the power, as shown in Equation 5, is then delivered to the ship rather than being provided by the ship. Power can be large in cases where $|\vec{v_\infty}|$ is large.

These drag devices have great promise for certain missions including outer solar system flybys or missions to the heliosphere boundary, and for braking systems for interstellar missions. By harvesting thrust power from outside sources, they can operate at levels of thrust power well beyond our current ability to provide propulsive energy storage aboard a spacecraft. Unfortunately, by the nature of a drag device, they can only accelerate "downwind", and so can only partially reduce propulsion requirements in cases such as outer solar system orbiters. Many desirable missions require thrust both for acceleration and for deceleration (stopping and starting a fast transit).

It is worth noting that the ideal Bussard Ramjet [12] while not a 'drag' device, would also fall in to this category of both the energy and the reaction mass being provided externally. The many practical difficulties in implementation of such a device have been discussed in the literature beginning with [13].

3 THE REMAINING OPTION

A morphological analysis (Zwicky box [4]) of the suite of propulsion devices shows that there is a remaining class of reaction devices: one in which the reaction mass is carried aboard the ship and is expelled using the power extracted from the flow of the surrounding medium. This approach does not appear in the common surveys of the propulsion art [4,11,14], and the first mention of it appears to be in Bond's discussion of the Ram-Augmented Interstellar Rocket [5], in which he points out that in the limit of high speed operation, the energy contribution of the rocket propellant becomes nearly negligible and that indeed the RAIR could then function with inert reaction mass. However, there is no reason to limit the application of this principle to that particular implementation – indeed, as noted in [5], the process of ram-compression of the interstellar medium to densities where RAIR operation is plausible introduces inefficiencies (parasitic drag) which make that particular implementation difficult (Fig.1).

The key element of the type of drive contemplated here is that if the interplanetary or interstellar medium is dense enough to provide meaningful drag using plasma techniques, then it can be a source of power as well as drag. The medium can do mechanical work on a system, thus extracting power from the 'wind': analogous to a ram air turbine in atmospheric flight. Since in doing so the vehicle experiences drag, the fundamental equations of this class of propulsion system must be examined to determine its performance and behavior. There is no need in general to compress the interstellar or interplanetary medium to operate a drive on these principles; one need only extract energy from it to expel onboard inert reaction mass.

3.1 Nomenclature of This Type of Drive

The nomenclature for such a device is not obvious. While it might be classified under the broad heading of 'jet propulsion' since it expels reaction mass, that classification also includes propellers, which are broadly recognized as different from rockets. As will be seen, the governing equations are also different from rockets (the rocket equation does not apply), so calling them some form of 'rocket' seems misleading. And since they produce thrust and consume propellant mass, 'sail' hardly seems appropriate. Following Zwicky, one might think of them as a 'dynamic-pressure-powered mass driver', but that is rather clumsy. Bond [5] suggests this as the high-speed, inert reaction mass limit of a ram-augmented interstellar rocket, but since in the general implementation, there is neither ram-pressure recovery, nor a rocket, nor augmentation, nor interstellar flight, that nomenclature seems ill-suited to the general case. This propulsive principle might be called a "wind drive", or, "ram drive", but using the common abbreviation q for dynamic pressure [15] suggests the name q-drive – which is the name used in the balance of this text.

3.2 Momentum and Energy Conservation

Fundamentally, as a propeller takes advantage of the fact that at low speed, it takes little energy to make thrust, the q-drive principle takes advantage of the fact that at high speed, a small drag device can extract a great deal of power. The power from a wind-harvesting device follows Equation 5, while the power required to expel stored reaction mass follows Equation 8. In the ideal case of no losses and no parasitic drag, this leads to the following fundamental equations (derived in the Appendix) for minimum use of reaction mass:

$$|\vec{v_e}| = \vec{v_\infty} \tag{9}$$

$$I_{SP} = \frac{1}{2}\vec{v_\infty} \tag{10}$$

$$\frac{(m_{ship(final)}+\Delta m_e)}{m_{ship(final)}} = \frac{(v_{\infty(initial)}+\Delta v)^2}{(v_{\infty(initial)})^2} \tag{11}$$

Contrast Equation 11 with Equation 6 and three dramatic differences are apparent, all favoring the q-drive principle in high velocity flight. First, in cases where $v_{\infty(initial)}$ is large compared to a rocket exhaust velocity ($\vec{v_e}$), the scaling is more favorable for the q-drive. Second, mass ratio for a q-drive scales with the square of velocity rather than with the exponential of velocity as in a rocket. Third, in cases where Δv is much less than $v_{\infty(initial)}$, as in most flight in the solar system due to the high

Source of Reaction Mass Source of Propulsive Energy	Carried Aboard Ship	Surrounding Medium
Carried Aboard Ship	rockets	propellers, ramjets, dipole drive
Surrounding Medium	RAIR with inert reaction mass, and this work	drag devices (sails)

Fig.1 Morphological box of propulsion methods.

velocity solar wind, the required mass ratio is even smaller (bearing in mind that the q-drive principle is only useful in situations where $v_{\infty(initial)}$ >>0).

Two examples help to illustrate the q-drive principle. Consider operating in a medium that is essentially at rest in the stationary reference frame, such as the interstellar medium in heliocentric coordinates. If given (through the use of some other propulsion system), an initial velocity v_{ship} of 600 km/s, which for zero wind speed is also $\vec{v_\infty}$ of 600km/s, using the q-drive principle with a mass ratio of 16 gives a final velocity of 2400 km/s. This rather startling velocity does not rely on an onboard nuclear reactor or energetic propellant; it is simply the result of momentum and energy exchange with the rest of the medium. The reaction mass is carried away by the surrounding medium and is at rest with respect to it, so the kinetic energy of the initial high-mass ship plus reaction mass has been concentrated into a final, low-mass ship at much higher velocity. It is worth noting that use of drag devices such as the Plasma Magnet sail purely in a drag configuration can produce heliocentric velocities of this magnitude, and that the abrupt deceleration of the solar wind in the termination shock at the heliopause then means that same heliocentric velocity, which tended towards $\vec{v_\infty}$ of zero within the solar wind now presents a high $\vec{v_\infty}$ in the interstellar medium.

The second example is a case relevant to maneuvering inside the solar system. Consider the solar wind to have a constant velocity of 450 km/s, and suppose a ship has been brought to a velocity radially outward from the sun of 150km/s (for example, by the use of a plasma magnet drag device). To brake from that outward velocity to achieve a state of rest in heliocentric coordinates is then a Δv of 150km/s, where the relative 'wind' speed $\vec{v_\infty}$ is initially 300 km/s and rises during the maneuver to 450 km/s. (When the vehicle is at rest in heliocentric coordinates, it has $\vec{v_\infty}$ equal to the wind speed.) In this case, the mass ratio required is 2.25 from Equation 11. By comparison, to achieve the same maneuver with the same mass ratio using a rocket, an exhaust velocity of 185 km/s would be required, which is far beyond any chemical rockets' capability, and if based on an onboard power plant, would require a very high power-to-mass ratio. By using the q-drive principle, the result can be achieved with inert reaction mass and with power harvested from the motion of the ship through the surrounding medium.

At first glance, the q-drive principle appears to offer "something for nothing". Propellant is expended but where does the energy come from? The answer is that the energy comes from the loss of velocity of the reaction mass to the surrounding medium. One may think of it as an inelastic collision between the expended reaction mass and the surrounding medium, where the resulting change in energy is carried away by the ship. In this sense, it is very reminiscent of the Oberth effect [16], in which there are also three masses involved: the ship, the exhaust mass, and a planet. The q-drive principle is much more flexible, however, since it uses the surrounding medium as the third mass, and so the q-drive is not restricted to operation near a gravitating body.

Finally, while the analogy is imperfect, this has some similarity to the method by which sailing vessels on Earth can sail upwind. In that case, the energy is derived from the motion of the surrounding air, and the "reaction mass" is provided by the action of the keel on the water. In space, we can achieve comparable results by expelling reaction mass from the vehicle.

4 EXAMPLE IMPLEMENTATIONS

The propulsive principle outlined in this paper could apply to high-speed atmospheric flight or to travel in the interplanetary or interstellar medium. However, to determine whether the q-drive principle has real engineering utility, some concept of how this principle can be embodied in hardware is helpful. Furthermore, the question of whether the acceleration achieved is useful for fast transits can only be assessed in the light of a hardware implementation. Realize that these examples are just that: guideposts for two ways to apply the q-drive principle to real hardware. The first example is presented only because it is physically very simple, and so enhances understanding of the physical principles. The second example may be a practical implementation, with acceleration > 0.02 m/s^2.

4.1 Continuous Mode, Electric Field Power Extraction

Flow of the solar wind or interstellar plasma over electrodes can be used to generate electrical power to expel reaction mass, following the q-drive principle. Flow of a neutral plasma across a tandem pair of grids, with the solar wind flowing over them, will develop a voltage difference from which power can be extracted. This principle is well known as a means of extracting power from conceptual fusion reactors [17], and its use in the reversed mode, applying power to make thrust, is noted in [9]. Because the Debye sheath formed around each conductor limits the amount of plasma intercepted, this approach requires high mass and offers low acceleration, but the principle of operation is helpful to understand. The velocity of the wind over the ship $\vec{v_\infty}$ produces electrical power. Extracting that power creates a voltage difference between the grids, which manifests as drag, precisely as in a windmill or ram air turbine operating in the air. Lower mass might be achieved by using a tandem set of radial wires similar to the "e-sail" [18].

In turn, the electrical power can be used to expel reaction mass. Any type of electrically powered thruster could be used, provided the reaction mass can be expelled at approximately the same exhaust velocity as the freestream velocity ($|\vec{v_e}| = \vec{v_\infty}$). While existing electric thrusters operating at ~$4x10^5$ m/s exhaust velocity are immature, they are plausible under known physical principles [19]. The expelled reaction mass ends up nearly at rest with respect to the solar wind, while the ship accelerates sunward (or reduces its outward velocity).

4.2 Pulsed Mode, Magnetic Field Power Extraction

For accelerations that enable fast transits, a method of extracting power from the solar wind is needed that provides a high drag-to-mass ratio, and it seems likely that a low parasitic drag is also important. In atmospheric applications, rotating devices (windmills, anemometers) are used to draw power from the wind, and magnetic field analogies of both are possible, but the relatively low lift-to-drag ratio of magnetic fields in plasma suggests these approaches may have high parasitic drag. A useful approach may lie in a linear, reciprocating motion of a magnetic field, where essentially all the drag goes into pushing on a moving field. High drag-to-mass is achievable using the plasma magnet approach [3].

The basic principle of the plasma magnet, illustrated in Fig. 2 overleaf, is that a rotating magnetic field, driven by alternating current in a crossed pair of coils, creates a circulating current in the plasma, and that current then expands in radius until it creates a dipolar magnetic field much larger than the physical coils.

If such a field is turned on and the generating coils are attached to a tether, the tether will be pulled by the solar wind, which could rotate the shaft of a conventional generator. Then, the field could be turned off, the tether reeled back in, and the cycle repeated. In principle this approach of mechanically moving the field coils in a reciprocating manner would extract power, and it illustrates the principle involved, but the mechanical motions would be too slow to provide adequate power-to-mass ratio. We need a more rapid motion of the field, which can be achieved by replacing the reciprocating motion of the coils carrying the magnetic field with the reciprocating motion of the magnetic field itself.

In the approach illustrated in Fig.3, a pair of plasma magnet generating coil sets are used, separated by a tether with wires to transfer power from one set of coils to the other. Initially, the windward coil set is energized and the solar wind pushes on it, transferring the energy in the dipole field to the leeward coils. During the power stroke, energy is extracted from the wind, which can be used to power an electric thruster to expel reaction mass. A third coil set, omitted from the illustration for clarity but located at the windward end with a closed (toroidal) configuration that does not generate a magnetic field outside the coils, receives the energy on the return stroke, so that drag is only pushing on the field during the power stroke. Then, the energy is again transferred to the windward coil, and the cycle repeats.

A detailed design would be required to estimate mass but a sizing study, based on peak currents in superconducting MgB_2 tapes at 20K [20-22] of 2.5 $x10^8$ A/m^2, suggests that accelerations in the 0.025-0.05 m/s^2 range may be feasible using this approach. The long tether, carrying oscillating currents in the 1 KHz range from end to end, modulated by a reciprocating frequency in the 20 Hz range, is admirably suited to form a Wideröe style [23] ion accelerator, thus providing an integrated method for converting the resulting electric power to thrust.

5 CONCLUSION

A new class of reaction drives appears capable of generating vehicle velocities greater than those practical for propeller or rocket devices. The basic principles of this drive are those employed in the "inert reaction mass, high velocity limit" of the Ram-Augmented Interstellar Rocket, but they do not require that particular implementation, nor do they require fusion technology, and by exploiting the solar wind, they are particularly useful for interplanetary flight. A conceptual design suggests that, by using plasma magnet techniques, such a drive could offer accelerations and mass ratios sufficient for rapid transits to the outer solar system.

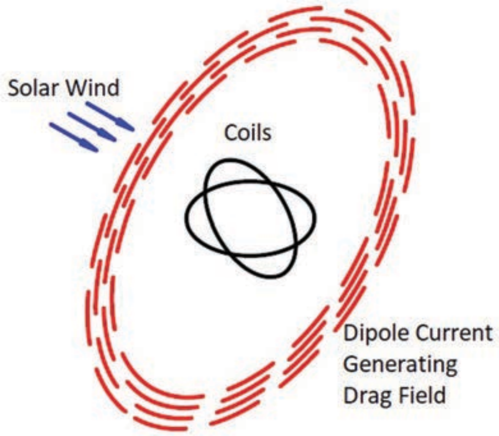

Fig.2 Operating principle of a plasma magnet.

To explore further, the analysis of the physics involved needs to be extended in two ways. First, the analysis needs to include the effects of efficiencies in power conversion and parasitic drag, to assess whether the approach is practical. Second, to extend the application of this technique for inner solar system missions, the theory needs to be extended to include thrusts that are not parallel to the drag vector, which would enable a wider range of maneuvers.

This paper begins to examine routes for embodying this type of reaction drive in hardware. To assess the achievable accelerations, designs will need to be carried to a level of detail at which masses can be estimated credibly.

Acknowledgments

While this work was developed without sponsorship from any organization, the author thanks the Tau Zero Foundation and the many researchers who have participated in the NASA Institute for Advanced Concepts for their work in advanced propulsion, elements of which inspired the investigation leading to this work. Thanks also to the reviewers at *JBIS* who suggested important prior art to include.

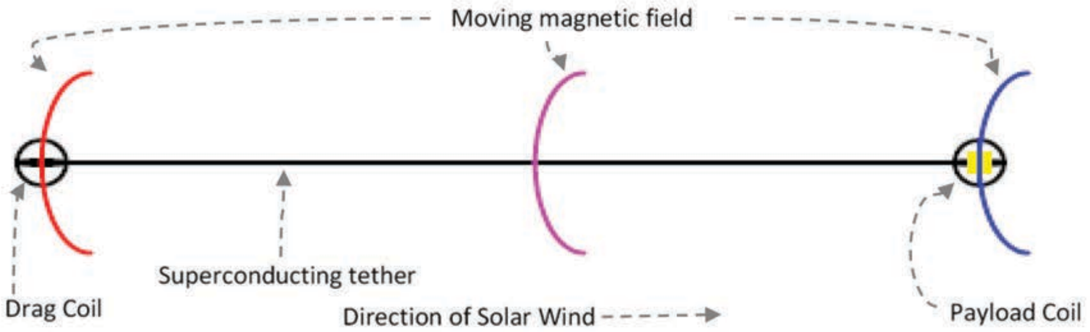

Fig.3 Oscillating magnetic piston for energy extraction from solar wind.

APPENDIX A

The derivation of the equations for the q-drive principle begins with, in the ship frame, recasting Equation 1 for the case where the drag comes from a 'windmill' extracting power from the wind, and thrust comes from the expulsion of reaction mass, as in Equation 7:

$$\vec{D} = -\frac{dm_{wind}}{dt}\Delta v_{wind} \qquad \text{(A1)}$$

$$\vec{T} = -\frac{dm_e}{dt}\overrightarrow{v_e} \qquad \text{(A2)}$$

We can then define a net thrust $\overrightarrow{T_{net}}$, which, due to our sign convention, is the sum (not the difference) of thrust and drag.

$$\overrightarrow{T_{net}} = \vec{T} + \vec{D} \qquad \text{(A3)}$$

Knowing that energy and momentum are both conserved in all reference frames, we begin in the ship frame, where momentum conservation gives rise to Eq. (A1) and (A2) while energy conservation gives us, using the same sign convention:

$$P_{drag} = -\frac{1}{2}\frac{dm_{wind}}{dt}[(\overrightarrow{v_\infty} + \Delta v_{wind})^2 - (\overrightarrow{v_\infty})^2] \qquad \text{(A4)}$$

If $\Delta v_{wind} \ll \overrightarrow{v_\infty}$ and $\overrightarrow{v_\infty} \gg 0$, the equations are simplified, since in a drive powered by the dynamic pressure of the passing medium, $\overrightarrow{v_\infty} \approx 0$ describes a condition where thrust drops to zero, these approximations are sound. Furthermore, for the 'drive' condition, in our sign convention, $\Delta v_\infty > 0$, as the acceleration from thrust, must be 'into the wind'. If thrust were negative, it would represent a drag device, in which case no reaction mass or use of the q-drive principle would be needed. Drag power is negative because it is power supplied to the ship. That allows simplification of Eq. (A4) to:

$$P_{drag} = -\left|\vec{D}\right|\overrightarrow{v_\infty} \qquad \text{(A5)}$$

Energy conservation for the thrust gives us:

$$P_{thrust} = \frac{1}{2}\frac{dm_e}{dt}(\overrightarrow{v_e})^2 \qquad \text{(A6)}$$

Which can be rewritten as:

$$P_{thrust} = \frac{1}{2}\left|\vec{T}\right|\overrightarrow{v_e} \qquad \text{(A7)}$$

While in actual cases it may be desirable to use propellant that adds energy to the available thrust power, for simplicity in the analysis consider the case where there is zero energy content in the propellant, where it is inert reaction mass. Again, neglecting unavoidable inefficiencies for simplicity, that implies that the power available for thrust is equal to the power derived from the drag, $P_{drag} + P_{thrust} = 0$.

This produces the simple and useful result, using Eq. (A5) and (A7) as the thrust and drag power, that:

$$\overrightarrow{v_e} = 2\frac{|\vec{D}|}{|\vec{T}|}\overrightarrow{v_\infty} \qquad \text{(A8)}$$

The relationship between thrust and drag is a design parameter; in a real system with less than perfect efficiencies it will be set by engineering considerations. In the ideal case, however, simply set a 'thrust fraction';

$$\varphi = \left|\frac{\overrightarrow{T_{net}}}{\vec{T}}\right| \qquad \text{(A9)}$$

The choice of φ which minimizes reaction mass can be determined by maximizing specific impulse:

$$I_{SP} = \frac{\overrightarrow{|T_{net}|}}{\left(\frac{dm_e}{dt}\right)} = \frac{\varphi|\vec{T}|}{\left(\frac{dm_e}{dt}\right)} \qquad \text{(A10)}$$

By substituting Eq. (A2) in to Eq. (A10), and then in turn substituting Eq. (A8) we get:

$$I_{SP} = \varphi\overrightarrow{v_e} = 2\varphi(1-\varphi)\overrightarrow{v_\infty} \qquad \text{(A11)}$$

Which has a maximum at $\varphi = 0.5$ producing the result, at that $\varphi = 0.5$ condition, which will be assumed throughout the rest of the analysis:

$$I_{SP} = \frac{1}{2}\overrightarrow{v_\infty} \text{ and } |\overrightarrow{v_e}| = \overrightarrow{v_\infty} \qquad \text{(A12)}$$

Substituting back in the definition of I_{SP} gives:

$$T_{net} = \frac{1}{2}\frac{dm_e}{dt}\overrightarrow{v_\infty} = m_{ship}\frac{dv_{ship}}{dt} \qquad \text{(A13)}$$

Observing that as long as v_{wind} is constant, $\Delta v_{wind} = \Delta\overrightarrow{v_\infty}$ and rearranging in terms of small changes in mass and velocity, and then realizing that every increment of exhaust mass is a decrement in ship mass:

$$2\frac{\Delta v_\infty}{v_\infty} = \frac{\Delta m_e}{m_{ship}} = -\frac{\Delta m_{ship}}{m_{ship}} \qquad \text{(A14)}$$

The same result can be derived by energy conservation in the rest frame, though the derivation is more complex.

To solve for the relationship between total mass expended and velocity change, recognize that Eq. (A14) is a differential equation and rearrange as:

$$\frac{\Delta m_e}{\Delta v_\infty} = 2\frac{m_{ship}}{v_\infty} \text{ or, equivalently } \frac{\Delta m_{ship}}{\Delta v_\infty} = -2\frac{m_{ship}}{v_\infty} \qquad \text{(A15)}$$

Which has a solution of the form $m_{ship} = C/v_\infty^2$ where C is a constant of integration, and m_{ship} a function of v_∞. The ratio of initial mass (before a maneuver) and final mass (after a maneuver) is then:

$$\frac{m_{ship(initial)}}{m_{ship(final)}} = \frac{\left(v_{\infty(final)}\right)^2}{\left(v_{\infty(initial)}\right)^2} \qquad \text{(A16)}$$

Realizing that for any maneuver using the q-drive principle, since v_∞ is always a headwind (positive) and acceleration is always in the direction of increasing v_∞ (if it were not, we would use a sail without need for expelling reaction mass), we can rewrite this as:

$$mass\ ratio = \frac{m_{ship(initial)}}{m_{ship(final)}} = \frac{\left(m_{ship(final)} + \Delta m_e\right)}{m_{ship(final)}} = \frac{\left(v_{\infty(initial)} + \Delta v\right)^2}{\left(v_{\infty(initial)}\right)^2}$$

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Received 5 April 2019 **Approved** 5 June 2019