The Cold Horn method for NF measurements, notes about 10 GHz applications.

The Cold Horn method for Noise Figure (NF) measurements is a kind of Y-factor methods with Cold and Hot loads at the receiver input. Sergey RW3BP developed it for 23 cm band at the end of 2000s[1]. The main goal was to get a better accuracy of measurements in comparison with conventional Y-factor method and tools that use solid state noise sources. For 23 cm the best NF one may achieve today is about 0.1-0.2 dB, so the problem of accurate measurements with uncertainties less than 0.1 dB seems to be important. For 10 GHz achievable NF is somewhat higher, about 0.6-0.7 dB for the best LNAs built today[2] with a tiny difference at the top of the list, less than 0.1-0.2 dB; contemporary low noise FETs at 10 GHz may achieve the parameter NF_{opt} less than 0.3 dB[3]. So, the accuracy of Noise Figure measurements seems important for 10 GHz too.

Errors of Y-factor method are widely investigated and reviewed[4]; and measurement of small NF is a common problem. The estimation of uncertainty for conventional tools with diode noise head is shown at Fig. 1. The calculator from Rohde&Schwarz is used[5], value of possible error ±0.25 dB seems unsuitably high for 10 GHz too; in addition, the calculator do not account possible sources of error like the gain error[6].

This work is about one of "less travelled roads" in NF measurements[4] and practical attempts to reduce NF uncertainty using Y-factor method.

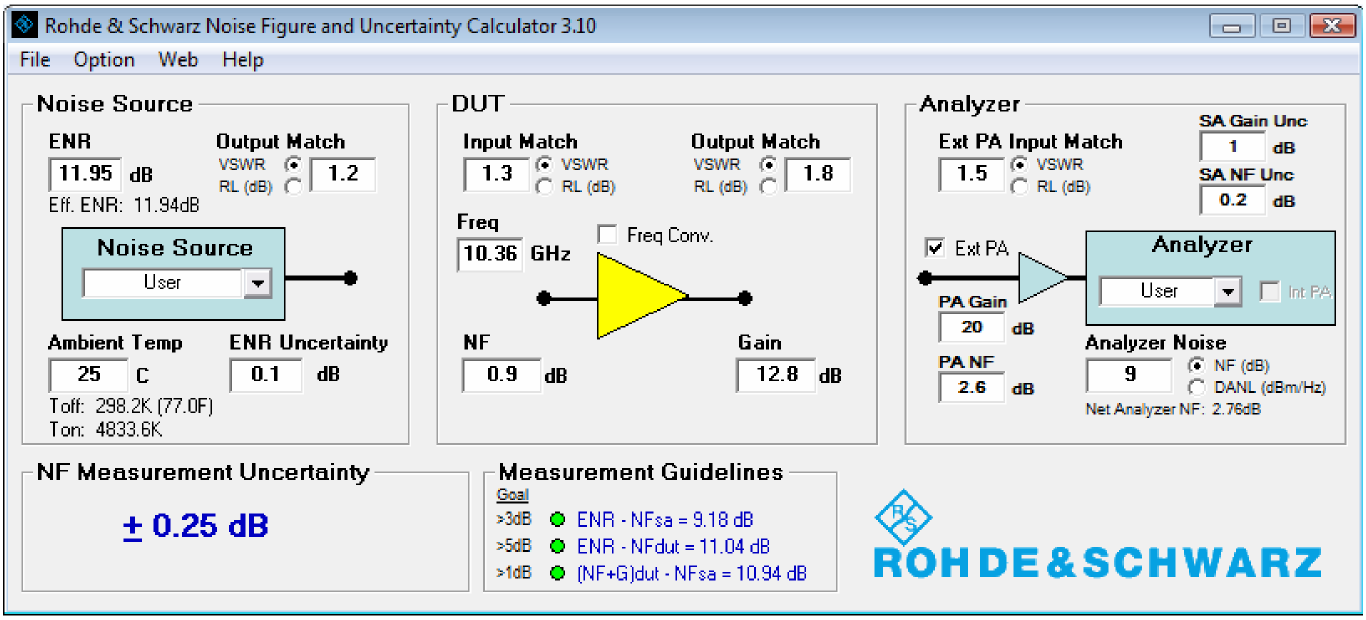


Fig. 1. The estimation of NF uncertainty at 10 GHz for typical LNA (NF=0.9 dB, Gain up to 13 dB and moderate VSWR at input and output. Noise head is characterized by ENR value with typical uncertainty δENR=0.1 dB, and output VSWR or refection coefficient.

As a way to reduce the interval of uncertainty, it is suggested to use lower temperatures for Cold and Hot sources. This way can be illustrated using the formula for RX noise temperature by Y-factor method,

$$T_{RX} = T_{hot}\frac{1}{Y-1} - T_{cold}\frac{Y}{Y-1}. \quad (1)$$

The Noise Figure and its uncertainty δNF are expressed via T_{RX} and uncertainty δT_{RX} as

$$NF = 10\cdot\log\left(1+\frac{T_{RX}}{T_0}\right), \quad \delta NF = \frac{10}{\ln 10}\frac{\delta T_{RX}}{T_0+T_{RX}}, \quad (2)$$

---

[1] See his notes at Russian VHF portal "Accurate Noise Figure measurements on 1296 MHz", http://www.vhfdx.ru/apparatura/accurate_noise_figure_measurements_1296_mhz.

[2] See measurements of Dominique HB9BBD (HB9CW) DUBUS 3/2017, v.46, page 124, presentation from EME meeting Örebro 2017 http://moonbouncers.org/Orebro2017/HB9CW%20Orebro%202017%20Measurments.pdf.

[3] After deembeding, i.e. from the FET's gate to drain.

[4] For fresh review see David Stockton GM4ZNX and Ian White GM3SEK, *Noise Figure Measurement – A Reality Check*, DUBUS 1/2013, page 74, DUBUS TECHNIK XII, page 235.

[5] For info about calculator see https://www.rohde-schwarz.com/us/applications/the-y-factor-technique-for-noise-figure-measurements-application-note_56280-15484.html. For today the latest version of Uncertainty calc R&S is 3.21, the core of this work has been done at 2013-2015 and I used older version 3.10 then.

[6] See Rainer Bertelsmeier DJ9BV, *How to Use a Noise Figure Meter*, DUBUS 4/1990, page 18, http://www.marsport.org.uk/dubus/archive/9004-2.pdf.

where $T_0 = 290\ \text{K}$ is the reference temperature; so, if one get lower with T_{RX} and its uncertainty the Noise Figure with uncertainty will be lower also. Y-factor in (1), Y - is a ratio of receiver outputs for Hot (noise temperature T_{hot}) and Cold (noise temperature T_{cold}) sources at the input, the receiver output should be measured in power units. For uncertainty δT_{RX} one can get

$$\delta T_{RX} = \sqrt{\delta T_{hot}{}^2 \frac{1}{(Y-1)^2} + \delta T_{cold}{}^2 \left(\frac{Y}{Y-1}\right)^2}\ , \tag{3}$$

assuming that errors for Hot and Cold temperatures, δT_{hot} and δT_{cold} are independent (uncorrelated), and if the Y is known exactly, $\delta Y = 0$ [7]. If the Y-factor is about 5 dB (that is typical for NF measurements, $Y \approx 3$), the contribution of δT_{cold} uncertainty in δT_{RX} becomes leading, and the contribution from δT_{hot} gets relatively suppressed. As a first step, one needs to reduce the T_{cold} uncertainty to get a lower δT_{RX}.

One should try to decrease T_{cold} down to tens of Kelvins; so, if this temperature is known with accuracy less 20 %, the absolute error interval δT_{cold} should also go down, and would not exceed several K. Cryogenic method to cool down the dummy load at RX input seems not convenient in practice; desirable temperatures about tens of K may be hardly achievable. Nevertheless, Cold Source could be realized using directional antenna (like microwave horn) with main beam looking upward, to the Cold Sky; then the noise temperature of antenna T_{ant} should be low, and antenna output may be used as Cold Source at RX input and $T_{cold} = T_{ant}$. Estimation by (2), (3) gives the contribution for NF uncertainty about ±0.04 dB (NF=0.7 dB, $Y \approx 3$) when δT_{cold} =±2 K.

One may note that even for higher values about 290 K the physical temperature can be measured with high accuracy; uncertainty of available thermometers do not exceed several K; a dummy load may be connected as a source at the RX input, and the noise temperature of this source would be exactly the physical (thermodynamic) one for the dummy load. In practice, this temperature should be corrected by mismatch losses at the RX input; and additional uncertainty appears in measurements because of the power reflections from RX input and source. Only a part of available power $P_{source} = k\,T_{source}$ is delivered from source to RX[8],

$$P_{in} = \frac{\left(1-|\Gamma_{RX}|^2\right)\cdot\left(1-|\Gamma_{source}|^2\right)}{|1-\Gamma_{RX}\,\Gamma_{source}|^2} P_{source}, \quad P_{in} = k\,T_{in}\,, \tag{4}$$

where Γ_{RX} - reflection coefficient of receiver with magnitude $|\Gamma_{RX}| = \frac{\mathrm{VSWR_{RX}}-1}{\mathrm{VSWR_{RX}}+1}$, Γ_{source} - reflection coefficient of the source with magnitude $|\Gamma_{source}| = \frac{\mathrm{VSWR}_{source}-1}{\mathrm{VSWR}_{source}+1}$, k - the Boltzmann constant. The numerator in (4), $\left(1-|\Gamma_{RX}|^2\right)\cdot\left(1-|\Gamma_{source}|^2\right)$ gives the reduction of source's temperature; for typical $\mathrm{VSWR_{RX}}$ =2 the temperature correction exceeds 11%; or more than 30 K down for 290 K source. The denominator $|1-\Gamma_{RX}\,\Gamma_{source}|^2$ in (4) defines the mismatch uncertainty; phases of reflection coefficients are unknown usually, but one can get the estimation for T_{in} uncertainty due to mismatch[8], $\delta T_{in} \approx \pm\, 2\cdot|\Gamma_{RX}|\cdot|\Gamma_{source}|\,T_{in}$, or expressing via T_{source}

$$T_{in} \pm \delta T_{in} \approx \left(1-|\Gamma_{RX}|^2\right)\cdot\left(1-|\Gamma_{source}|^2\right)\cdot\left(1\pm 2\cdot|\Gamma_{RX}|\cdot|\Gamma_{source}|\right) T_{source}\,, \tag{5}$$

and using the magnitudes of reflection coefficients only. For typical VSWR_{RX} =2 and VSWR_{source} =1.1 the uncertainty is about ±5.5%, or more than ±15 K for source 290 K. I think it is too much if we are looking for accuracy in measurements about 0.1 dB or better; I also should note that uncertainty δT_{cold} =±15 K gives the

[7] See Appendix 3 for general case, $\delta Y \neq 0$.

[8] Les Besser, Rowan Gilmore, *Practical RF Circuit Design for Modern Wireless Systems, Volume 1, Passive Circuits and Systems*, Artech House, Boston, London, 2003, page 64. Authors called "mismatch loss" the whole relation P_{source}/P_{in}.

contribution to total uncertainty of Noise Figure almost ±0.3 dB at NF=0.7 dB, as it follows from estimation by (2), (3) for $Y \approx 3$.

Temperature of Cold Horn

The temperature of horn could be computed if temperature of media where its beam directed is known. A many of factors affect the temperature of Cold Horn. If the main beam of horn is directed upward, T_{cold} will be defined by:

1. Temperature of Cosmic Microwave Background (CMB);
2. A small part of T_{cold} is expected due to the noise from occasional objects in the Sky like solar noise, galaxy noise etc. They are localized sources usually;
3. Environment and ambient temperature; it should be noted that the horn collects a noise from ground and nearby objects by back- and side-lobes. Therefore, a shape of horn's directivity pattern affects too. A horn antenna with good noise parameters should be selected, and back- and side-lobes should be suppressed as good as possible.
4. Losses of signal in atmosphere and temperature of atmosphere layers.

Temperature of CMB is well known, $T_{CMB} \cong 2.7\,\mathrm{K}$. Localized sources of radio noise in the space (galaxy noise etc) are expected small at 10 GHz; the possible contribution from the solar radio flux can be reduced or excluded if the horn does not see the Sun by its main beam. To decrease contribution from localized sources in the space one should choose the horn with a wider main beam. I can take into account the impact from cosmic sources besides CMB by introducing an additional uncertainty; I estimate it by the value ±1.5 K, the result for cosmic noise should be $T_{space} \pm \delta T_{space}$ =4.2±1.5 K[9]. It is interesting to note that oscillations of horn temperature are observed during a day at 23 cm, see RW3BP text[1].

Next problem is a noise from surroundings of the horn. This noise is considered as thermal one. I should denote two different situations for practical reasons, they are schematically depicted at Fig. 2: left picture – the horn is placed near a wall with ambient temperature about 290 K, right – the horn looking to zenith, the ground is assumed warm with 290 K also. The best choice is to set 45 degrees of elevation when a warm wall is present, then the main beam of horn could be chosen wider. In practical measurements the left picture Fig. 2 corresponds to a model when the horn placed and looking to the sky from window in laboratory, for example. Recommended linear sizes of wall and ground if they are made artificially should be about 3-4 m or higher for 10 GHz[10]; the horn should be placed at the corner of wall and ground planes.

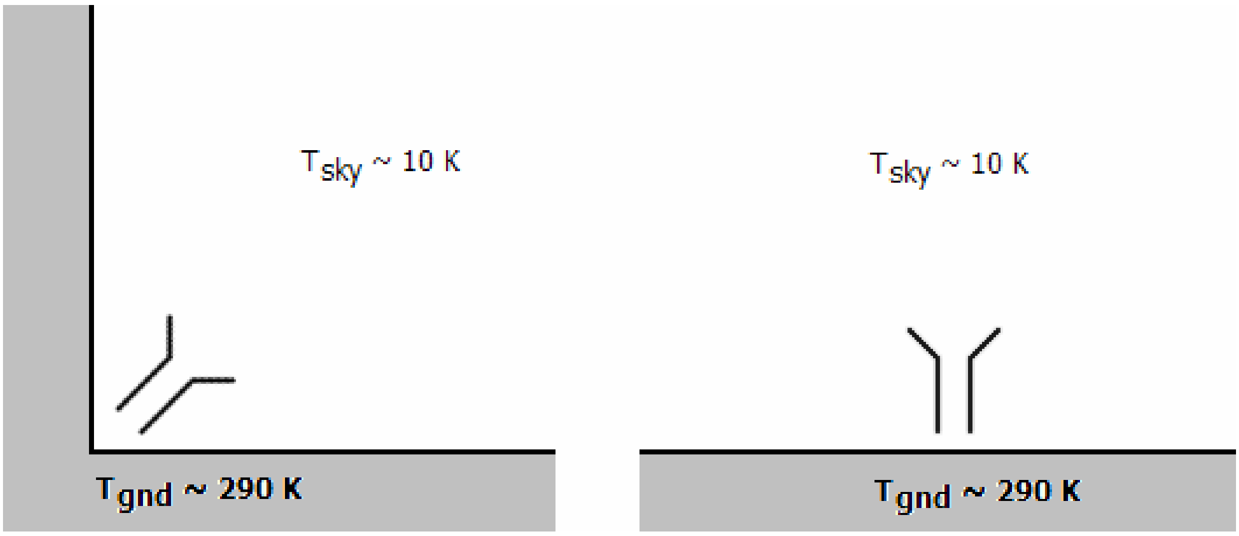


Fig. 2. Two possible situations in Cold Horn measurements: left – the horn placed near a wall with ambient temperature about 290 K, right – the horn looking to zenith, no any wall present. I is assumed the Sky has a noise temperature about 10 K at 10 GHz.

[9] Note that the lowest limit of cosmic temperature should be T_{CMB} exactly.

[10] I can imagine a variety of possible artificial walls. They could be useful for EMI protection, as a shield from the Sun etc.

Two horn's positions are different also by additional noise that the horn collects from warm wall and ground. When the wall present there is also a difference between horn polarizations; for horizontal polarization (E-field directed along the corner of wall) this noise usually lower than for vertical one due to peculiarities of the far field pattern. I have calculated noise parameters for several horns using HFSS. As a noise parameter I used the noise temperature of horn in working position when temperature of the sky $T_{SKY} = 0$; it can be defined by integration over the far sphere,

$$T_{horn} = T_{cold}\big|_{T_{SKY}=0} = \frac{1}{4\pi} \oint_{sphere} T(\theta,\varphi) \cdot G(\theta,\varphi)\, d\Omega, \quad d\Omega = \sin\theta\, d\theta\, d\varphi \qquad (6)$$

where $T(\theta,\varphi) = T_{amb} = 290\text{ K}$ if θ, φ pointed to the ground or warm wall (otherwise $T(\theta,\varphi) = T_{SKY} = 0$), so the noise from horn's surroundings has been taken into account only. The integration (6) was performed numerically using HFSS output for the horn's directivity $G(\theta,\varphi)$, horns are assumed lossless. Results of calculations are summarized in Table 1, detailed information is collected in Appendix 1.

Table 1. Noise parameters of horns.

Horn	T_{horn} , K, hor	T_{horn} , K, vert	T_{horn} , K, to zenith	description
Pyramidal short	15.27	21.13	6.98	Appendix 1, part A
Pyramidal long, by W1GHZ	7.77	19.48	6.27	Appendix 1, part B
Skobelev short, by W1GHZ	2.57	4.6	0.57	Appendix 1, part C
Skobelev long, by RA3AQ & RW3BP	2.03	4.32	1.22	Appendix 1, part D

The most of my measurements were made from laboratory's window using short Skobelev horn[11], Appendix 1, part C, see also photos at Fig. 3; it has a wide main beam, small back- and side-lobes providing very good noise characteristics. For the best match of this horn with waveguide a short tuning stub in WR90 section was mounted additionally.

Fig. 3. 10 GHz Skobelev horn for NF measurements.

The next is a problem of atmospheric losses and noise; the nature of atmospheric noise is also thermal. A sky temperature seen by the horn near ground[12] can be expressed in convenient form like

[11] Thanks to Andrey RD4HI (sk) for well manufactured horn.

[12] Following to ITU recommendations P.372, *Radio noise*, http://www.itu.int/rec/R-REC-P.372/en; I used the text of September 2013 edition.

$$T_{sky} = T_{eff}\left(1-10^{-\frac{A}{10}}\right) + T_{space}\,10^{-\frac{A}{10}}, \qquad (7)$$

where A - pass losses in dB, T_{eff} - effective temperature of atmosphere, recommended value for practical use[12] - $T_{eff} = 275\ \text{K}$. The pass losses depend on elevation[13],

$$A = \frac{A_{90^o}}{\sin\psi}, \qquad (8)$$

where ψ - the elevation angle. Zenith attenuation A_{90^o} can be calculated using the data for weather near ground only (air temperature, relative humidity, air pressure). Formula (8) should be used for elevation angles >5 deg, otherwise the algorithm of calculations is more complicated. It is important to note that mentioned procedures give the accuracy about 10% and good applicable for clear weather; the density of fogs and clouds is usually unknown or hardly to estimate it with good accuracy.

Obviously, T_{sky} depends on elevation, $T_{sky} = T_{sky}(\psi)$. A horn with wide beam receives the sky radiation from many of elevation angles simultaneously. Therefore, the total horn temperature can be calculated by integrations over full sphere,

$$T_{cold} = \frac{1}{4\pi}\oint_{full\ sphere} T(\theta,\varphi)\cdot G(\theta,\varphi)\,d\Omega = \frac{1}{4\pi}\left(T_{amb}\int_{gnd,wall} G(\theta,\varphi)\,d\Omega + \int_{sky} T_{sky}(\psi)\cdot G(\theta,\varphi)\,d\Omega\right),\ d\Omega = \sin\theta\,d\theta\,d\varphi, \qquad (9)$$

where the first integral is left part taken when θ,φ are looking to ground or warm wall, and second one – to the sky, the value of T_{amb} is usually known for current weather, and the elevation angle ψ can be expressed via angles θ,φ. These integrations are performed numerically using HFSS output for the horn's directivity $G(\theta,\varphi)$. Calculations by (9) should be done every time, and for every weather environment and conditions occur[14]. Measurements should be made when the weather outside is clear, otherwise the impact of atmosphere on Cold Horn temperature is hardly predictable.

In addition, there are some losses in horn's waveguide (local losses); they will rather small if connections are made as short as possible, but able to add to horn's temperature up to 1-2 K and should be taken into account.

Uncertainty of T_{cold}

Next, I should estimate the uncertainty of T_{cold} for Cold Horn. Possible sources of uncertainty are listed in Table 2, estimations for δT were made from the most of knowledge about them, and I think the total uncertainty may be even overestimated. Calculations correspond to Cold Horn Fig. 3 directed to the sky from window in laboratory room, elevation – 45 deg, horizontal polarization.

Table 2. Estimations of T_{cold} uncertainty. Cold Horn – Appendix 1, part C, warm wall present, elevation – 45 deg, horizontal polarization.

Weather (current, dated April 2014):	20^o C (293 K), humidity 29%, pressure 1023 hPa, clear, Moscow
Cosmic noise:	$T_{space} \pm \delta T_{space}$ = 4.2 ± 1.5 K
Atmospheric attenuation (zenith):	0.048 dB (by ITU methodic)
Sky noise (including atmosphere):	T_{sky} = 9.41 K (by integration using horn's directivity in sky direction)
- from uncertainty of attenuation:	δT_{atm1} = 0.6 K (taken as 10% from atmospheric noise)
- from uncertainty of atmospheric effective temperature T_{eff} :	δT_{atm2} = 0.6 K (taken as 10% from atmospheric noise)
- additional uncertainty from errors at small elevation angles:	δT_{atm3} = 0.3 K (see text)

[13] See ITU recommendations P.676 *Attenuation by atmospheric gases*, http://www.itu.int/rec/R-REC-P.676/en; I used the text of September 2013 edition, Annex 2.
[14] Ask me if you are interested in the details of calculations. I also could help to compose a computational tool for your horn and environments.

Total noise of Cold Horn:	T_{cold} = 11.91 K (by integration using horn's directivity over full sphere)
- uncertainty for the noise added from ground and warm wall:	δT_{gnd} = 0.8 K (estimated as ~ 1/3 form ambient noise's contribution)
- possible error of numerical calculations:	δT_{num} = 0.5 K
Total for uncertainties:	$\sum_{RSS} \delta T \approx 2$ K (RSS summation)
Total result for Cold Horn:	$T_{cold} \pm \delta T_{cold}$ = 11.91 ± 2 K

The simple formula (8) is applicable at elevations >5 deg; fortunately, the contribution of atmosphere to the total noise of Cold Horn is suppressed at small elevations due to horn's directivity, so, errors of calculations should be also low. I estimate the contribution from low elevations about 0.6 K (10% of atmospheric contribution), and possible additional error should be taken about half of that, ~ 0.3 K. The value of uncertainty for ambient noise was taken rather high, about 1/3 from contribution; the nature of ground or warm walls as thermal radiators is not well known usually, and may be far from Blackbody model.

All listed components of uncertainty were summed as independent (uncorrelated) by RSS[15] manner,

$$\delta T_{total} = \sqrt{\delta T_1^2 + \delta T_2^2 + \ldots + \delta T_n^2} = \left(\sum_{i=1}^{n} \delta T_i^2 \right)^{1/2}, \tag{10}$$

if number of different components is n. This manner can also be referenced as statistical summation in contrast to direct sum of errors, or sum of absolutes,

$$\delta T_{total}^{abs} = |\delta T_1| + |\delta T_2| + \ldots + |\delta T_n| = \sum_{i=1}^{n} |\delta T_i|, \tag{10a}$$

which is useful to estimate the max possible uncertainty if some correlations between components are present.

The value $T_{cold} \pm \delta T_{cold}$ = 11.91 ± 2 K from Table 2 could be considered as typical for Moscow and for specified surroundings and feed position. The most of my measurement days gave close results; deviations did not exceed 0.5-1 K, and I suppose the similar could be observed in other climates too.

Hot Source and notes on measurements

The most of my measurements were made with waveguide devices, so I have chosen a dummy load at ambient temperature to serve as the Hot Source, see Fig. 4. The low VSWR, about 1.1 or less for 10 GHz, should be an advantage, so the mismatch uncertainty will be not too high even for ambient temperatures about 290 K.

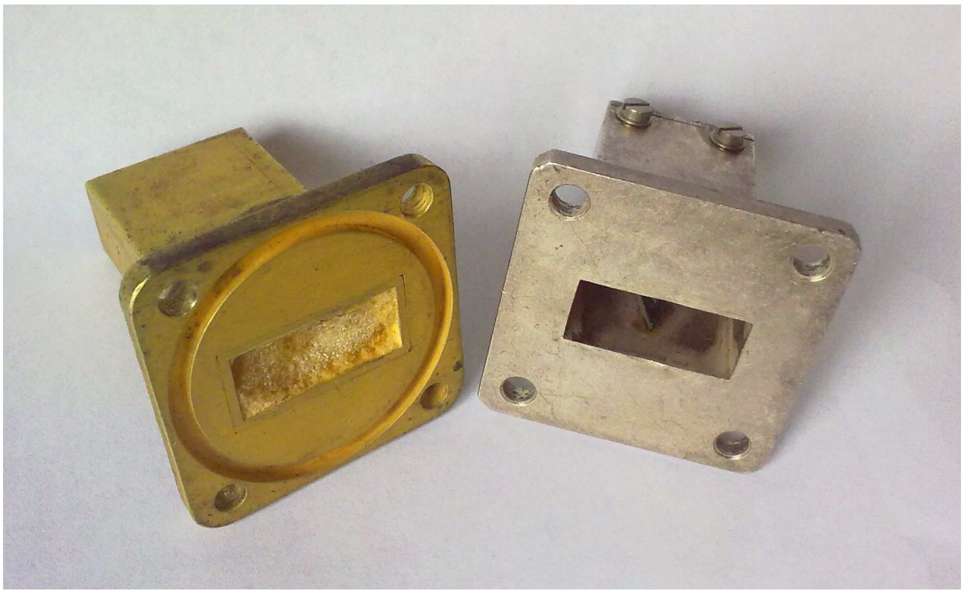


Fig. 4. Dummy loads, waveguide size WR90.

Block diagrams at Fig. 5 illustrate the process, sequence of measurements and structure of equipment used for. Comments about parts of the system are following:

1. Device Under Test (LNA, RX module 10 GHz, etc). One needs to know the input VSWR of DUT for proper estimation of measurement uncertainty. The cold and hot sources, i.e. properly placed horn and dummy load (matched WR90 terminator) are changed to measure RX output levels for Y-factor ratio.

---

[15] Root Sum of Squares.

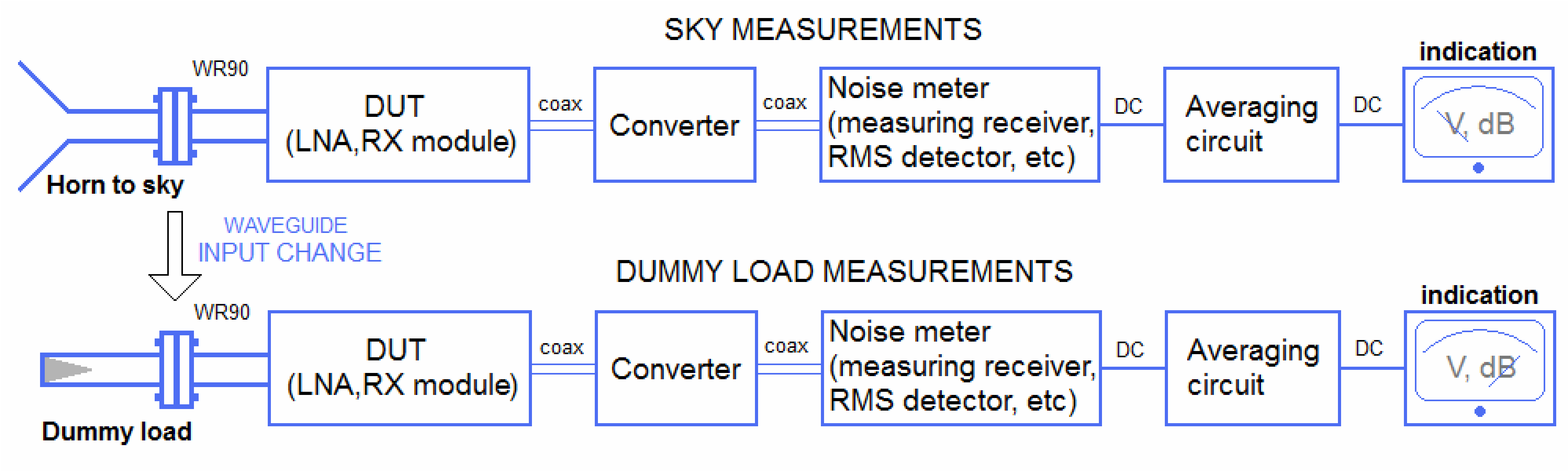


Fig. 5. Schematic (block diagrams) for Noise Figure measurements by Cold Horn.

One may use a waveguide SPDT switch for quick changeover, but losses in the switch should be known with good accuracy; they impact on DUT NF immediately. Moreover, VSWR of DUT should be measured with the switch installed at the input; and just this VSWR should be used for calculations of mismatch losses and uncertainty.

I practiced a simple manual change from horn to dummy load using screwdriver; but it requires at least several minutes, and a gain drift of measurement system may appear and become significant. This leads to additional errors in Y-factor, and, therefore, errors in final result. What to do with the gain drift? See Appendix 2 where I have noted my experience.

2. Converter is needed to shift the noise spectrum from 10 GHz to lower frequencies for better accuracy of detection. Moreover, some frequency selection and limiting of receiving bandwidth also needed; and, as a possible solution, one may use for that a good IF filters at lower frequencies. I would recommend to use an isolator at the input of converter to provide a good load for DUT and better stability.

3. About measuring receiver. As a possible solution, one can use high accuracy true RMS detectors with linear voltage output. A good way is to use specialized IF modules-radiometers for Radio Astronomy[16]; this solves also the next problem – averaging and indication. Up to 2015-2016 I used old measuring receiver with BW=120 kHz, IF=704 MHz with known range of linearity (instrumental error about 0.2 dB) and using averaging circuits at receiver's output. Next, as a modern replacement, I used SDR receiver BladeRF x40 Nuand with specially written Noise Meter program[17] enjoying higher measurement bandwidth (up to 28 MHz), possibility to use higher IF up to 3.8 GHz, clearly done and controlled averaging with predefined accuracy, and convenient indication.

[16] See IF modules by CT1DMK, http://www.cupidotech.com/prod.html, http://www.qsl.net/ct1dmk/rad2.html.
[17] Written on LabVIEW and using MATLAB scripts enclosed to BladeRF.

4. Averaging circuits are needed if RMS detector or similar with voltage output is used. The receiver bandwidth defines a spectrum of noise at output; the power of this noise should be properly measured. A noise with limited spectrum is no more the White Noise; close samples of signal may be correlated, and such noise may be characterized by correlation time. For practice there is a minimal time between independent (or uncorrelated) samples of noise signal; and when the bandwidth is lower such samples should be rarer, and to collect required number of independent samples for RMS averaging one needs a longer time. The relative accuracy when the noise power (or temperature) is measured with averaging time τ can be expressed as[18]

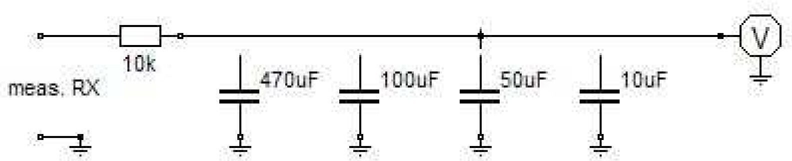


Fig. 6. Averaging RC circuit at the measuring receiver output.

$$\frac{\delta T}{T} \approx \frac{1}{\sqrt{\tau \Delta f}}, \qquad (11)$$

where Δf - noise or RX bandwidth. I used a simple RC circuit at the detector output for averaging, see Fig. 6; a higher capacity in the circuit leads to better accuracy, but the measurement time and transient response will be longer also. The estimation of measurement time can be done using the duration of transient response of averaging circuit,

$$\tau_{meas} \approx -\frac{RC}{2} \ln \frac{\delta T}{T} \qquad (12)$$

for relative power accuracy $\frac{\delta T}{T}$. The value of RC may be chosen reaching a compromise between better stability of voltage readouts and shorter transient response, recommended value $RC \approx 2\tau$ or higher.

There are true RMS detectors with specially recommended averaging filters suitable for their schematic. One should also note that detectors with linear voltage output (not logarithmic) are correct for use if an averaging circuit will be connected after; the cause is that average and logarithm do not commute, result depends on the sequence of these operations.

5. An indication may be performed using just high impedance voltmeter if the RMS type detector with averaging circuit is used. As one can see, there are another ways to solve the problem of noise power measurement and indication.

NF and uncertainty calculator

Computations of NF with uncertainty are cumbersome a little, and I have collected them in special calculator[19]. It is based on Excel spreadsheet and uses VBA forms and Basic code in macros see Fig. 7, so the macros should be enabled in Excel. See below for my notes about functions, features and sequence of actions; inner procedures of the calculator described shortly also. Equations and formulas for procedures in the calculator are collected in Appendix 3.

1. Noise Figure is calculated according equation (1); computations of uncertainty uses an extended version of (3) including Y-factor uncertainty, see Appendix 3. Previously calculated (without correction to local losses and mismatch) T_{cold} and T_{hot} should be entered in the main form; this form is called by "T_{rx} and NF calculations" button. For T_{cold} one enter a value by integration over the sky and environments according (9) with uncertainty estimation; for T_{hot} one enters the data according thermometer on the dummy load or nearby with uncertainty estimation.

[18] See for instance Thomas L. Wilson, Kristen Rohlfs, Susanne Hüttemeister, *Tools of Radio Astronomy*, Springer-Verlag Berlin Heidelberg 2009, pages 68-69.

[19] Can be downloaded from http://www.vhfdx.ru/faylyi/view-details/radiolyubitelskie-raschetyi/coldhorn, or ask me for latest version. Any questions how to use are highly welcome.

Y-factor should be measured previously too; the value with uncertainty should be entered in the main form. There are options to enter Y as a linear ratio or in dB; the linear ratio can be calculated using voltage outputs from measuring receiver detector, see Fig. 8; just enter readouts from voltmeter. Uncertainty of Y-factor is calculated form individual uncertainties of voltages.

2. Losses in horn and connecting waveguide (local losses) can be accounted next; corresponding functions are called by button "Attenuation & losses". One should enter the temperature of waveguide and attenuation; see secondary forms at Fig. 8.

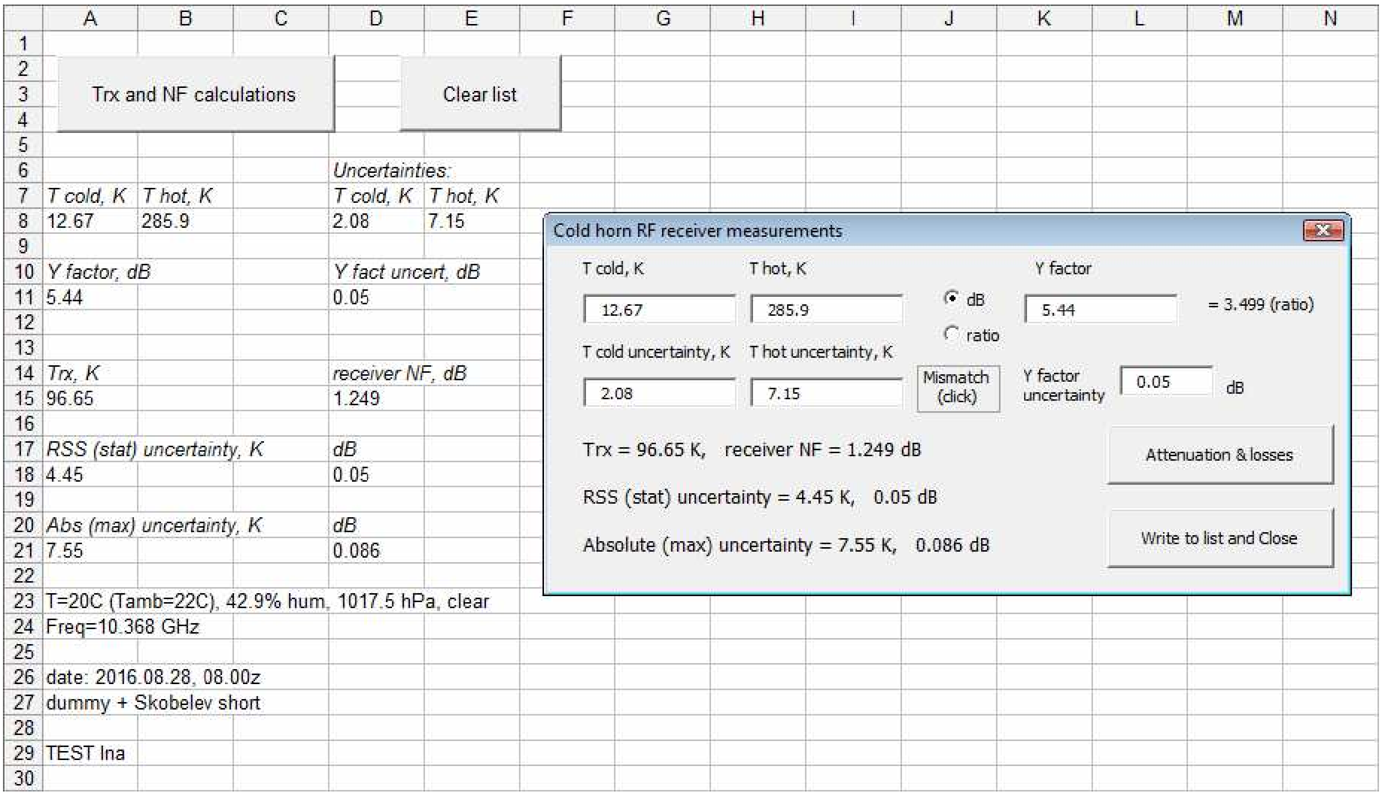


Fig. 7. Spreadsheet of Cold Horn calculator. Results for Test LNA 10 GHz.

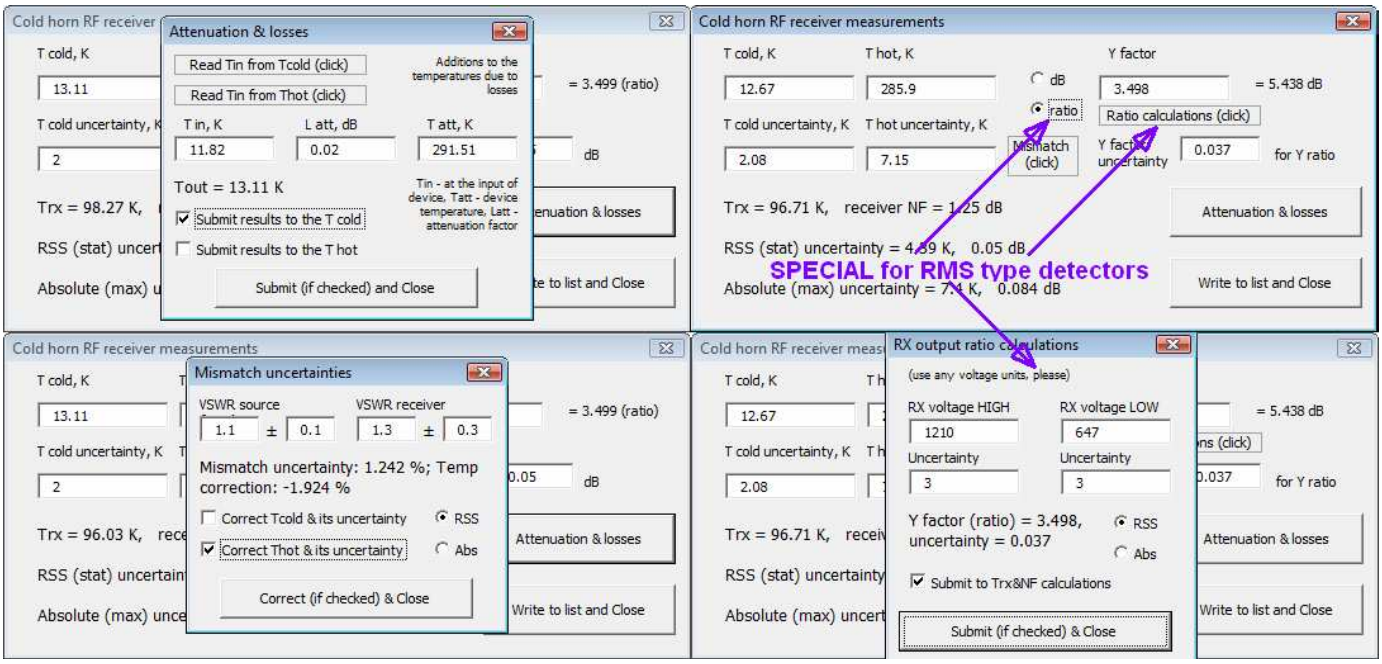


Fig. 8. Secondary forms of Cold Horn calculator.

3. Next the mismatch correction should be done. Mismatch form is called by clicking the area right to T_{hot} uncertainty. Corrections to T_{cold} , T_{hot} and their uncertainties will be implemented if corresponding checkbox is marked; I recommend to treat T_{cold} , T_{hot} separately and carefully checking the VSWRs to avoid gain

mistakes in calculations[20]. Mismatch corrections are based on equation like (5); additionally, VSWR tolerances can be entered if they are known, see Appendix 3.

4. After that one can look to results for NF and uncertainty. Data and results can be copied to Excel spreadsheet by "Write to list and Close".

The RSS summation of uncertainties (errors) is default everywhere in secondary forms; and if the RSS uncertainty needed only, see second output string at the main form. "Absolute (stat) uncertainty" at the main form correspond to the direct sum of T_{cold} , T_{hot} and Y-factor uncertainties. One can choose also the direct summation when computing uncertainties at secondary forms separately; then the mismatch uncertainty will be added to T_{cold} , T_{hot} uncertainties directly.

Test LNA 10 GHz

A special LNA was manufactured for Cold Horn testing with single stage NE32584c FET inside, see Fig. 9. All bias voltages are supplied from outside using high performance EMI filters. The input was matched to WR90 waveguide using tuning screws; input VSWR was tuned as low as possible (less 1.2-1.3), and lowest Noise Figure is not considered as priority. Results for this LNA are shown at spreadsheet Fig. 7. As one can see, the value of uncertainty about 0.05 dB is achievable.

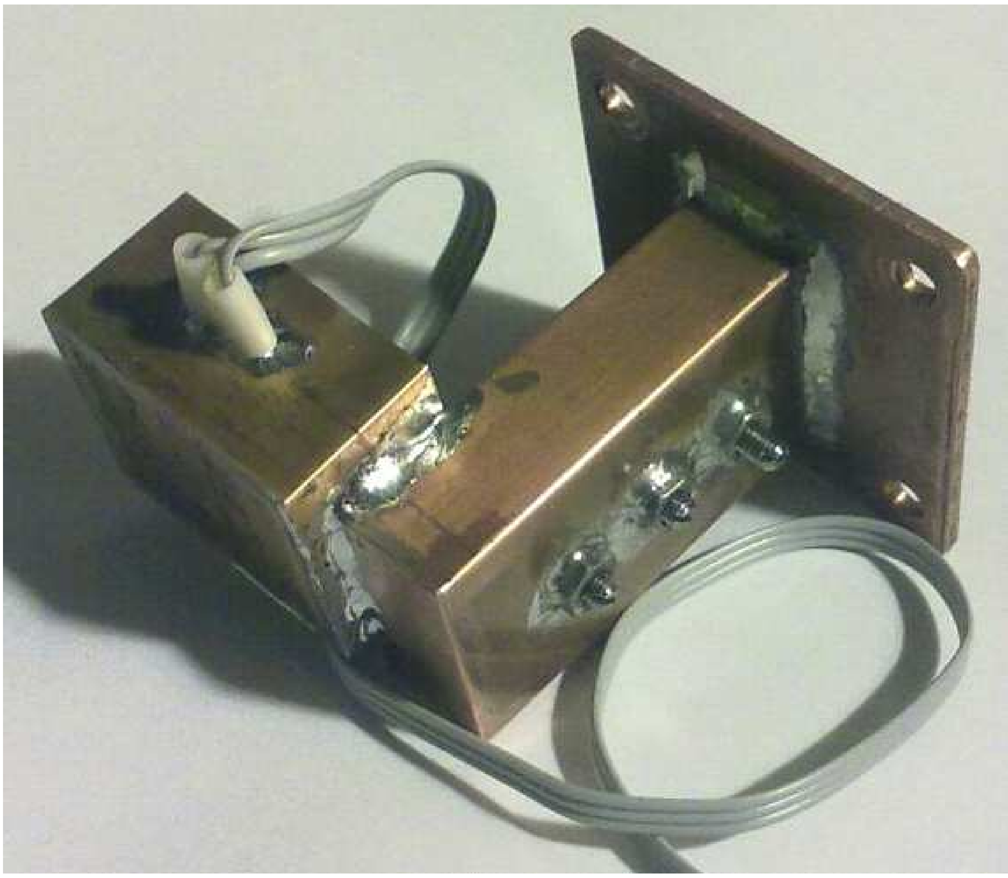


Fig. 9. The test LNA 10 GHz.

The Cold Horn procedure gives the Noise Figure for the whole system only. The gain of LNA is about 13 dB, so the impact from the next device on the Noise Figure is expected significant. The Noise Figure of the device itself can be obtained using cascaded Friis's equation[21], but NF or noise temperature of the next cascade should be known. This procedure is also called as deembeding; expected addition to Noise Figure uncertainty would be about 0.02 dB if the next stage NF is not exceed 2-2.5 dB with uncertainty up to 0.3 dB.

---

[20] There is no the gain error[6] in the procedure above as it is, but one need an information about VSWRs for proper corrections.

[21] See (13)-(15) in Application Note https://www.rohde-schwarz.com/us/applications/the-y-factor-technique-for-noise-figure-measurements-application-note_56280-15484.html; see (5.24) in Thomas L. Wilson, Kristen Rohlfs, Susanne Hüttemeister, *Tools of Radio Astronomy*, Springer-Verlag Berlin Heidelberg 2009, page 88. In addition, one can use free AppCad software, https://www.broadcom.com/appcad, NoiseCalc option in menu.

10 GHz devices without input flange or connector

The Cold Horn method is suitable for them too. Similar device is shown at photo Fig. 10 this is 10 GHz LNA reconstructed from old satellite converter "Cambridge" with circular polarization[22]. Feedhorn is not removable; therefore, to switch between cold and hot loads with the procedure above is not possible.

A hot source should be performed without using a dummy load. I used sheets of SHF absorbing material; they can be placed in front of horn's aperture. The best absorber available for me was black foam leafs with reflectivity about -26 dB. A small box can be made from these, so, all directions where sky was seen by the feed would be closed by absorber making the "hot" source. The hot temperature T_{hot} should be about ambient by thermometer readouts, but I suppose with a higher uncertainty taken; I used as initial value of δT_{hot} about 5 K. Reflections of this device are hardly to define; I taken a typical value corresponding to input VSWR=2. VSWR=1 was entered for "cold" source; for the VSWR of "hot" source I used a value corresponding to reflectivity -26 dB, i.e. about 1.1.

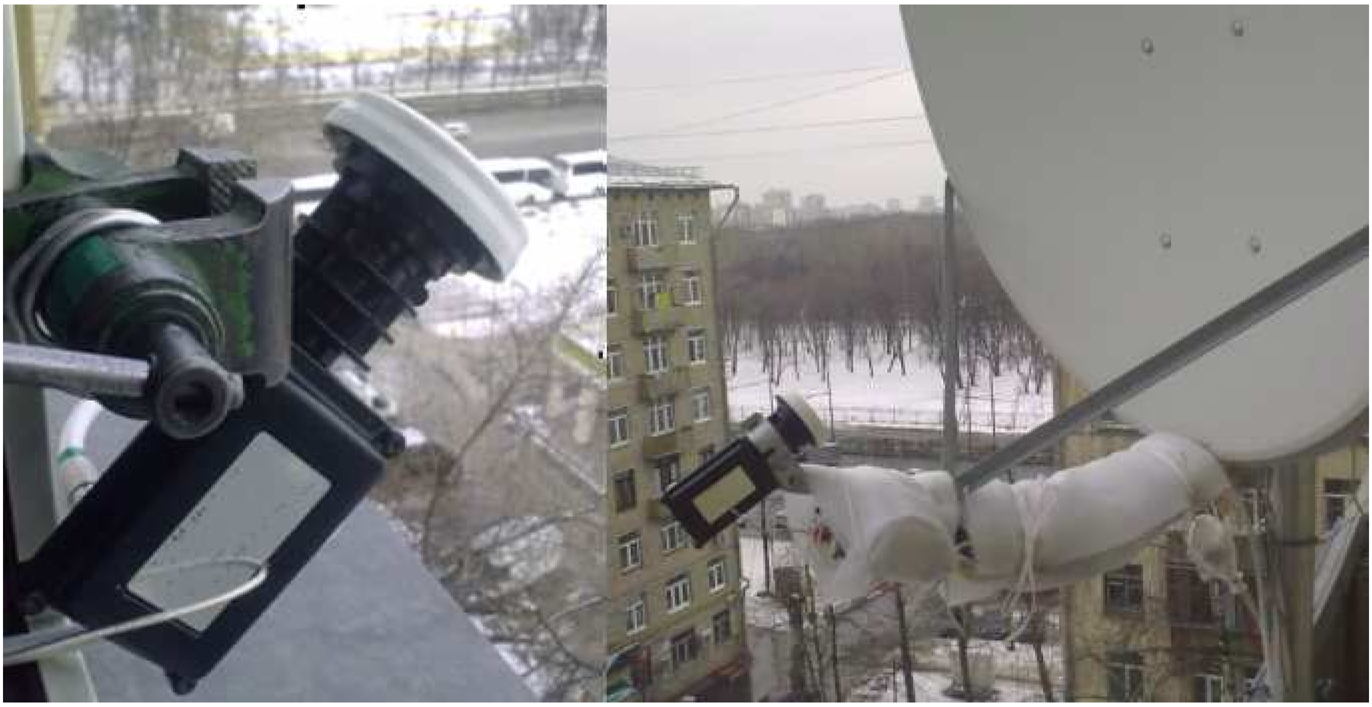


Fig. 10. LNA with permanently attached horn made from old Sat converter "Cambridge", circular polarization.

The cold temperature T_{cold} can be calculated using procedures described above, but taking into account the circular polarization of horn. Contribution of ambient thermal noise in corner position (with elevation 45 deg) was computed via linear polarizations; they differ by collected noise yet, but the half of horizontal and vertical powers should contribute to circular one. Circular polarizations, RHCP and LHCP should not differ by collected noise in corner position.

My result for this device NF=0.9±0.15 dB, and several measurements within couple of days in January-February 2014 confirmed this result; uncertainty is somewhat higher in comparison with my test LNA, but this is a good application of the Cold Horn method. This LNA was used for lunar monitoring at 10368 MHz. Some details about feedhorn are in Appendix 4.

Waveguide source

A more application of the Cold Horn is the laboratory noise source. It should be calibrated to use in measurements. Procedure of Cold Horn is rather complicated, implies a waiting of good weather, every measurement requires a verification of the horn position and checking the environment, recalculation of the

[22] Thanks to Alexey RA3AES for many of his surplus devices.

horn temperature etc. I think a laboratory source could be useful when a boring rainy weather outside for many days.

A variant of such source is shown on Fig. 11; this is a homemade device with reverse-biased diode (base-emitter junction of BFQ67 transistor in my case, right part) matched to waveguide at 10 GHz by screws. Central part – is a variable attenuator with max attenuation about 20 dB. Device under test (LNA) is attached at the left part. The reference plane is on the left flange of attenuator; available Excess Noise Ratio (ENR) at the reference plane is about 11-12 dB (with 20 dB attenuation).

Fig. 11. Waveguide noise source: central part – attenuator ~ 20 dB, right part – solid state noise generator, left part – LNA under test.

The source can be calibrated using previously measured system with test LNA. Expected uncertainty of NF by Cold Horn for test LNA should not exceed 0.1 dB; this allows to calibrate the source with accuracy better 0.1-0.2 dB. The known NF of receiver with test LNA could be reproduced using this source and conventional Y-factor method adjusting the value of ENR. The calibration accuracy could be calculated by

$$\delta_{ENR,dB} = \sqrt{\delta^2_{NF,dB} + \left(\frac{\ln 10}{1-10^{-Y_{dB}/10}}\right)^2 \delta^2_{Y_{dB}} + 2\,\delta^2_{VSWR,dB}}, \quad \delta_{VSWR,dB} = \max\left[\pm 20\log\left(1 \pm |\Gamma_{RX}|\cdot|\Gamma_{source}|\right)\right], \qquad (13)$$

where all uncertainties expressed in dB. Equation (13) was derived from simplified formula for Noise Figure when $T_{cold} \approx T_0$,

$$NF = ENR_{dB} - 10\log\left(10^{Y_{dB}/10} - 1\right) \quad \text{(dB)}$$
$$ENR_{dB} = 10\log\left(\frac{T_{hot} - T_{cold}}{T_0}\right), \quad T_{cold} \approx T_0, \qquad (14)$$

and just adding the mismatch term with reflection coefficients $\Gamma_{RX}, \Gamma_{source}$. The Y-factor of conventional measurements should be known; Y-factor uncertainty should be estimated also to get the calibration accuracy, its contribution to $\delta_{ENR,dB}$ may be significant and even leading. I have paid a less attention above to the problem of accurate noise measurement including long averaging and checking the linearity errors, but here it appears more significant.

Also, the best practice is to provide the same ambient temperature for Cold Hold and conventional measurements while calibrating the source; also note that ENR value usually referenced to $T_{cold} = T_0 = 290\ \text{K}$.

Remarks to conclusion

Similar ideas and practices are widely used in Radio Astronomy and Deep Space Network projects for calibration of radio telescopes and receivers[23]; and there are good fundamentals for Cold Horn method. The

[23] See for instance DESCANSO Book Series, Macgregor S. Reid, *Low-Noise Systems in the Deep Space Network*, 2008, https://descanso.jpl.nasa.gov/monograph/series10_chapter.html, see chapter 2.4 and next.

procedure seems rather complicated, especially after using contemporary automatic Noise Figure Analyzers, but results with lower uncertainty could be expected and the method itself is a much promising.

There are people (besides Sergey RW3BP and me) who also used Cold Horn to measure NF, and I should say a word of appreciation to:

Ivan RA3WDK, he used the Cold Horn at 5.7 GHz and 10 GHz[24] in 2009-2015. Ivan noted the method is good for LNA tuning, and results coincide with measurements by conventional NF analysers, but he estimates the minimal NF uncertainty about 0.15-0.2 dB;

Victor UA9FAD, he experimented with long Pasternack horn PE9852-20 and dualmode RA3AQ horn[25] at 24 GHz in 2015-2016, and there are projects with feed temperature calculations at 24 GHz also[26].

My appreciations also for many of people including Sergey RW3BP for always valuable attention to the work, Anatoly UA4HTS and his conference VHF-Volga where this work was first presented, and my neighbors for long patience while they were observing strange devices installed on the window of apartment building in Moscow (window of my laboratory up to summer 2017).

Dimitry Fedorov, UA3AVR,

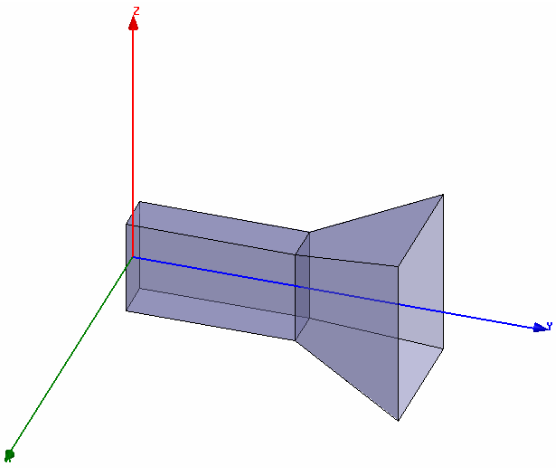
this work dated 2013-2017.

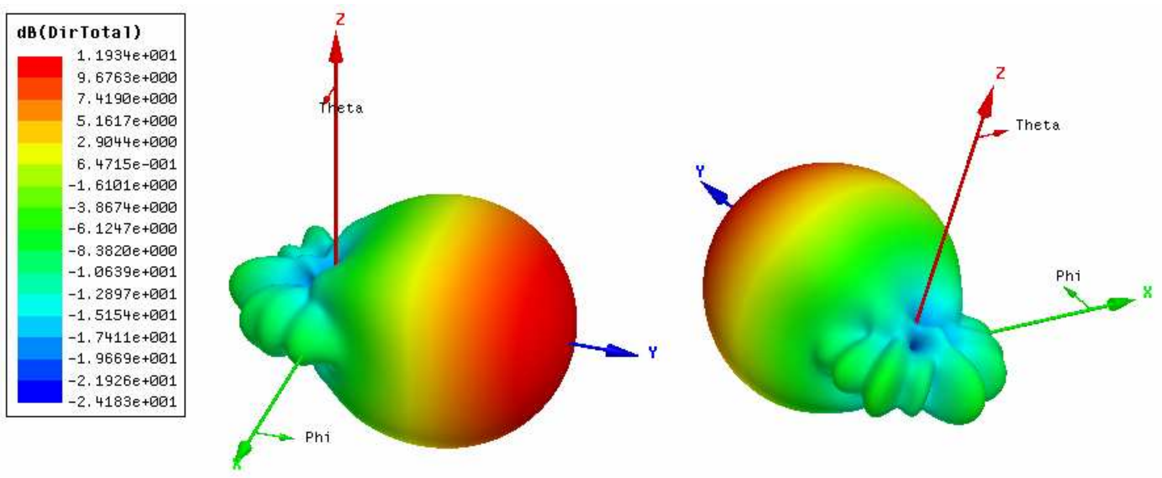
[24] See his web pages with LNA projects, http://ra3wdk.qrz.ru/LNA.htm.
[25] See http://www.vhfdx.ru/faylyi/start-download/shemyi-i-opisaniya/ra3aq-round-septum-with-dual-mode-flare-horn-pdf.
[26] Ask me if 24 GHz temperature calculations are interesting.

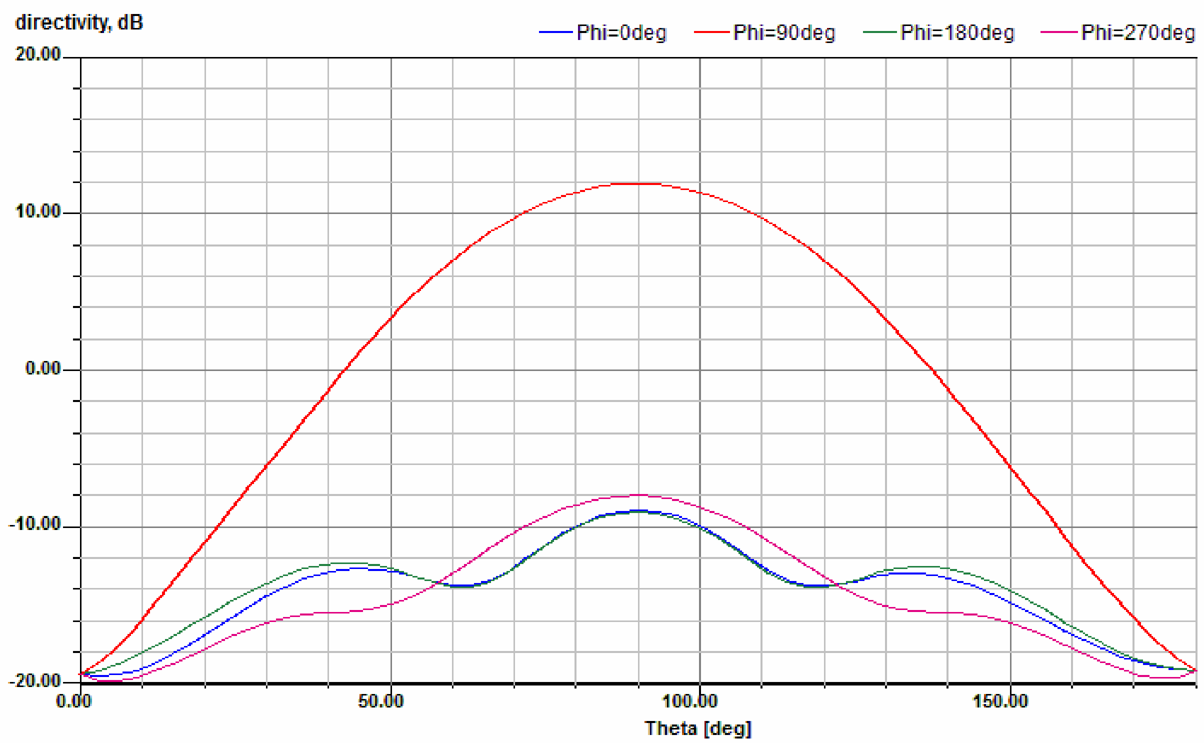
Appendix 1

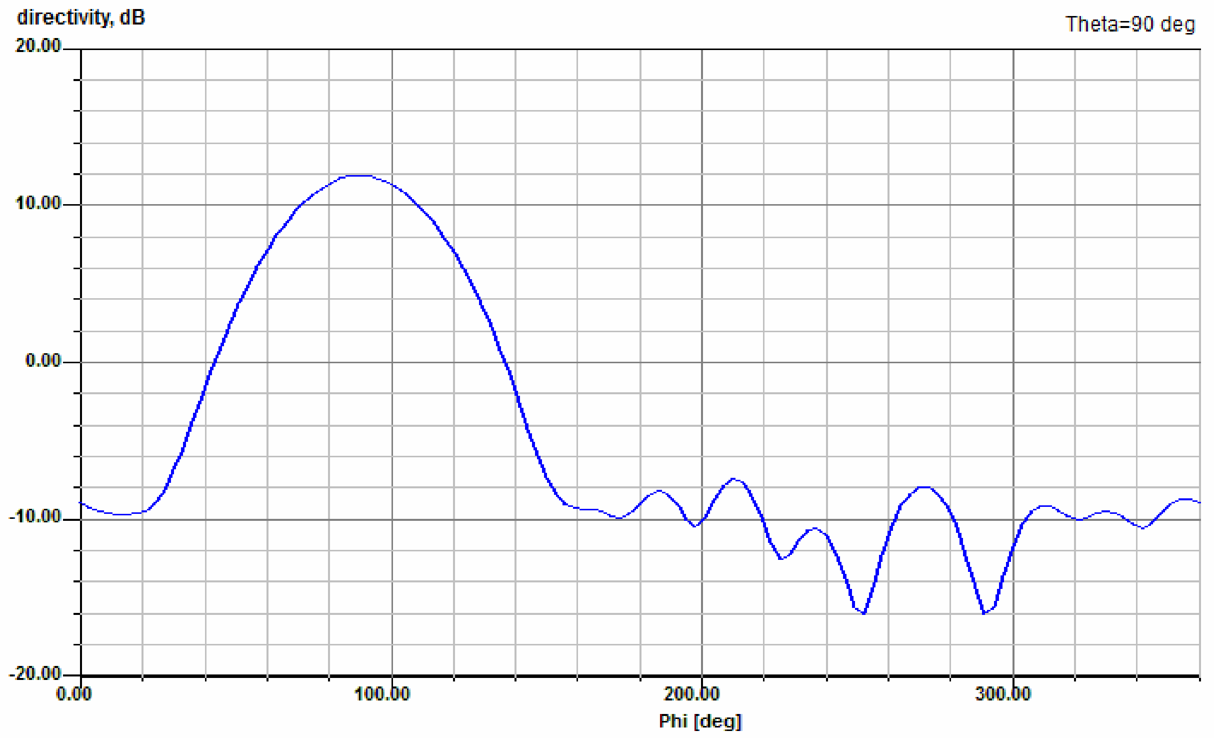
Part A. Pyramidal horn (short).

Horn data:	WR90 (23x10)	
axial length	28	mm
aperture H	41	mm
aperture E	32	mm

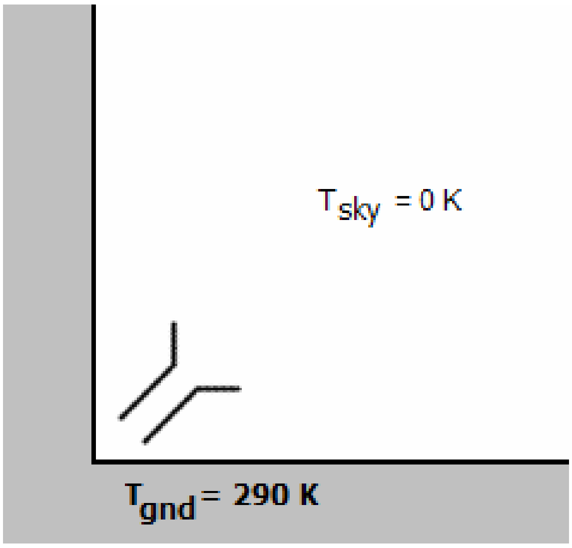


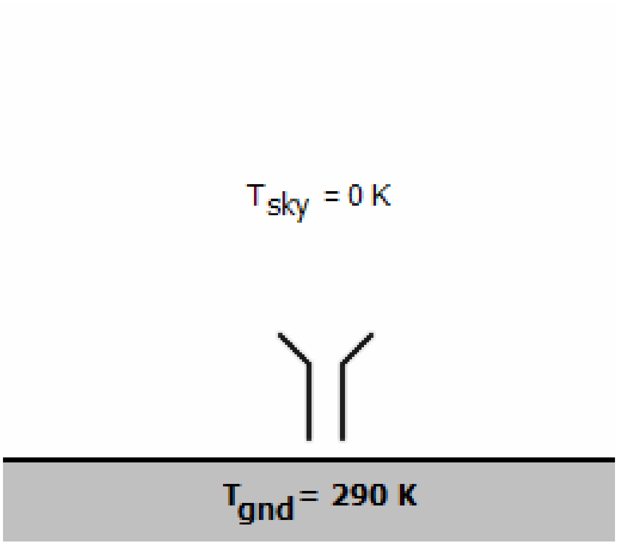






Noise parameters (T_{sky} = 0):





T_{horn} = 15.27 K (Hor pol)
T_{horn} = 21.13 K (Vert pol)

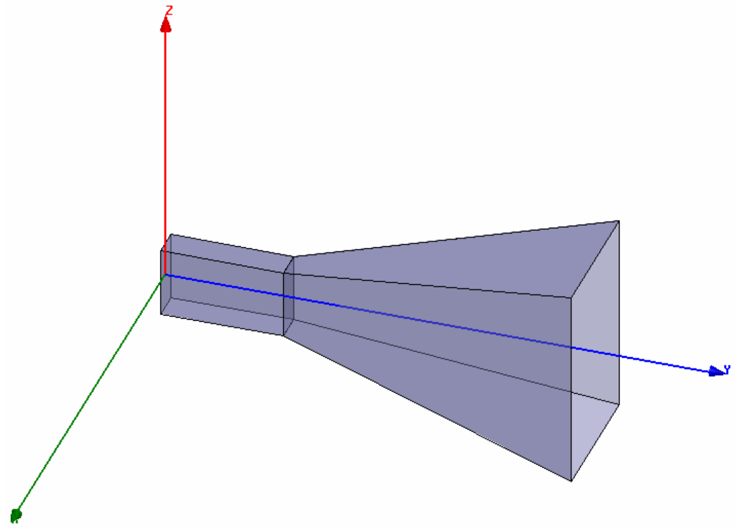
T_{horn} = 6.98 K

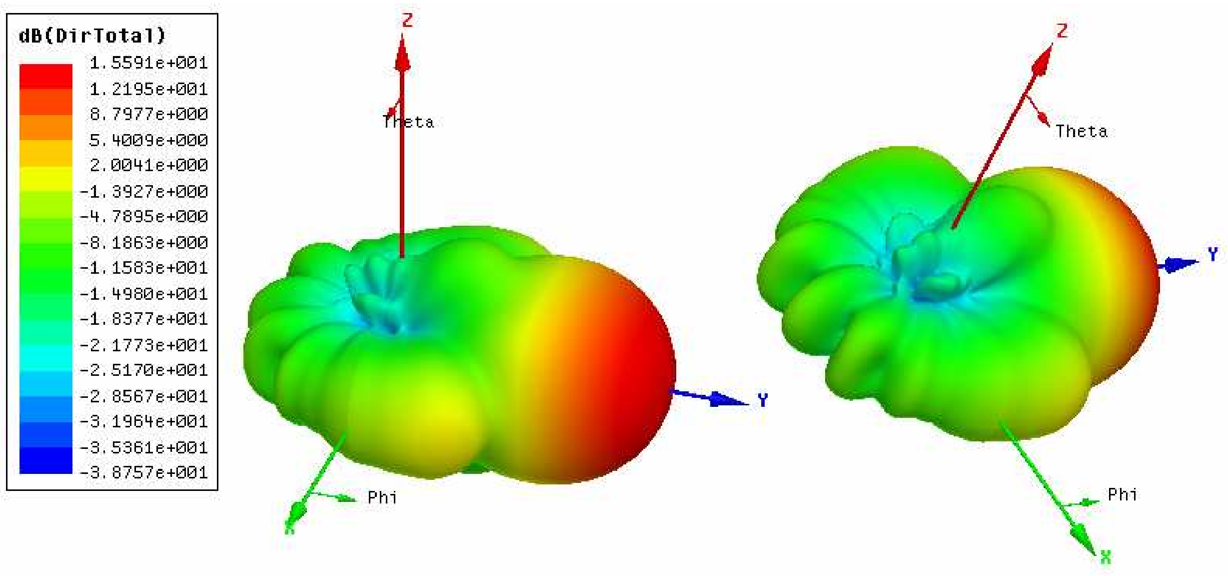
Here and after:
T_{horn} – the noise temperature of horn without losses when T_{sky} = 0 K and ambient temperature T_{gnd} = 290 K; horizontal polarization (Hor pol) – along the corner of wall and ground, vertical one (Vert pol) across the corner, elevation of horn – 45°.

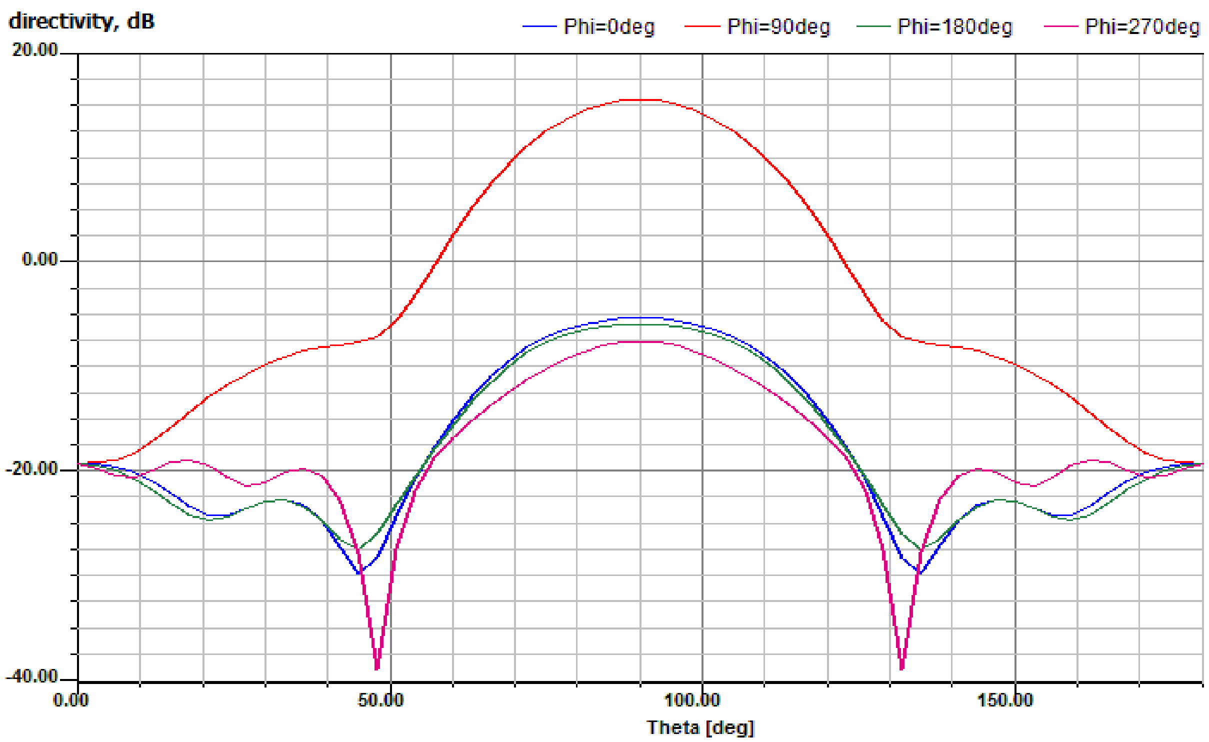
Part B. Pyramidal horn (long).

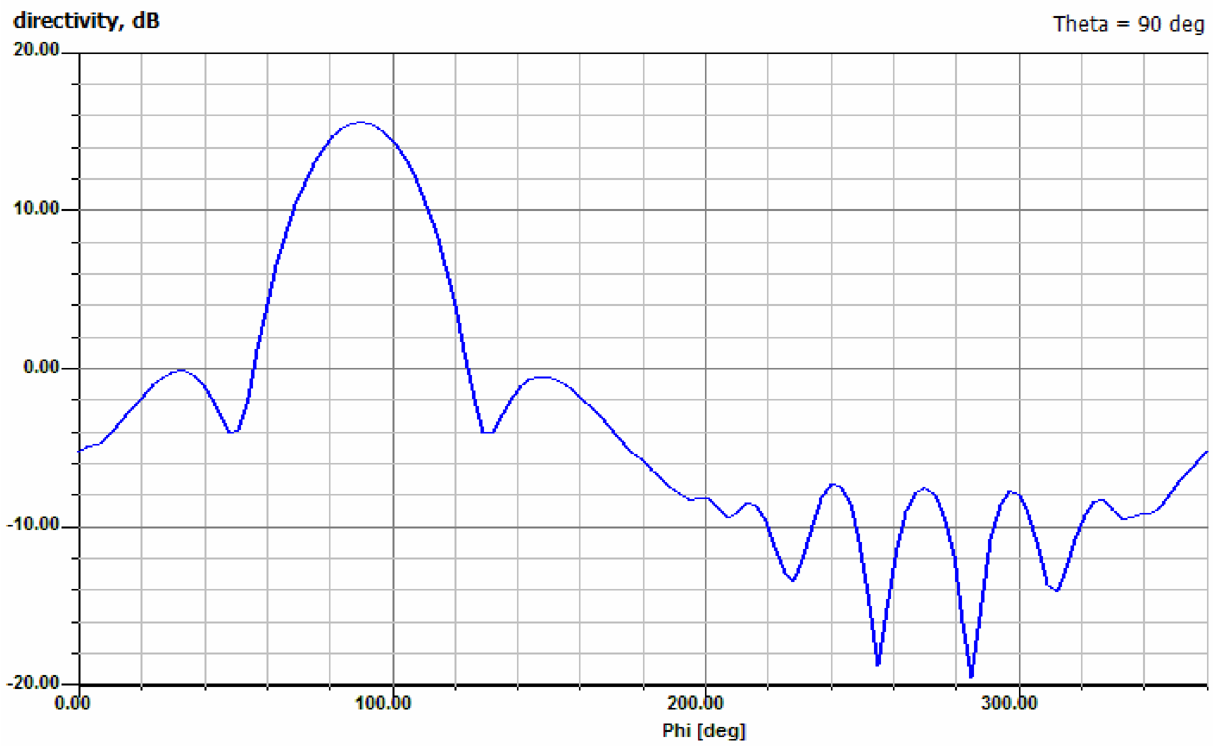
Horn data*:	WR90 (23x10)	
axial length	100	mm
aperture H	67	mm
aperture E	47	mm

*sizes by W1GHZ software Hdl_Ant for parabolic dish f/d=1.

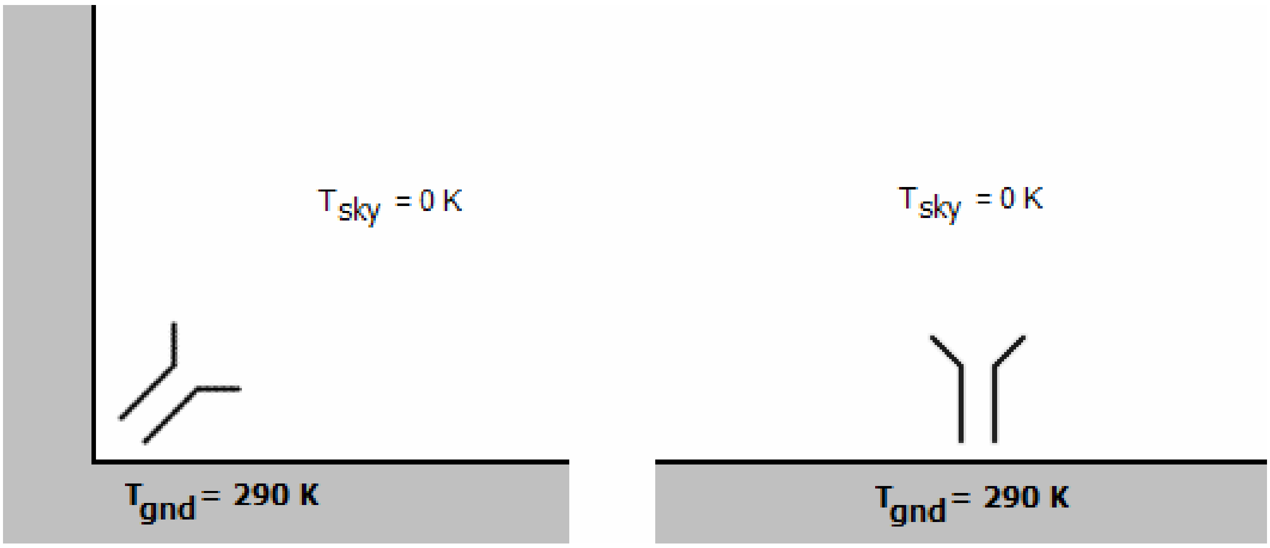








Noise parameters (T_{sky} = 0):



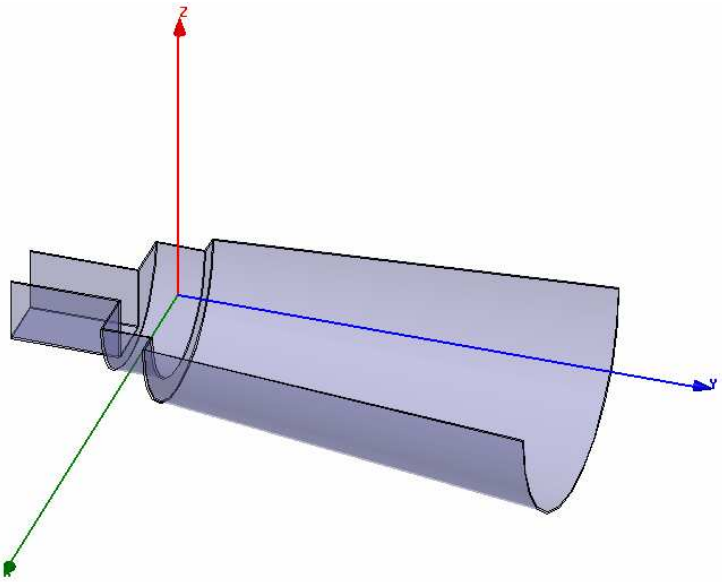
T_{horn} = 7.77 K (Hor pol)
T_{horn} = 19.48 K (Vert pol)

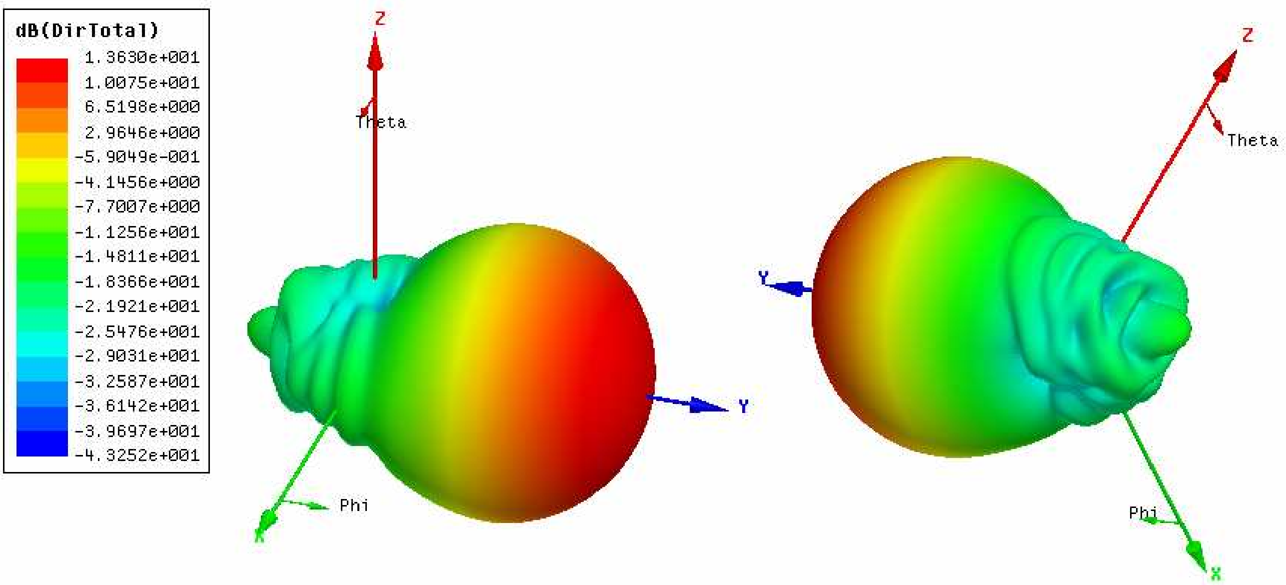
T_{horn} = 6.27 K

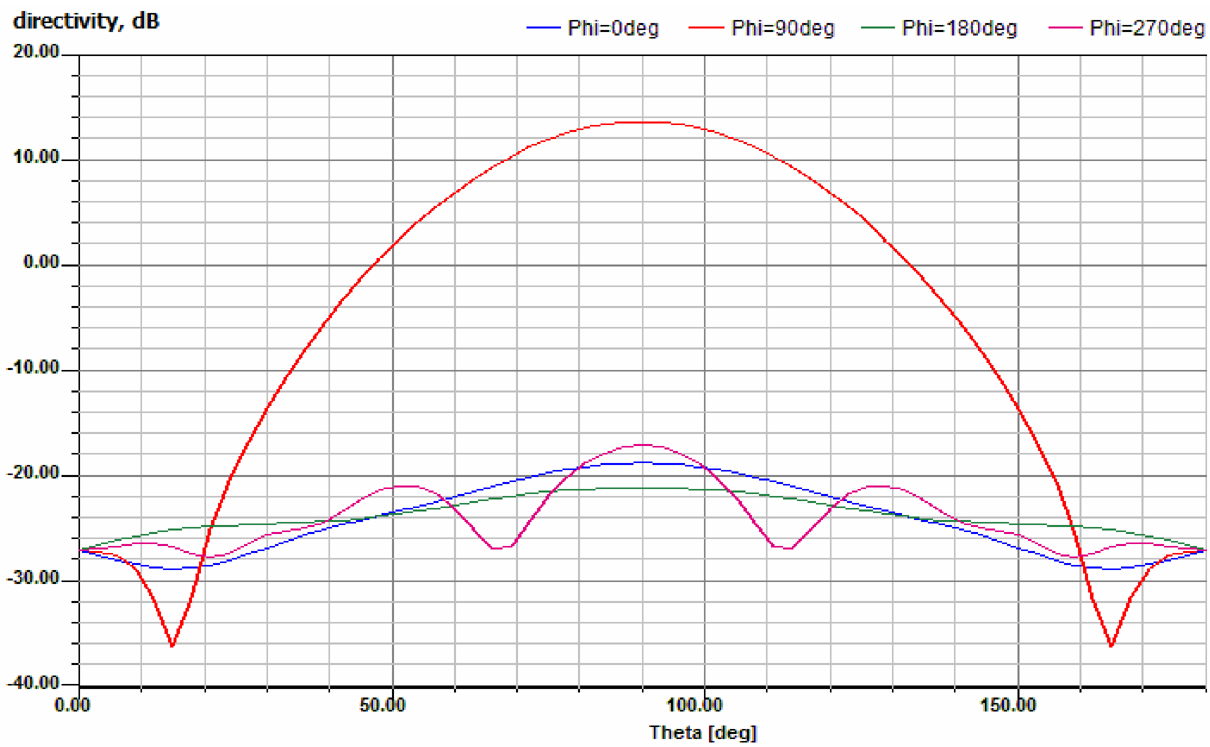
Part C. Skobelev horn (short).

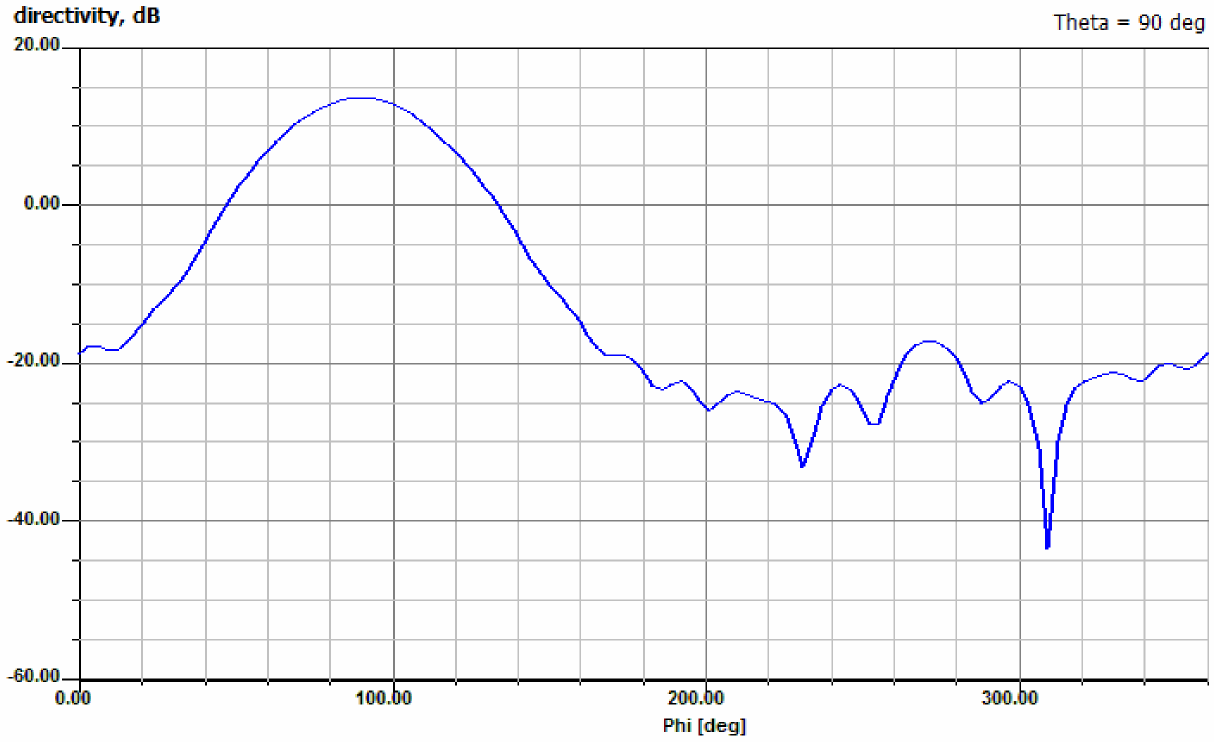
Horn data*:	WR90 (23x10)			
flare len	71.71	mm	2.48	λ
flare diam1	52.05	mm	1.8	λ
flare diam2	37.59	mm	1.3	λ
dual Mode len	9.00	mm		
dual Mode diam	29.38	mm	1.016	λ

*sizes by W1GHZ, best f/b and sidelobes, http://www.w1ghz.org/antbook/conf/optimized_dualmode_feedhorns.pdf.

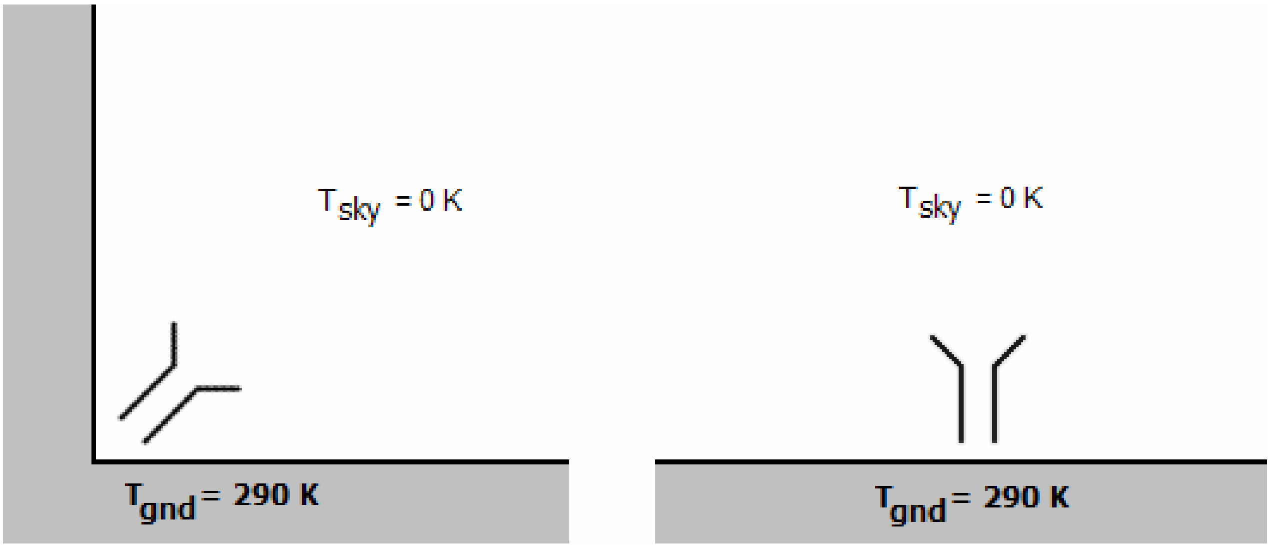








Noise parameters (T_{sky} = 0):



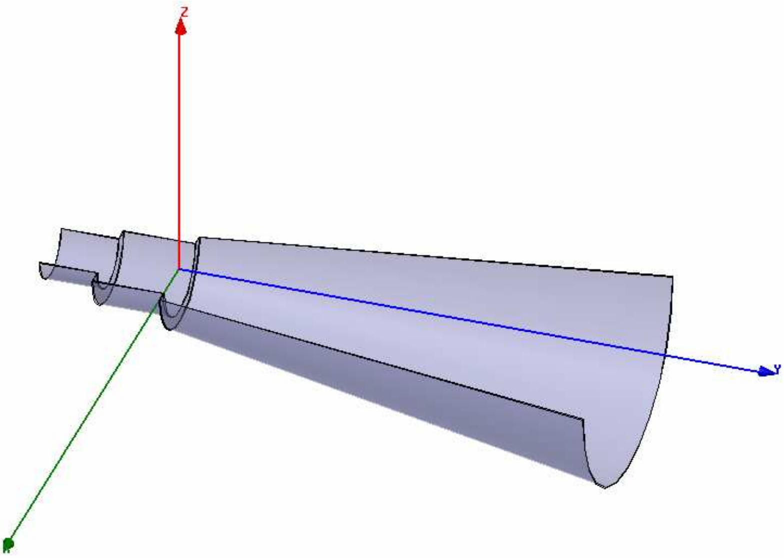
T_{horn} = 2.57 K (Hor pol)
T_{horn} = 4.6 K (Vert pol)

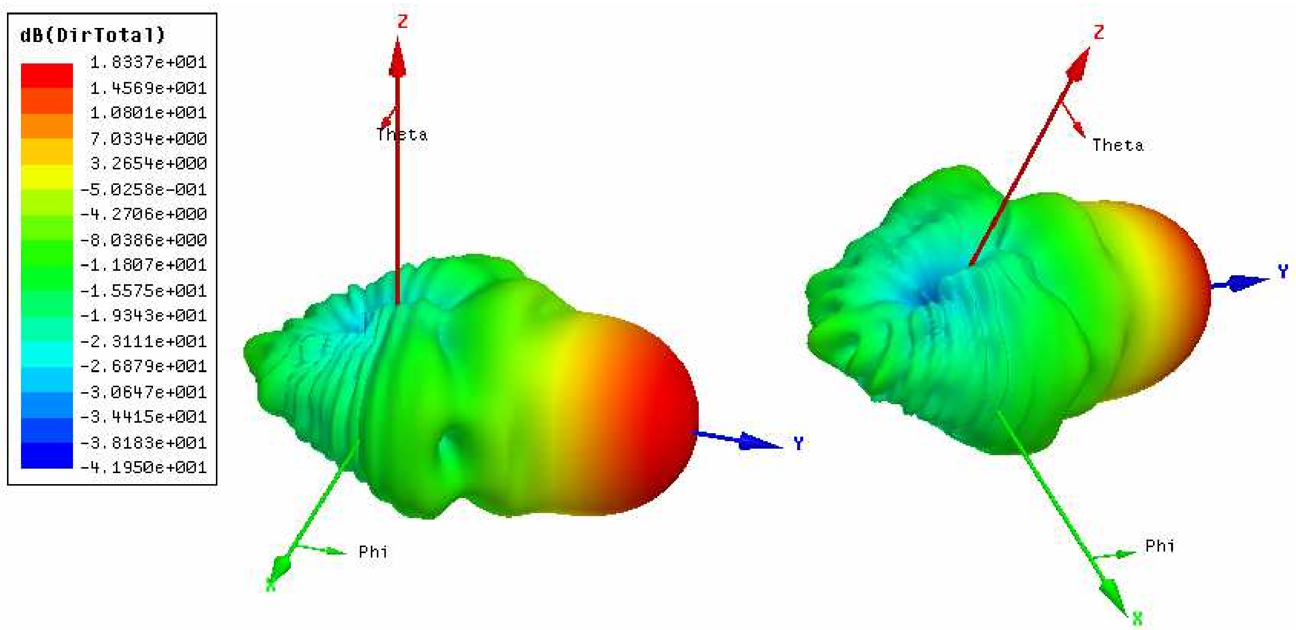
T_{horn} = 0.57 K

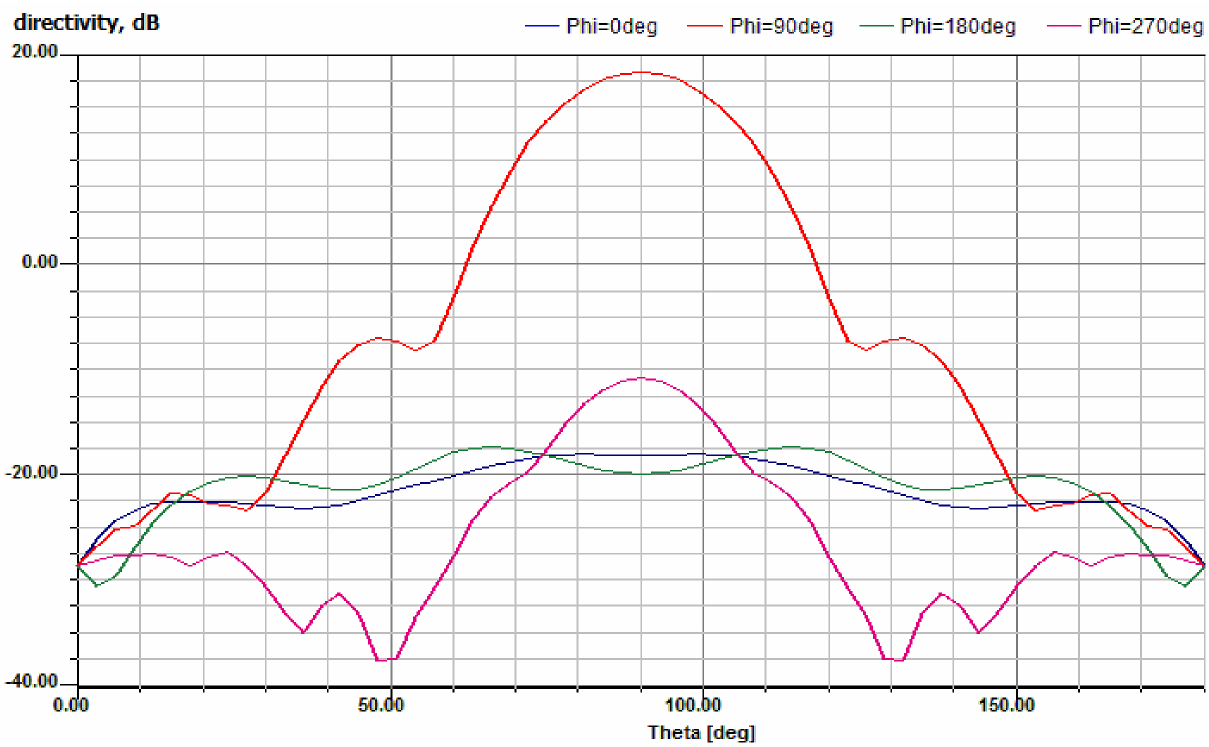
Part D. Skobelev horn (long, by RA3AQ&RW3BP).

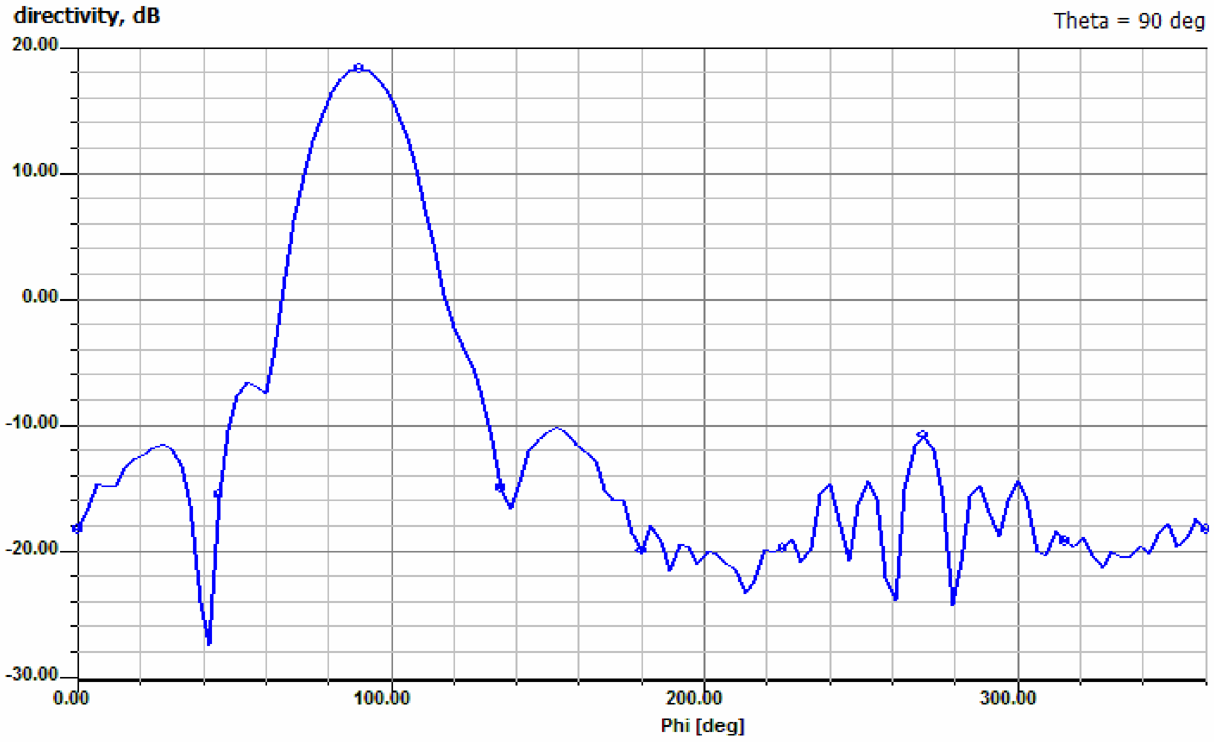
Horn data*:		
flare len	5.01	λ
flare diam1	3	λ
flare diam2	1.31	λ
dual Mode len	0.8	λ
dual Mode diam	1.04	λ

*see sizes also http://www.vhfdx.ru/faylyi/view-details/shemyi-i-opisaniya/holodnyiy-rupor-dlya-izmereniya-koeffitsienta-shuma.

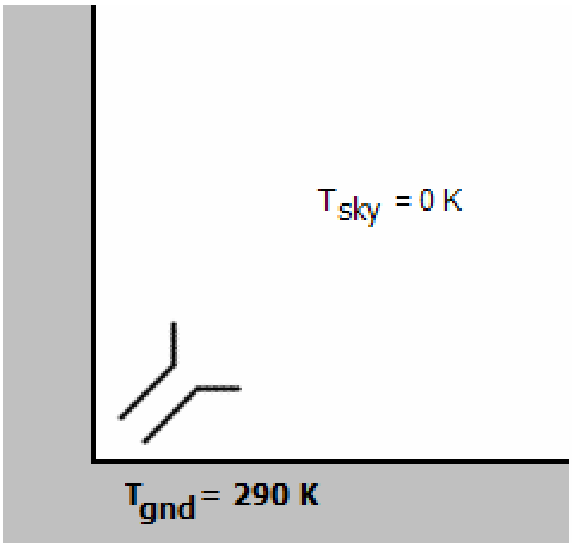


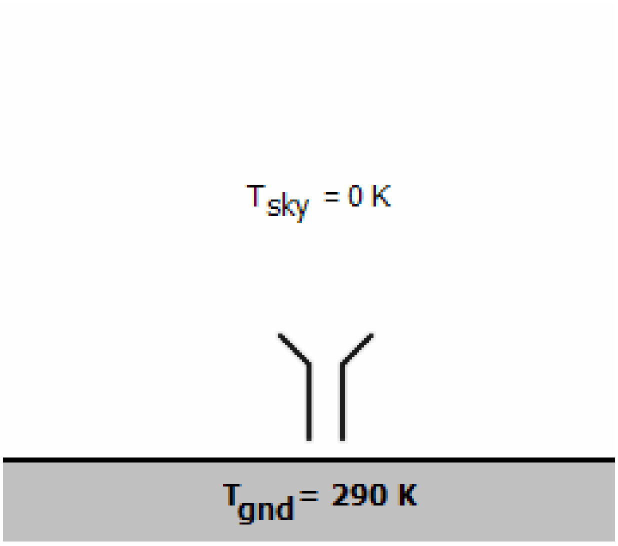






Noise parameters (T_{sky} = 0):





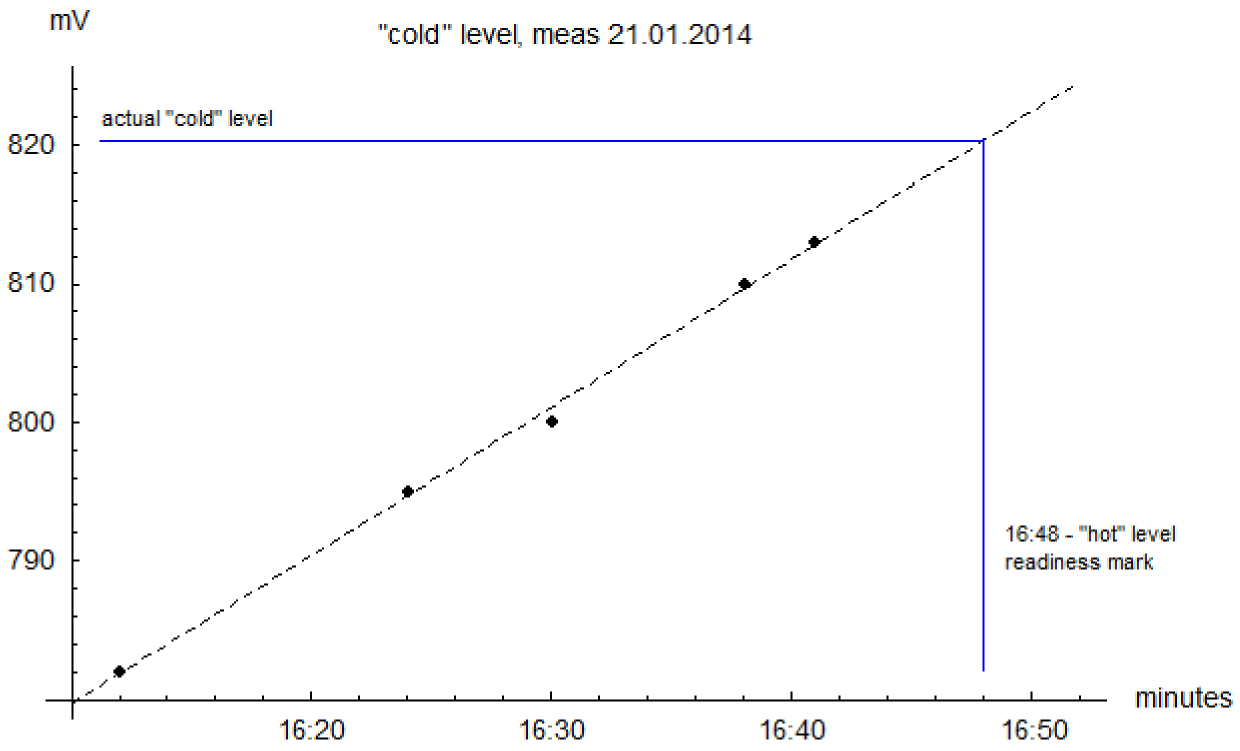
T_{horn} = 2.03 K (Hor pol)
T_{horn} = 4.32 K (Vert pol)

T_{horn} = 1.22 K

Appendix 2

If the gain drift appears and become significant for time intervals about 5-7 minutes additional procedures are needed to refine the measurement data. Following procedure was applied for refining data from the system with output detector, averaging circuit and voltage indicator.

1. I performed several measurements with Cold Horn at the DUT input, up to 4-5 during a half of hour. Readouts were marked on the plot, as it shown below; time marks were done also.



2. Next, I interpolated the dots; interpolation curve is a simple straight line usually, but some assurance is needed about and a plot showing the curve and readouts is very useful for. The line corresponds to the gain drift; and this line can be extrapolated further, so, one can predict a future levels of "cold" readouts if so could be done.

3. After, I installed the Dummy Load; changeover require at least several minutes, and the measurement is done about 5-7 minutes later, the time of "hot" readout is marked also.

4. One should define the actual results for "cold" readout using mentioned extrapolation, as it shown on the plot. The time of "hot" readout should be considered as a measurement time. A real gain of the system at this moment is not matter because one only needs the ratio of "hot" and "cold" readouts to define an Y-factor.

5. For measurement uncertainty, one can use an averaged or RMS deviation of "cold" dots from interpolation line. One could also add the instrumental uncertainty of voltmeter to "hot" and "cold" readouts and specific linearity error of detector; but I should warn about possible overestimations. Relative voltmeter errors are expected the same in a wide range, and will not affect on the Y-factor ratio.

Appendix 3

ColdHorn calculator performs computations based on equations and formulas:

- for NF and uncertainty –

$$NF = 10 \cdot \log\left(1 + \frac{T_{RX}}{T_0}\right), \quad \delta NF = \frac{10}{\ln 10}\frac{\delta T_{RX}}{T_0 + T_{RX}}, \quad T_0 = 290\ \mathrm{K}, \quad T_{RX} = T_{hot}\frac{1}{Y-1} - T_{cold}\frac{Y}{Y-1}$$

- for Y-factor (Y_{dB} in dB) –

$$Y = 10^{Y_{dB}/10}, \quad \frac{\delta Y}{Y} = \frac{\ln 10}{10}\cdot \delta Y_{dB},$$

- the common rule for δy uncertainty (RSS) , $y = f(x_1, x_2, \ldots, x_n)$ –

$$\delta y = \sqrt{\left(\frac{\partial f}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\delta x_2\right)^2 + \ldots + \left(\frac{\partial f}{\partial x_n}\delta x_n\right)^2}$$

- the common rule for δy uncertainty (Abs) , $y = f(x_1, x_2, \ldots, x_n)$ –

$$\delta y = \left|\frac{\partial f}{\partial x_1}\delta x_1\right| + \left|\frac{\partial f}{\partial x_2}\delta x_2\right| + \ldots + \left|\frac{\partial f}{\partial x_n}\delta x_n\right|$$

- for T_{RX} uncertainty (RSS) –

$$\delta T_{RX} = \sqrt{\delta T_{hot}^{\ 2}\frac{1}{(Y-1)^2} + \delta T_{cold}^{\ 2}\left(\frac{Y}{Y-1}\right)^2 + \delta Y^2 \frac{(T_{hot} - T_{cold})^2}{(Y-1)^4}}$$

- for T_{RX} uncertainty (Abs) –

$$\delta T_{RX} = \left|\frac{\delta T_{hot}}{Y-1}\right| + \left|\delta T_{cold}\frac{Y}{Y-1}\right| + \left|\delta Y \frac{T_{hot} - T_{cold}}{(Y-1)^2}\right|$$

- for RX detectors with voltage output –

$$Y = \frac{V_{hot}^2}{V_{cold}^2}, \ \delta Y = 2\sqrt{\left(\frac{V_{hot}\ \delta V_{hot}}{V_{cold}^2}\right)^2 + \left(\frac{V_{hot}^2\ \delta V_{cold}}{V_{cold}^3}\right)^2}\ \text{(RSS)}, \ \delta Y = 2\left(\left|\frac{V_{hot}\ \delta V_{hot}}{V_{cold}^2}\right| + \left|\frac{V_{hot}^2\ \delta V_{cold}}{V_{cold}^3}\right|\right)\ \text{(Abs)}$$

- for correction due to local losses L_{att} in horn and waveguide with temperature T_{att} (L_{att} in dB) –

$$T_{cold,\,hot}\Big|_{corrected} = T_{cold,\,hot}\cdot 10^{-L_{att}/10} + T_{att}\cdot\left(1 - 10^{-L_{att}/10}\right)$$

- for mismatch correction of temperatures –

$$T_{cold,\,hot}\Big|_{corrected} = \left(1 - |\Gamma_{source}|^2\right)\cdot\left(1 - |\Gamma_{RX}|^2\right)\cdot T_{cold,\,hot}$$

- for magnitudes of reflection coefficients –

$$|\Gamma_{source}| = \frac{VSWR_{source} - 1}{VSWR_{source} + 1}, \quad |\Gamma_{RX}| = \frac{VSWR_{RX} - 1}{VSWR_{RX} + 1}$$

- for correction of uncertainty due to mismatch (RSS, $T_{cold,\,hot}$ taken *before* mismatch correction) –

$$\delta T_{cold,\,hot}\Big|_{corrected} = \left(\left(\left(1 - |\Gamma_{source}|^2\right)\cdot\left(1 - |\Gamma_{RX}|^2\right)\cdot \delta T_{cold,\,hot}\right)^2 + 4\cdot|\Gamma_{source}|^2\cdot|\Gamma_{RX}|^2\left(\left(1 - |\Gamma_{source}|^2\right)\cdot\left(1 - |\Gamma_{RX}|^2\right)\cdot T_{cold,\,hot}\right)^2 + \right.$$

$$\left. + \left(\left(1 - |\Gamma_{source}|^2\right)\cdot\frac{\partial|\Gamma_{RX}|^2}{\partial VSWR_{RX}}\cdot \delta VSWR_{RX}\cdot T_{cold,\,hot}\right)^2 + \left(\left(1 - |\Gamma_{RX}|^2\right)\cdot\frac{\partial|\Gamma_{source}|^2}{\partial VSWR_{source}}\cdot \delta VSWR_{source}\cdot T_{cold,\,hot}\right)^2\right)^{1/2}$$

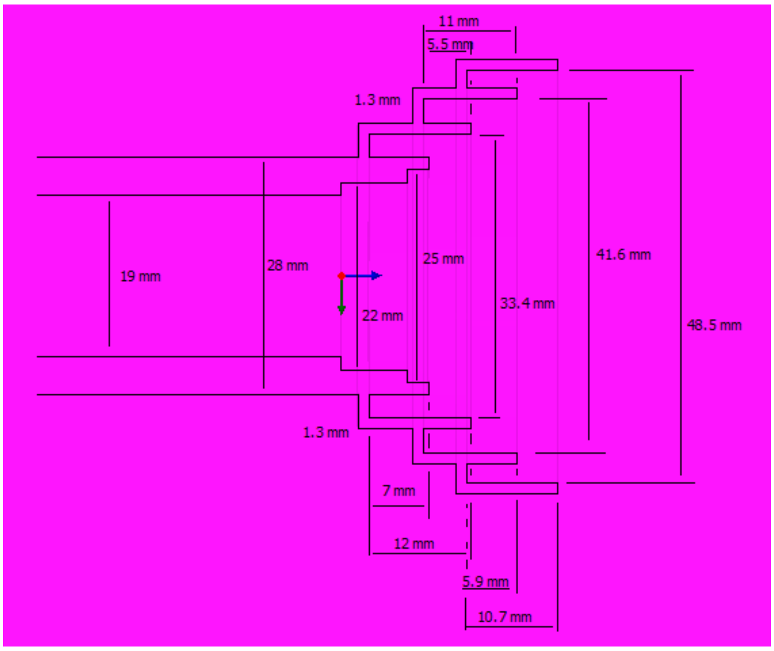
- for correction of uncertainty due to mismatch (Abs, $T_{cold,\,hot}$ taken *before* mismatch correction) –

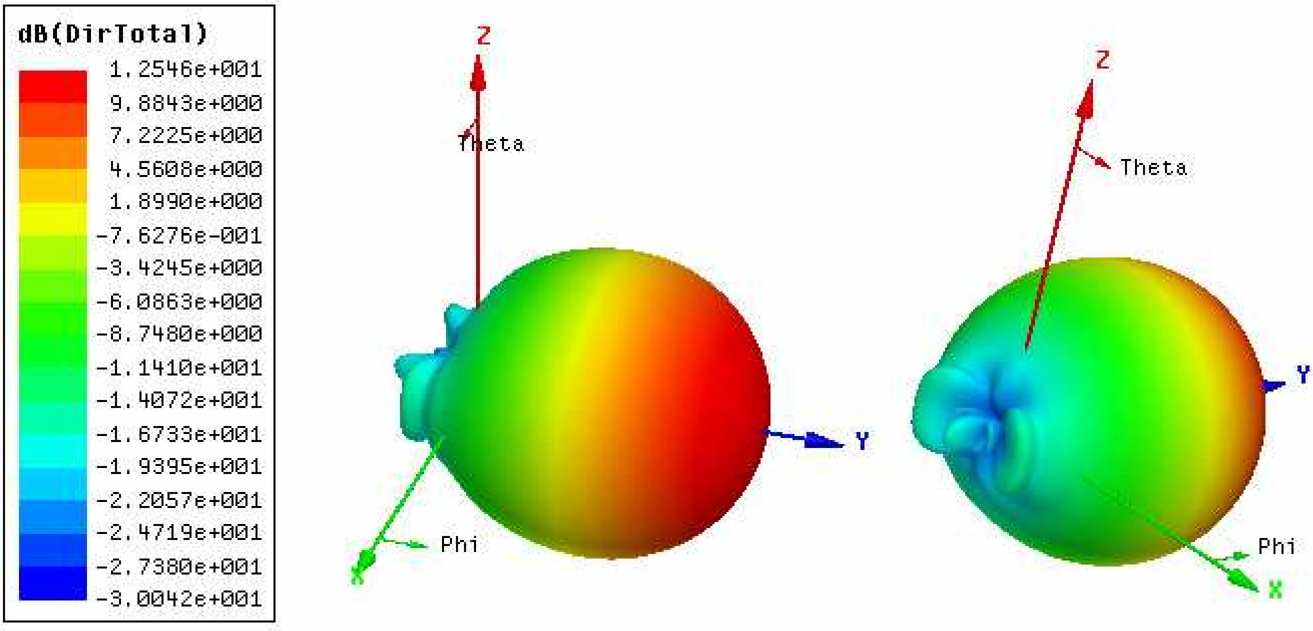
$$\delta T_{cold,\,hot}\Big|_{corrected} = \left|\left(1 - |\Gamma_{source}|^2\right)\cdot\left(1 - |\Gamma_{RX}|^2\right)\cdot \delta T_{cold,\,hot}\right| + 2\cdot|\Gamma_{source}|\cdot|\Gamma_{RX}|\cdot\left(1 - |\Gamma_{source}|^2\right)\cdot\left(1 - |\Gamma_{RX}|^2\right)\cdot T_{cold,\,hot} +$$

$$+ \left(1 - |\Gamma_{source}|^2\right)\cdot\left|\frac{\partial|\Gamma_{RX}|^2}{\partial VSWR_{RX}}\cdot \delta VSWR_{RX}\right|\cdot T_{cold,\,hot} + \left(1 - |\Gamma_{RX}|^2\right)\cdot\left|\frac{\partial|\Gamma_{source}|^2}{\partial VSWR_{source}}\cdot \delta VSWR_{source}\right|\cdot T_{cold,\,hot}$$

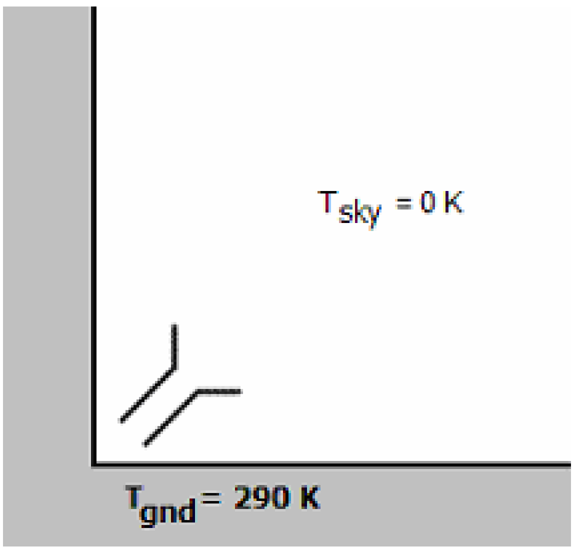
Appendix 4

LNA with circular polarization, Cambridge feedhorn





Noise parameters ($T_{sky} = 0$):



T_{horn} = 11.66 K (Circ pol, RHCP or LHCP)
T_{horn} = 8.29 K (Hor pol)
T_{horn} = 15.03 K (Vert pol)