An Introduction to Kinetic Inductance Detectors

Detection de Rayonnement à Très Basse Température 2012

Scope of this lecture

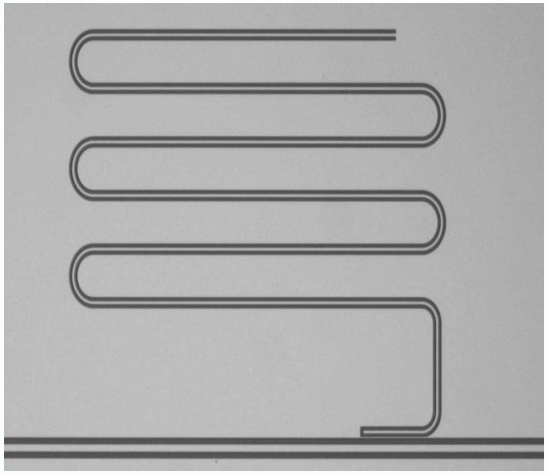
- An introduction to the Kinetic Inductance Detector
- A brief history of Kinetic Inductance Detector
- Complex Impedance in Superconductors
- Concept of an energy gap 2Δ
- Quasi-Particle Lifetimes
- Basic concepts in microwave theory
- Resonator Theory
- The Lumped Element Kinetic Inductance Detector (LEKID)
- Optical coupling to a LEKID
- Optimizing a practical LEKID geometry
- Measuring performance of a LEKID
- Readout Electronics and multiplexing
- Instruments using KIDs
- Conclusion

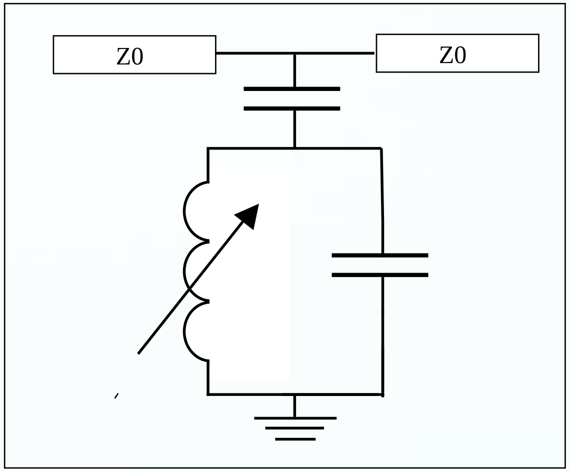
Useful Literature

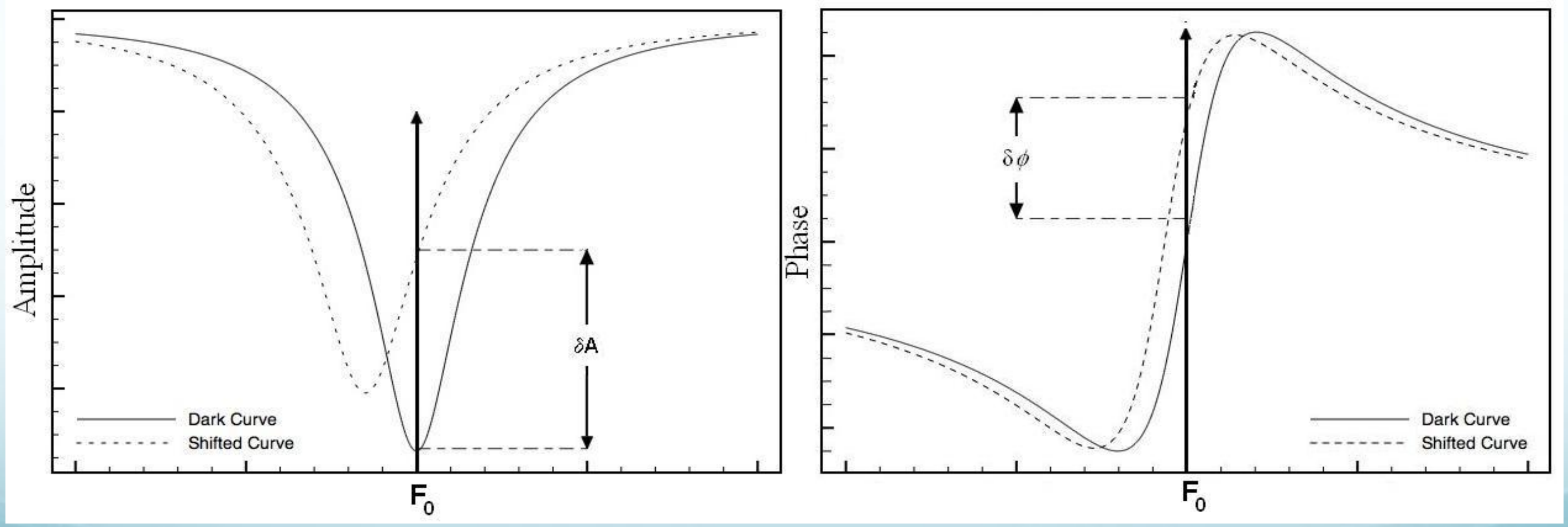
This lecture will cover the basic principles of kinetic inductance detectors more information can be found in the following literature”

- Lumped Element Kinetic Inductance Detectors , Doyle PhD thesis http://www.astro.cardiff.ac.uk/~spxsmd/
- Course notes to follow
- A.C Rose-Innes and H.E. Rhoderick “Introduction to Superconductivity”
- T Van Duzer and C.W Turner, “Principles od Superconducting Devices and Circuits”
- D.M. Pozar, “Microwave Engineering”

The quarter wave resonator







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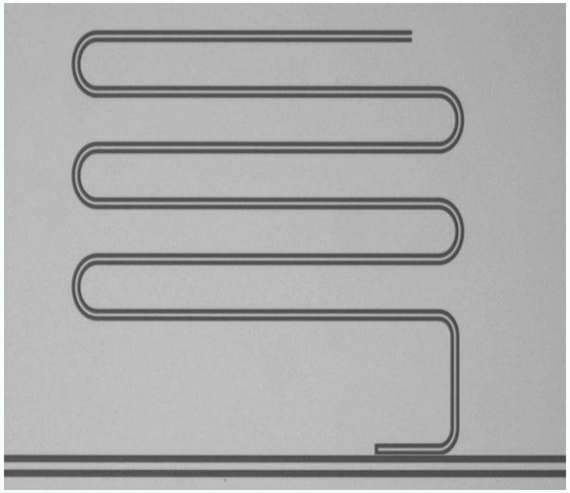
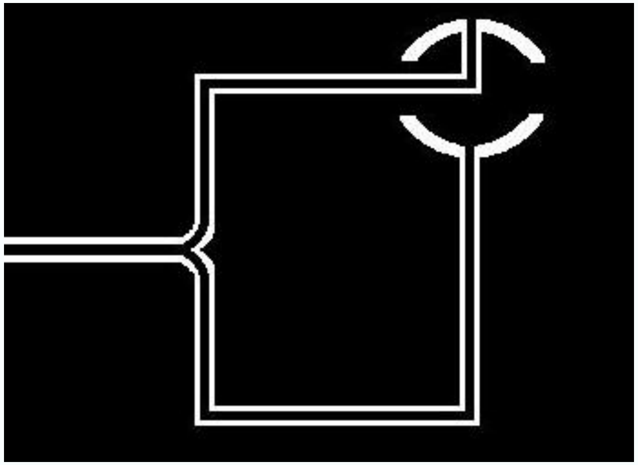
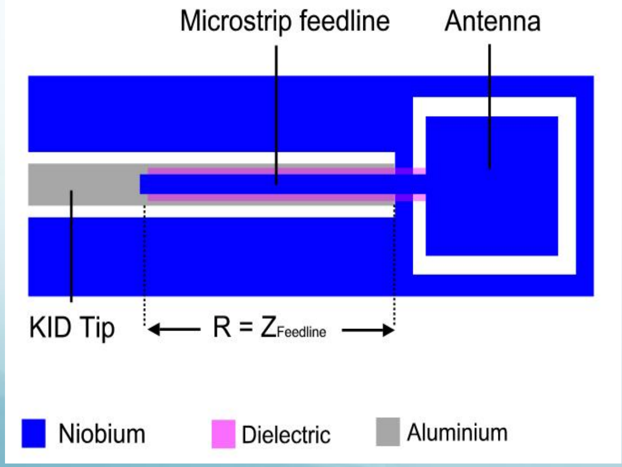
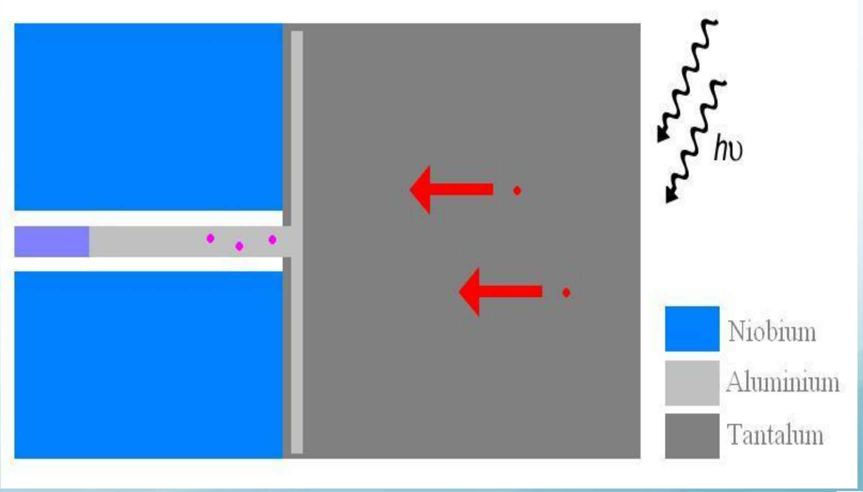
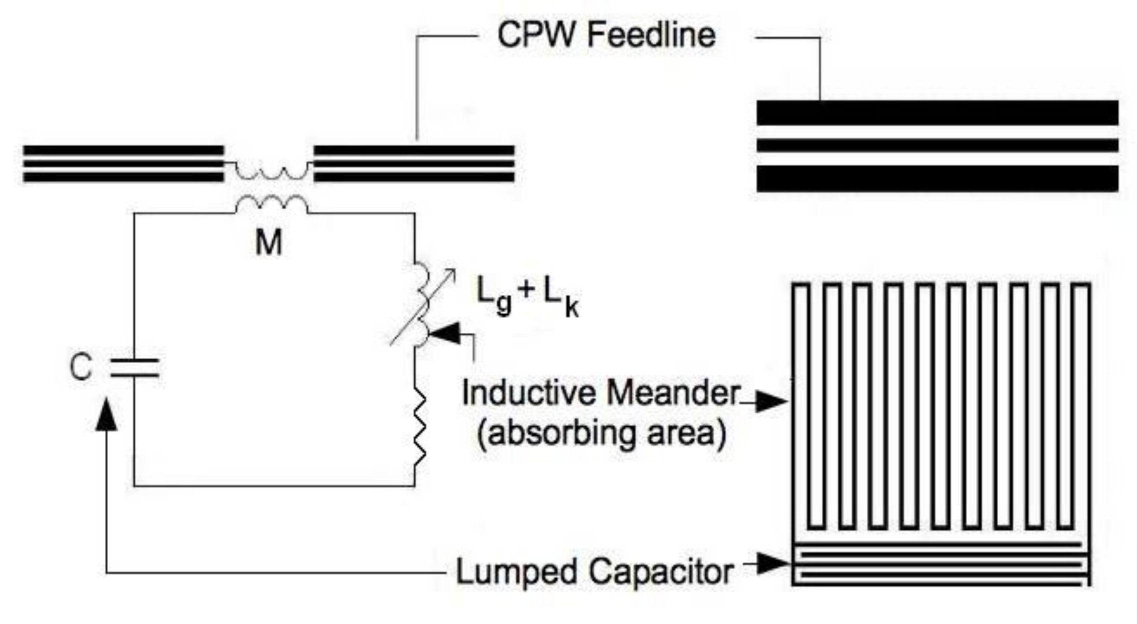


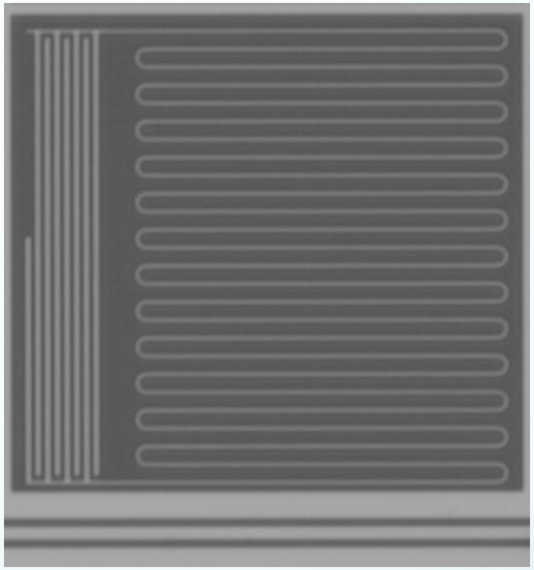


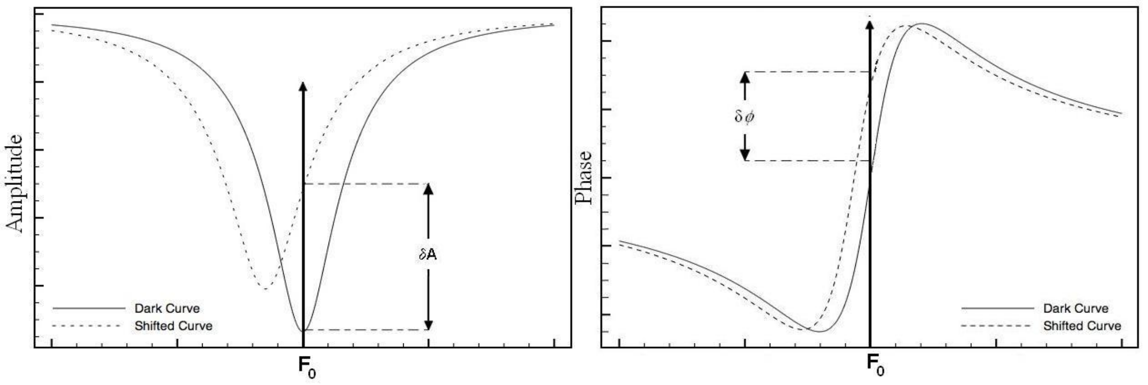




The Lumped Element Kinetic Inductance Detector







Why use KIDs

- Very easy to fabricate, in some cases single deposition and etch step.
- Very sensitive and should reach the photon noise limit for most applications
- Can be very broad band
- Highly multiplexed in the frequency domain so making large arrays is simple
- Relatively in-sensitive to micro-phonics and EMI

A Brief History of Kinetic Inductance Detectors

2002 Idea of micro-resonators to be used a photon detector by sensing a change in Kinetic inductance (9th Low temperature detectors workshop).

2003 Nature paper (Day et al) released presenting results from observed X-ray pulses and expected NEP measurements for mm/submm devices.

2005 Mazin PhD thesis published first in depth study of Kinetic Inductance detectors.

March 2007 Lumped element Kinetic Inductance Detector (LEKID) first proposed

2007 Caltech demonstrated a16 pixel antenna coupled camera on the CSO.

July 2007 first light on LEKID. 200um cold blackbody detected.

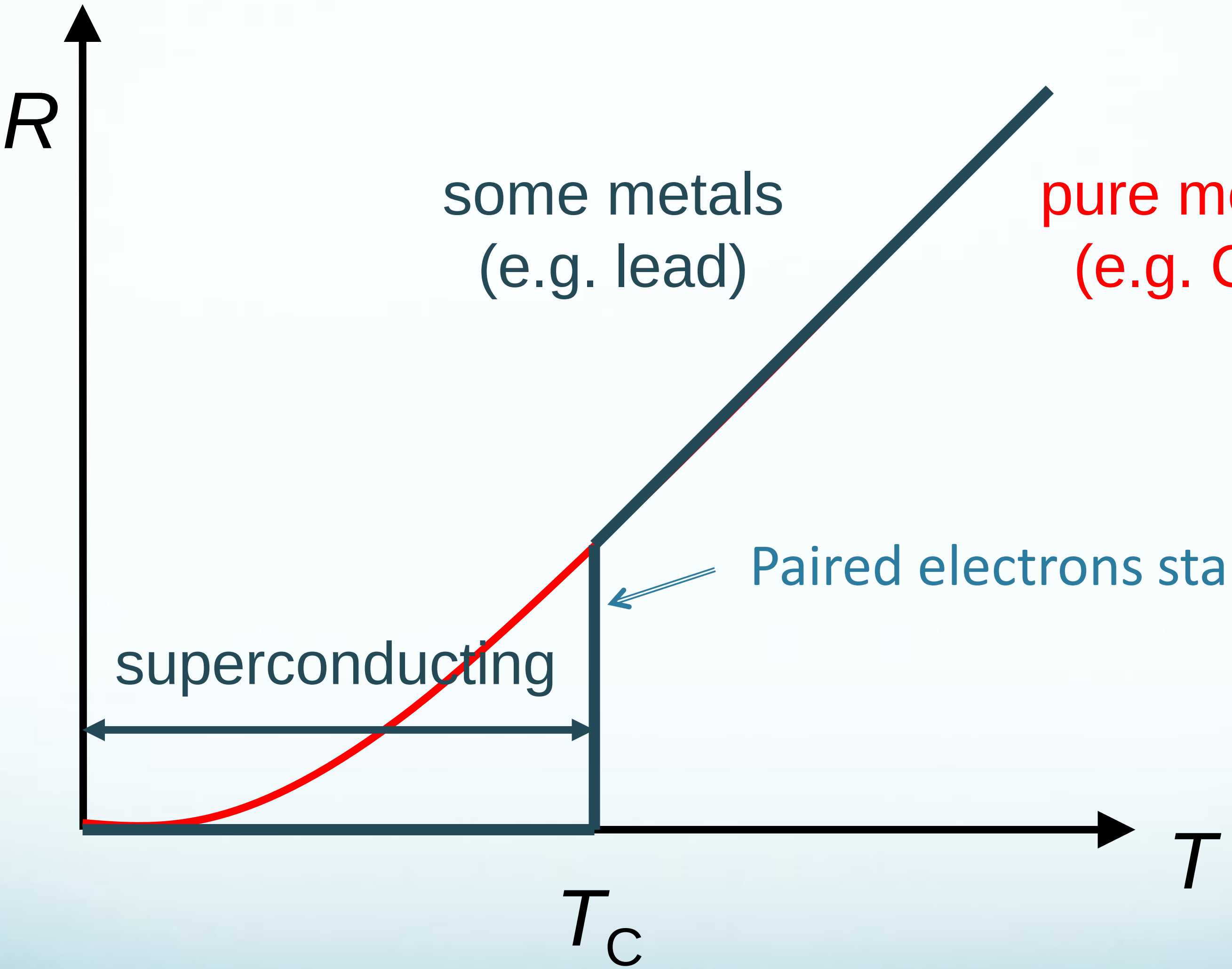
October 2009 antenna coupled MKID and LEKID demonstrate 2mm observations on the IRAM telescope.

2009 - 2010 SRON presenting electrical NEPs for antenna coupled MKIDs in the 10^{-19} W/Hz$^{0.5}$ range.

2010 Caltech JPL present TiN material with measured electrical NEP of MKID resonators of 3×10^{-19}. This material is ideal for the LEKID.

2010 NIKA team return to IRAM with a 2 channel system (150 GHz 220GHz) and demonstrate the best on sky sensitivity of a KIDs camera to date (only a factor of two within the photon noise limit)

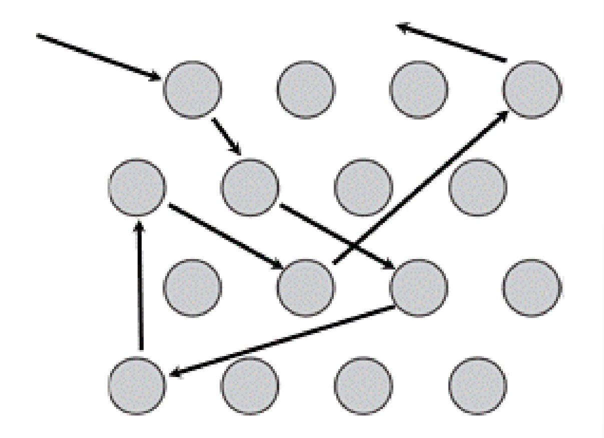
The complex conductivity of Superconductors



Physical properties radically change at T_c but to continue to vary down to zero Kevin

The conductivity of normal metals (Drude model)

$$\sigma_n = \frac{\sigma_0}{1 - j\omega t}$$



$$\sigma_n = \frac{n_n e^2 \tau}{m(1+\omega^2\tau^2)} - j\frac{n_n e^2 \omega \tau^2}{m(1+\omega^2\tau^2)}$$

$m = 9.1x10^{-31}$
$n_n \approx 10^{29}$

τ is typically of order 10^{-14}s and at practical frequencies, say f=5GHz ($\omega = 3x10^{10}$) leaving $\omega^2\tau^2 << 1$ and the imaginary term negligible.

$$\sigma_n = \frac{n_n e^2 \tau}{m(1+\cancel{\omega^2\tau^2})} - j\frac{n_n \cancel{e^2 \omega \tau^2}}{m(1+\cancel{\omega^2\tau^2})} \Rightarrow \sigma_n = \frac{n_n e^2 \tau}{m}$$

Conductivity of non-scattering electrons

The phenomena of superconductivity arises from the non-scattering properties of the superconducting electron population n_s.

Consider the Drude model where $\tau \rightarrow \infty$ to denote a non-scattering electron population n_s.

$$\sigma_s = \frac{n_s e^2 \tau}{m(1+\omega^2\tau^2)} - j\frac{n_s e^2 \omega \tau^2}{m(1+\omega^2\tau^2)}$$

$$\sigma_1 = \frac{n_s e^2}{\frac{m}{\tau} + m\omega^2\tau} \xrightarrow{\tau\rightarrow\infty} 0$$

$$\sigma_2 = -j\frac{n_s e^2 \omega}{\frac{m}{\tau^2} + m\omega^2} \xrightarrow{\tau\rightarrow\infty} -j\frac{n_s e^2}{m\omega}$$

$$\boxed{\sigma_2 = -\mathrm{j}\frac{n_s e^2}{\omega m}}$$

The first London equation

We can write the current density **J** in a superconducting volume as the response to the superconducting electrons to an electric field **E**.

$$\mathbf{J} = \sigma \mathbf{E}$$

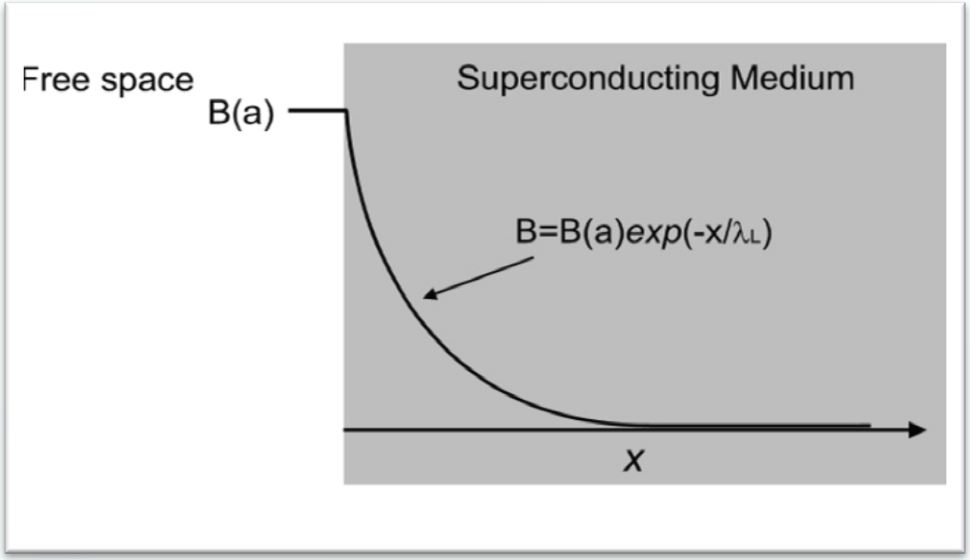
Consider the response of a superconducting to a time varying (AC) field $E = E_0 \cos(\omega t)$:

$$\mathbf{J_s} = -\mathbf{j}\frac{n_s e^2}{\omega m}\mathbf{E} = -\mathbf{j}\frac{n_s e^2}{\omega m} E_0 \cos \omega t$$

$$\boxed{\frac{d\mathbf{J}}{dt} = \frac{n_s e^2}{m}\mathbf{E}}$$

First London equation: The electrodynamics of a perfect conductor

The London penetration depth



For a non-scattering electron volume

$$\dot{\mathbf{B}}(\mathrm{x})=\dot{\mathbf{B}}(s)exp\left(\frac{-\mathrm{x}}{\sqrt{m/\mu_0 n_s e^2}}\right)$$

For a **superconducting** electron volume

$$\mathbf{B}(\mathrm{x})=\mathbf{B}(s)exp\left(\frac{-\mathrm{x}}{\sqrt{m/\mu_0 n_s e^2}}\right)$$

Applying Maxwell's equations to a perfect conductor displays diamagnitism of AC magnetic fields.

London and London suggested a set of constitutive conditions to Maxwell's equations so that both DC and AC fields as expelled from the bulk of a superconductor.

The field decays to 1/e of its value at the surface within the London penetration depth λ_L.

$$\boxed{\lambda_{\mathrm{L}} = \sqrt{\frac{m}{\mu_0 n_s e^2}}}$$

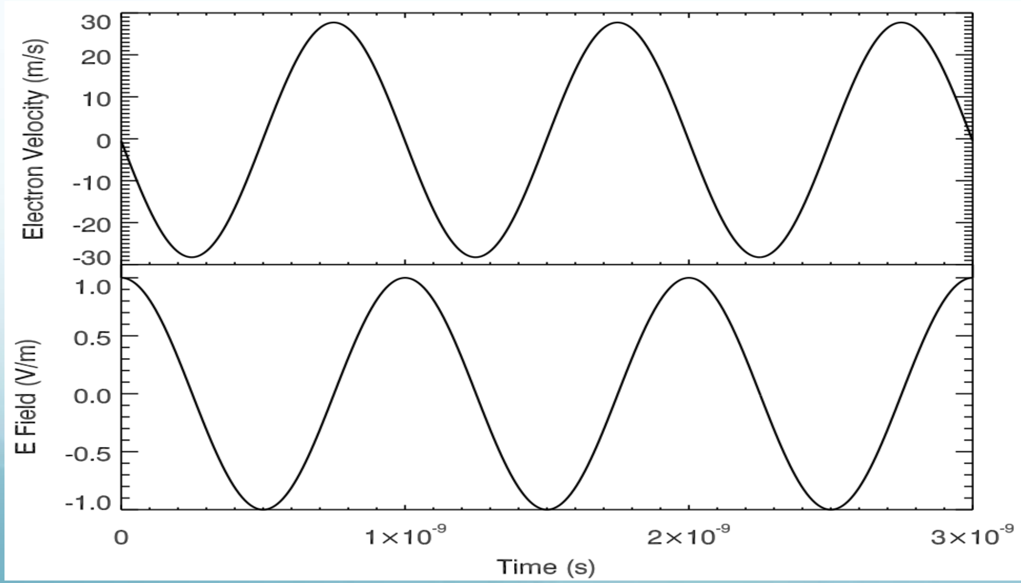
The London Penetration depth

The complex nature of σ_2

The conductivity of the non-scattering electrons (σ2) is complex. This implies that the super-current (J_s) is not in phase with the applied field **E**. Consider a field applied to a single non-scattering electron:

$$E = E_0 \cos \omega t \qquad F = -Ee = ma \rightarrow a = \frac{-eE_0 \cos \omega t}{m}$$

$$V_e = \int_0^{\pi/2\omega} \frac{-eE_0}{m} \cos \omega t = \left[\frac{-eE_0}{\omega m} \sin \omega t\right]_0^{\pi/2\omega} = \frac{-eE_0}{\omega m} \qquad P_e = \frac{-eE_0}{\omega}$$

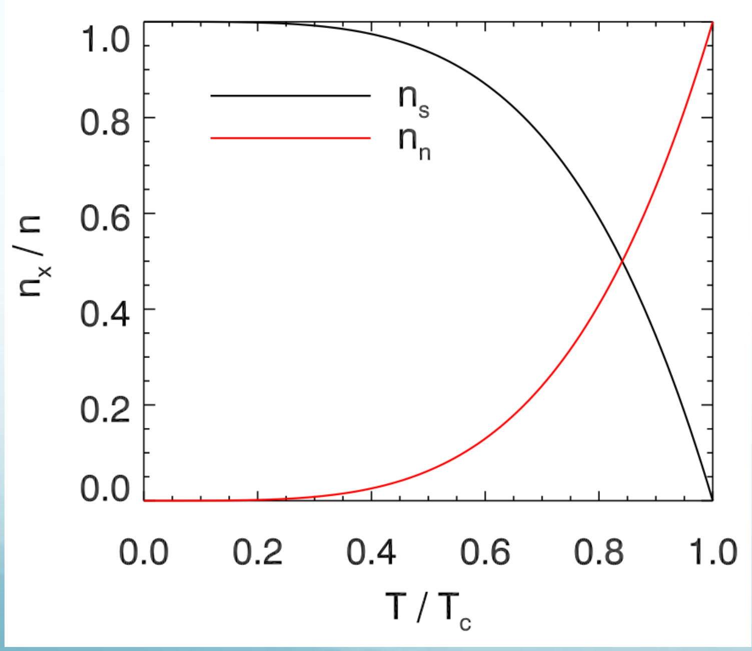


The electron will accelerate with the field gaining velocity and momentum. When the field is reversed the electron must first loose this momentum before changing direction. The current lags the field by 90° due to the kinetic energy stored within the electron – hence **KINETIC INDUCTANCE.**

The two fluid model

In 1934 Gorter and Casimir put forward the concept the two fluid model to explain the electrodynamics of a superconductor.

$$\frac{n_s}{n} = 1 - \left(\frac{T}{T_c}\right)^4 \qquad n_{qp} = n - n_s$$

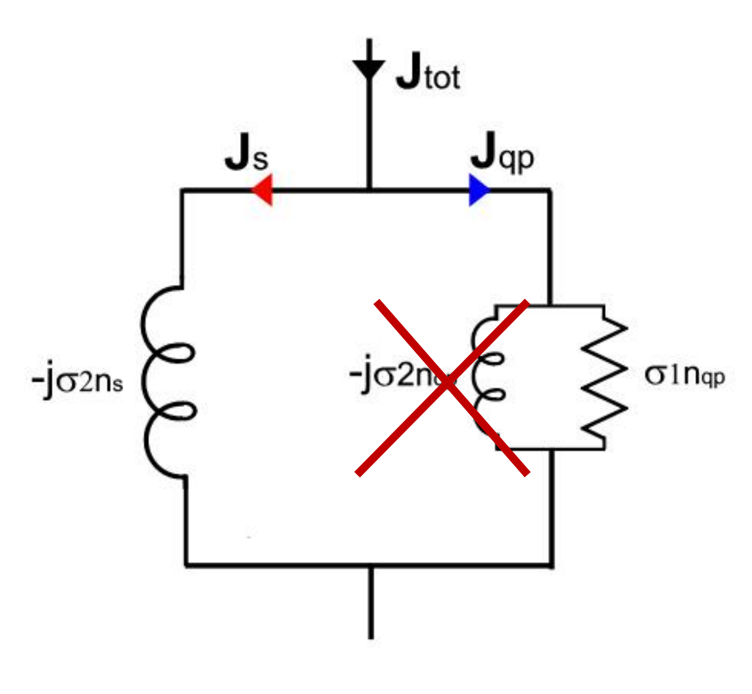


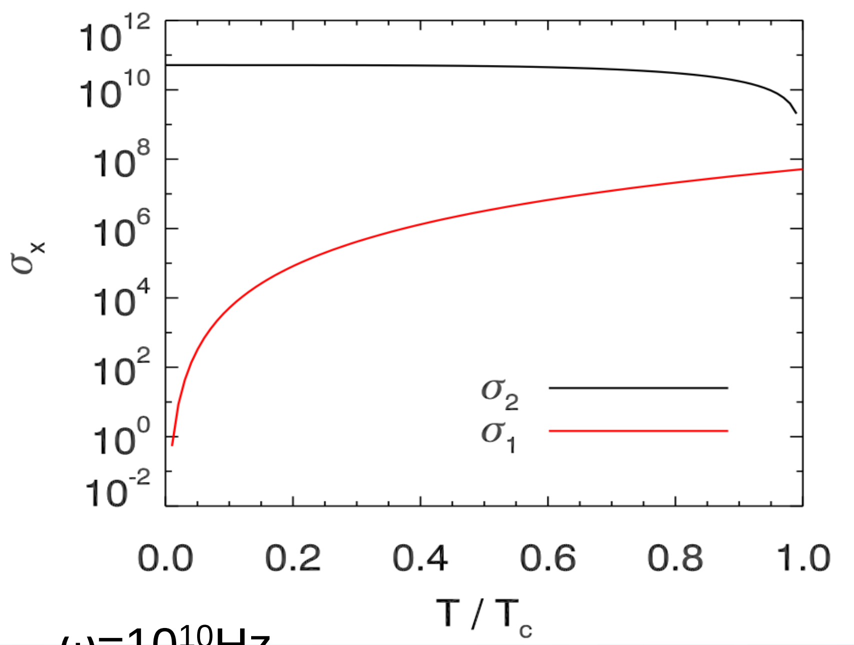
A a superconductor passes through Tc the free electrons begin to pair up to form what is know as Copper pairs. As the temperature is lowered from Tc to zero Kelvin, more electrons from non-scattering pairs until all of the free electron population is paired a T=0

The two fluid model

$$\sigma_1 = \frac{n_n e^2 \tau}{m}$$

$$\sigma_2 = -\mathrm{j}\frac{n_s e^2}{\omega m}$$





$\omega = 10^{10}$Hz

The normal state conductivity (σ_1) due n_n, reduces rapidly as we cool from T_c were as σ_2 increases. The preferred AC current path shift to σ_2 as we move towards T=0. DC resistance is always zero as any resistive path from σ_1 is shorted by the infinite conductivity of σ_2 if ω=0

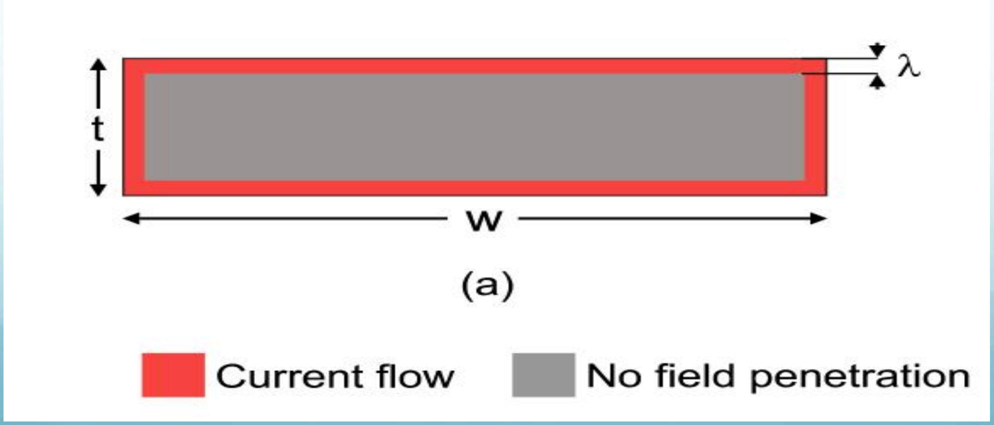
Total Internal Inductance

So far we have derived σ1 and σ2 which give the conductivities per unit volume of a scattering and non-scattering electron population. Recall the second London equation:

$$\mathbf{B}(x)=\mathbf{B}(s)exp\left(\frac{-x}{\sqrt{m/\mu_0 n_s e^2}}\right)$$

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

This property of a superconductor states that a magnetic field and hence current is limited to flow within a thin volume defined by λ_L of the surface. Looking at the cross-section of a superconducting strip give us the following model:



Total Internal Inductance

We can calculate a value of the Kinetic Inductance (L_k) by equating the kinetic energy stored in the super-current to an inductor.

The Kinetic Energy per unit volume is:

$$\mathrm{KE} = \frac{1}{2} n_s m v_s$$

The current density per unit volume is:

$$\mathbf{J}_s = -n_s e v_s$$

$$\lambda_L^2 \mu_0 = \frac{m}{\mu_0 n_s e^2}$$

We can write the kinetic energy per unit volume in terms of λ_L

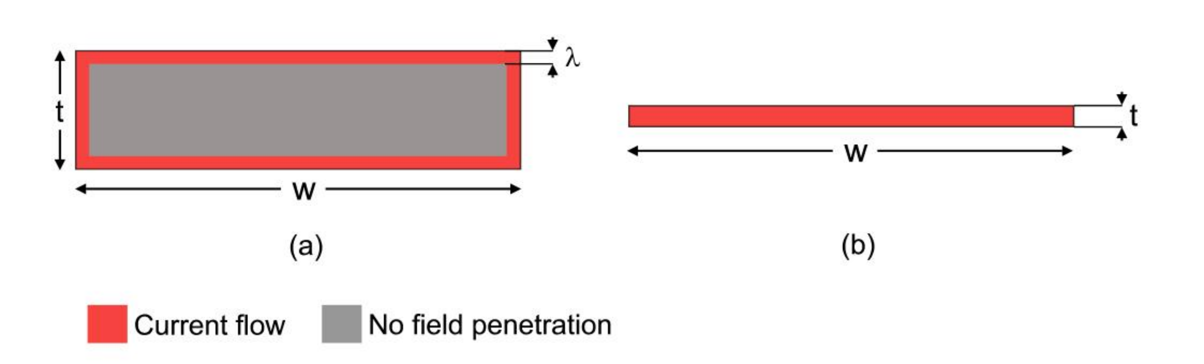
$$\mathrm{KE} = \frac{1}{2} \frac{m}{n_s e^2} \mathrm{J}_s^2 = \frac{1}{2} \mu_0 \lambda_L^2 \mathrm{J}_s^2$$

Total Internal Inductance

As stated before the current can only flow I within the volume defined by λ_L.

$$\mathrm{KE} = \frac{1}{2}\frac{m}{n_s e^2}\mathrm{J}_s^2 = \frac{1}{2}\mu_0\lambda_L^2\mathrm{J}_s^2 \qquad \mathrm{KE} = \frac{1}{2}L_k I^2 = \frac{1}{2}\mu_0\lambda_L^2\int_s J_s\,ds$$

Recalling that the current density J=I/A we can look at L_k for the following two cases of a thick and thin superconducting film of width W:



(a) $t \gg \lambda_L$ Area $\approx 2W\lambda_L$

(b) $t \ll \lambda_L$ Area $\approx Wt$

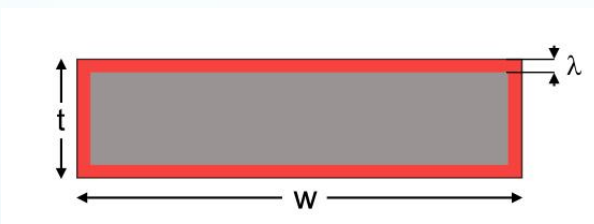
Total Internal Inductance

Equating the energy stored in and inductor with the KE for each case gives the kinetic inductance L_k per unit length for the strip.

$$\boxed{\mathrm{KE} = \frac{1}{2} L_k I^2 = \frac{1}{2} \mu_0 \lambda_L^2 \int_s J_s \, ds}$$

For the case where $\lambda_L << t$

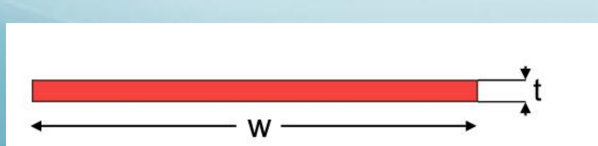
$$\int_s J_s \, ds = \mathrm{I/2W}\lambda_L$$



$$\frac{1}{2} L_k I^2 = \frac{\mu_0 I^2}{8W\lambda_L^2} 2W\lambda_L \rightarrow L_k = \frac{1}{2}\frac{\mu_0 \lambda_L}{W}$$

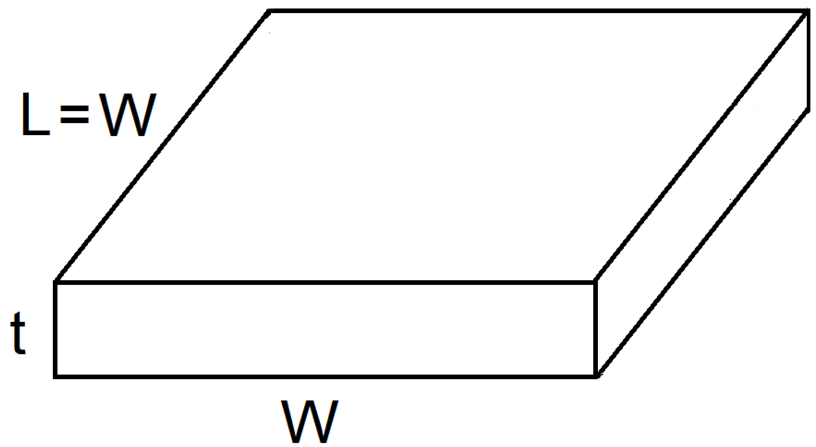
For the case where $\lambda_L >> t$

$$\int_s J_s \, ds = \mathrm{I/Wt}$$



$$\frac{1}{2} L_k I^2 = \frac{\mu_0 I^2 \lambda_L^2}{2W^2 t^2} Wt \rightarrow L_k = \frac{\mu_0 \lambda_L^2}{Wt}$$

Square Impedance



It is useful to write L_k in terms of a square impedance. Here we simply look at the impedance of a strip where the length is equal to its width (a square patch). Is the case of calculating resistance the above example becomes:

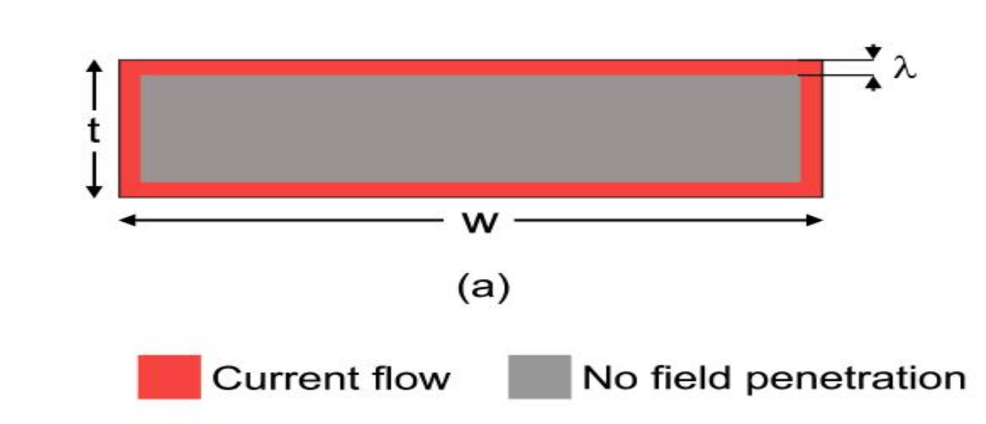
$$R = \rho \frac{L}{A} = \rho \frac{L}{Wt} \xrightarrow{L=W} sq = \rho \frac{W}{Wt} = \rho \frac{1}{t} \quad (\Omega/sq)$$

$$L_k = \frac{\mu_0 \lambda_L}{2W} \rightarrow L_k = \frac{\mu_0 \lambda_L}{2} \quad (H/sq)$$

$$L_k = \frac{\mu_0 \lambda_L^2}{Wt} \rightarrow L_k = \frac{\mu_0 \lambda_L^2}{t} \quad (H/sq)$$

Internal Magnetic Inductance (L_m)

There is also a inductive term associated with the with the magnetic energy stored within the current flowing within the the volume defined by λ_L. This internal magnetic inductance also varies with $\lambda_{L.}$

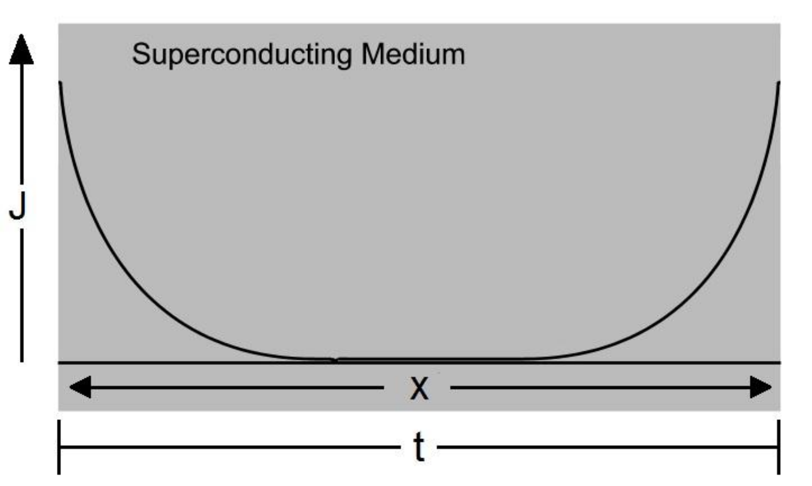


Derivation is lengthy but can be found in the literature

$$L_m = \frac{\mu_0 \lambda_L^2}{4W}\left[\cosh\left(\frac{t}{2\lambda_L}\right) - \left(\frac{t}{2\lambda_L}\right)\right]\operatorname{cosec}^2\left(\frac{t}{2\lambda_L}\right)$$

L_k and L_m for practical film thicknesses

Quite often we are working between the limits of $t \ll \lambda_L$ and $t \gg \lambda_L$. In this case we need to perform the surface integrals for current over the entire film cross-sectional area and take into account any variations in current density.

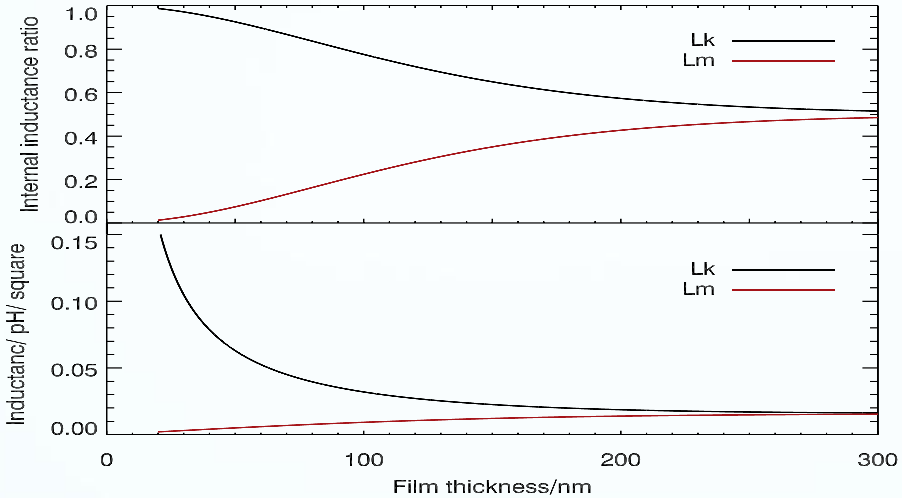


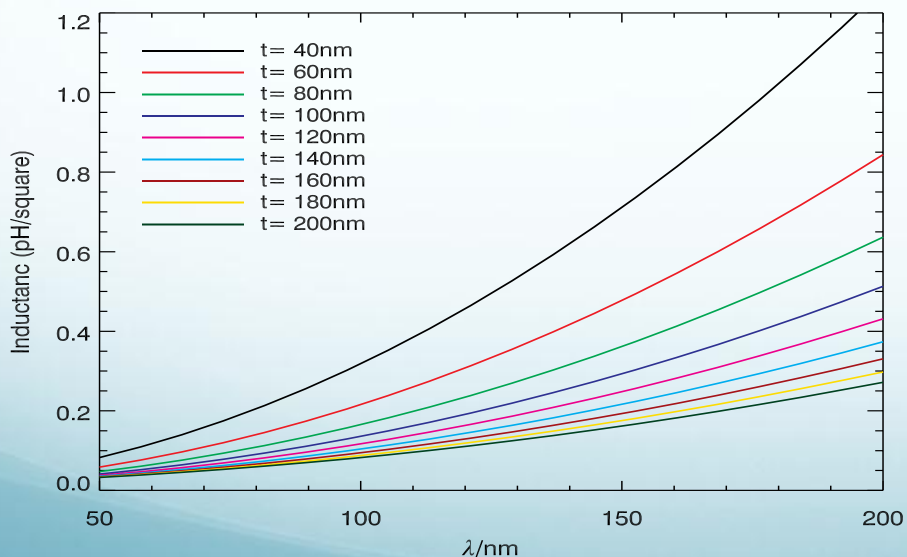
$$L_k = \frac{\mu_0 \lambda_L^2}{4W}\left[\coth\left(\frac{t}{2\lambda_L}\right) + \left(\frac{t}{2\lambda_L}\right)\right]\mathrm{cosec}^2\left(\frac{t}{2\lambda_L}\right)$$

$$L_m = \frac{\mu_0 \lambda_L^2}{4W}\left[\coth\left(\frac{t}{2\lambda_L}\right) - \left(\frac{t}{2\lambda_L}\right)\right]\mathrm{cosec}^2\left(\frac{t}{2\lambda_L}\right)$$

$$\boxed{L_{int} = L_k + L_m = \frac{\mu_0 \lambda_L}{2}\coth\left(\frac{t}{2\lambda_L}\right)}$$

Total Internal Inductance





Typical film thicknesses used for KIDs are of the order of 40nm or less. Kinetic inductance Lk dominates in this regime and Lm can generally be ignored.

Resistance of a superconducting strip

As with LK we need to derive the resistance of a superconducting strip in terms of the current density across the strip.

$$RI^2 = Real\ part \int_S \frac{J^2}{\sigma_1 - j\sigma_2} ds = \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2} \int_s J^2\, ds$$

The surface integral can be written in terms of L_k derived earlier

$$\int_s J^2\, ds = \frac{L_k I^2}{\mu_0 \lambda_L^2} \rightarrow R = \frac{L_k}{\mu_0 \lambda_L^2} \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2}$$

$$\lambda_L^2 \omega = \frac{\omega m}{\mu_0 n_s e^2} = \frac{1}{\mu_0 \sigma_2}$$

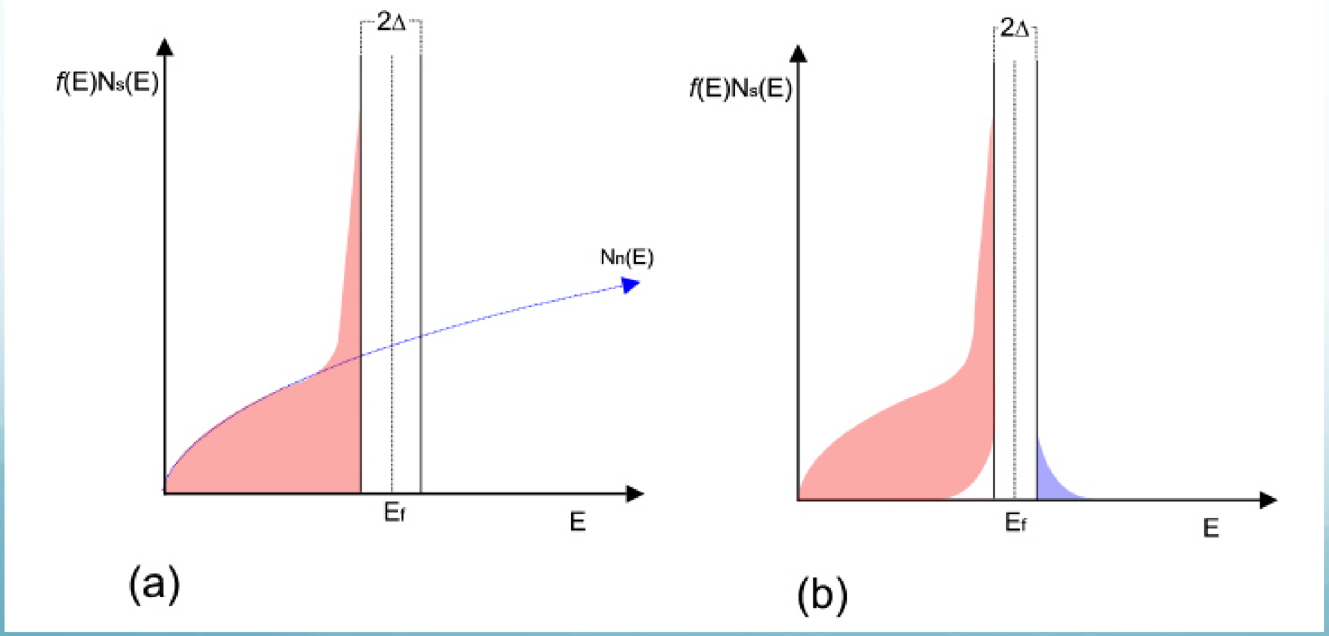
$$\boxed{R = L_k \omega \frac{\sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2} \xrightarrow{\sigma_1 \ll \sigma_2} R = L_k \omega \frac{\sigma_1}{\sigma_2}}$$

The concept of an energy gap

The non-scattering electron population consists of paired electrons known as Cooper pairs. These pairs are bound together with a binding energy 2Δ.

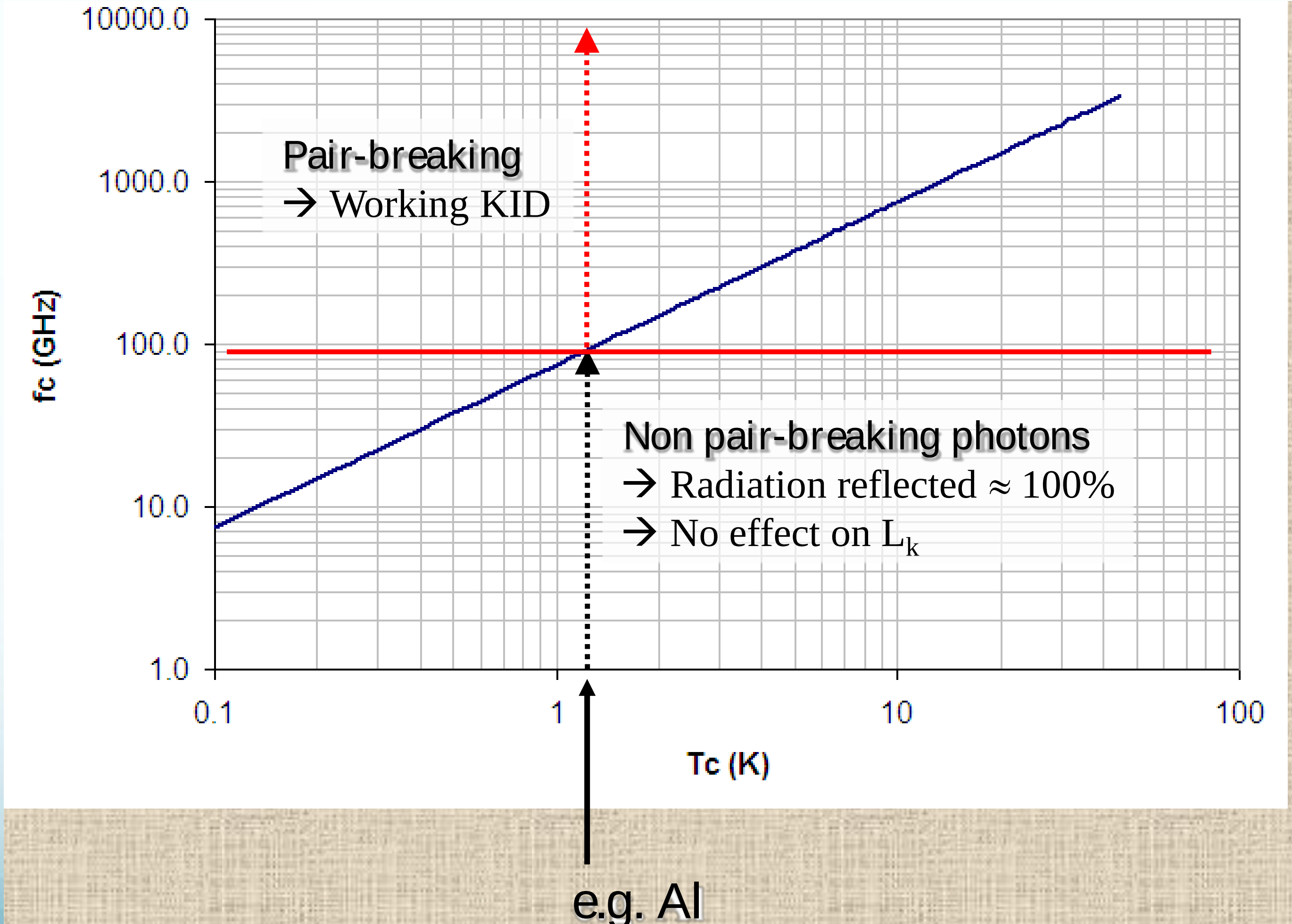
In a superconducting volume at zero K all the free electrons are paired and exist at the Fermi energy

Cooper pairs can be split by either thermal agitation (phonon E>2Δ) or by a light (E=hν> 2Δ) to form quasi-particles (non paired normal state electrons)



$$2\Delta(0) \approx 3.5 k_B T_c$$

The concept of an energy gap



Examples:

Ti → $f_c \approx$ 40GHz

Al → $f_c \approx$ 90GHz

Re → $f_c \approx$ 130GHz

Ta → $f_c \approx$ 340GHz

Nb → $f_c \approx$ 700GHz

NbN → $f_c \approx$ 1.2THz

…

Quasi-particle lifetime

A Cooper pair can be split, by say absorbing a photon of energy hf>2Δ. The time for the quasi-particle to recombine to form a Cooper pair is dictated by the quasi-particle lifetime given by Kaplan theory:

$$\frac{1}{\tau_{qp}} = \frac{\sqrt{\pi}}{\tau_0}\left(\frac{2\Delta}{k_B T_c}\right)^{\frac{5}{2}}\left(\frac{T}{T_c}\right)^{\frac{1}{2}} e^{\frac{-\Delta}{k_B T}}$$

Here τ_0 is a material dependent property

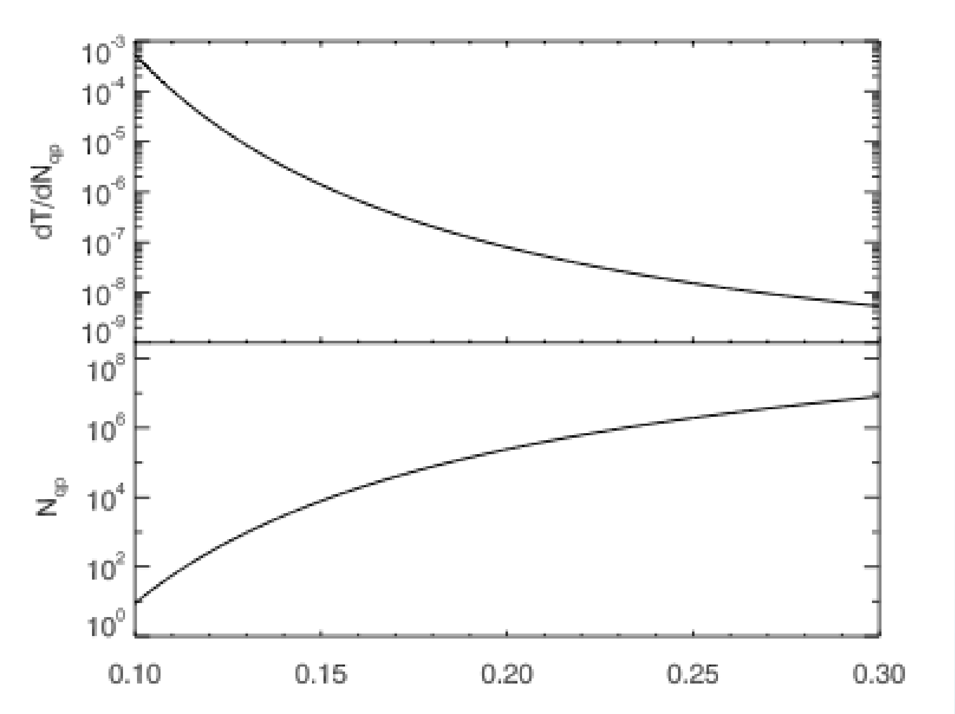
Metal	T_c/K	$\tau_0 \times 10^9 s$
Aluminium	1.19	438
Tantalum	4.48	1.78
Niobium	9.2	0.149
Tin	3.75	2.3
Zinc	0.875	780

As T is reduced the number of quasi-particle is reduced also. This means that once a pair is split it takes longer for the two quasi-particle to find a state where they have equal and opposite momenta to reform.

Quasi-Particle population

For a given temperature the quasi-particle population can be calculated using:

$$n_{qp} = 2N(0)\sqrt{2\pi k_B \Delta(0)}\; e^{\frac{-\Delta(0)}{k_B T}}$$



Effective change in T with change in N_{qp} for a 1000μ^3 volume of Aluminium.

Number of quasi-particles in a 1000μ^3 volume of Aluminium.

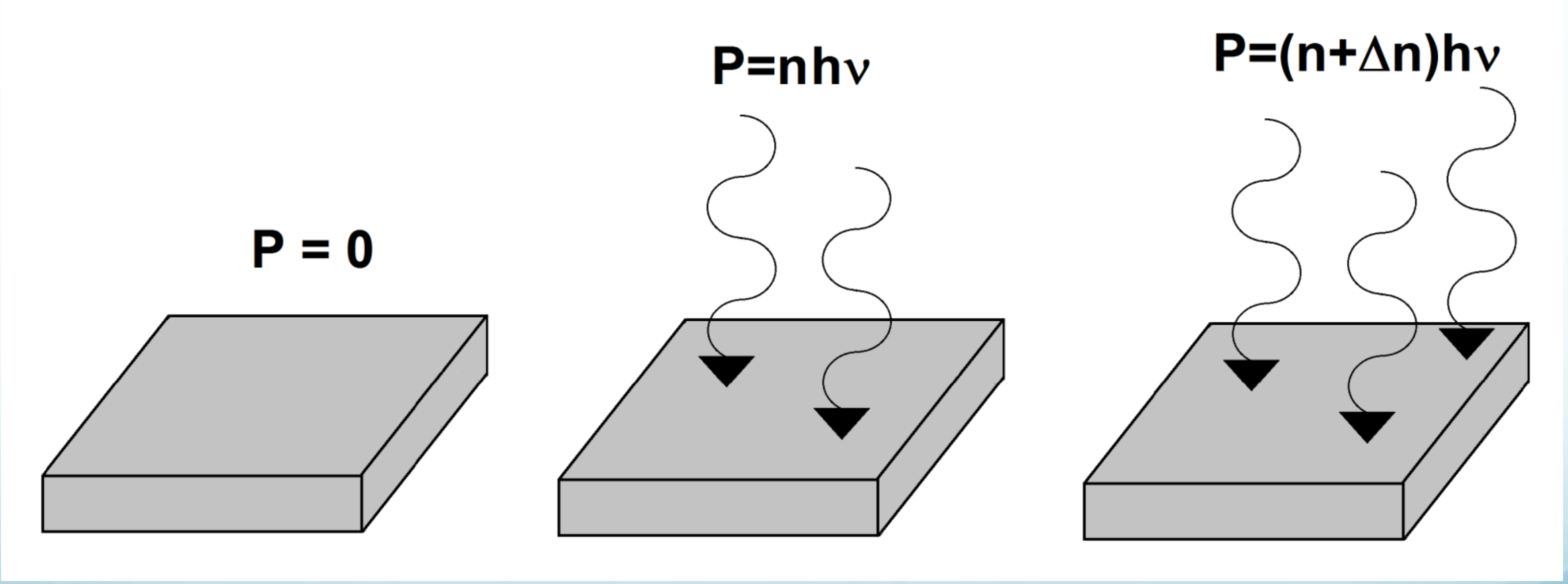
NOTE at low Temperature the effective $\Delta T/dN_{qp}$ is large!

Excess Quasi-particle population

Generally in mm and submm astronomical applications we measure the difference in power incident on a detector. It is useful therefore to look at the excess quasi-particle population within a super conducting volume.

For small amounts of optical power $\tau_{qp}(T)$ is constant so we can approximate the excess quasi-particle population as:

$$N_{xs} = \frac{\eta P \tau_{qp}}{\Delta}$$



$N_{qp} = n_{qp}(t) \times V$

$N_{qp} = (n_{qp}(t) \times V) + N_{xs}$

Mattis-Bardeen Integrals

The Mattis-Bardeen integrals take into account the existence of a band gap and the average distance between the two electrons in a Cooper pair

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty}[f(E) - f(E+\hbar\omega)]g(E)dE \quad + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta}[1 - f(E+\hbar\omega)]g(E)dE$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega,-\Delta}^{\Delta}\frac{[1-2f(E+\hbar\omega)][E^2+\Delta^2+\hbar\omega E]}{[\Delta^2-E^2]^{\frac{1}{2}}[(E+\hbar\omega)^2-\Delta^2]^{\frac{1}{2}}}dE$$

Can be approximated by:

$$\frac{\sigma_1}{\sigma_n} = \frac{2\Delta(T)}{\hbar\omega}\exp(-\Delta(0)/k_BT)K_0(\hbar\omega/2k_BT)[2\sinh(\hbar\omega/2k_BT)]$$

$$\frac{\sigma_2}{\sigma_n} = \frac{\pi\Delta(T)}{\hbar\omega}[1 - 2\exp(-\Delta(0)/k_BT)\exp(-\hbar\omega/2k_BT)I_0(\hbar\omega/2k_BT)]$$

Here K_0 and I_0 are modified bessel functions

Important results

$$\lambda_\mathrm{L} = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

The London penetration depth varies with Cooper pair density n_s

$$L_{int} = L_k + L_m = \frac{\mu_0 \lambda_L}{2} \coth\left(\frac{t}{2\lambda_L}\right)$$

The total internal inductance Lint therefore varies with pair density

$$R = L_k \omega \frac{\sigma_1}{\sigma_2}$$

There is also a resistive term associated with the quasi-particle population n_{qp}

$$\frac{1}{\tau_{qp}} = \frac{\sqrt{\pi}}{\tau_0}\left(\frac{2\Delta}{k_B T_c}\right)^{\frac{5}{2}}\left(\frac{T}{T_c}\right)^{\frac{1}{2}} e^{\frac{-\Delta}{k_B T}}$$

Pairs, once broken recombine on a timescale of order τ_{qp}

$$n_{qp} = 2N(0)\sqrt{2\pi k_B \Delta(0)}\; e^{\frac{-\Delta(0)}{k_B T}}$$

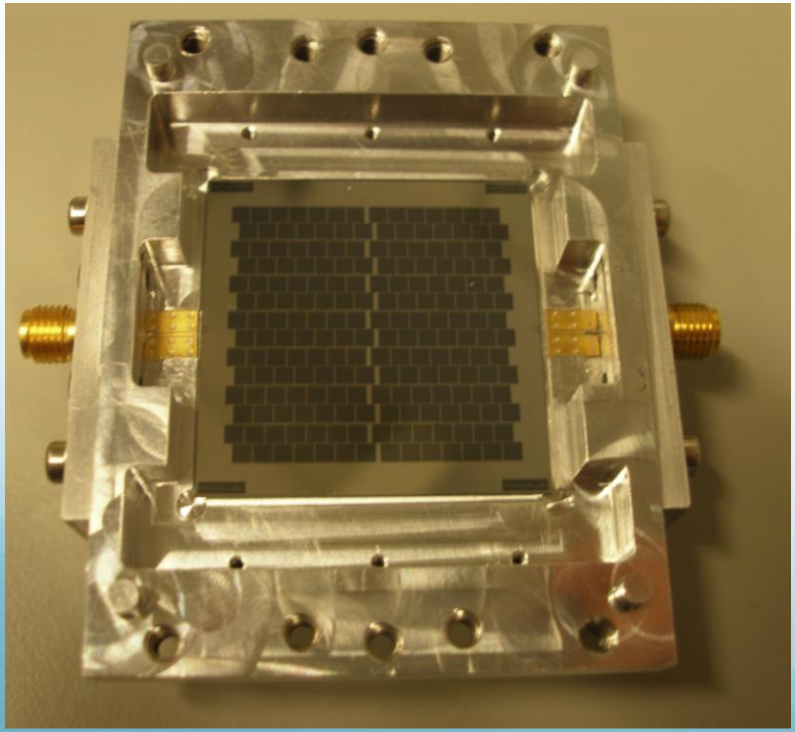
$$N_{qp} = n_{qp} Volume$$

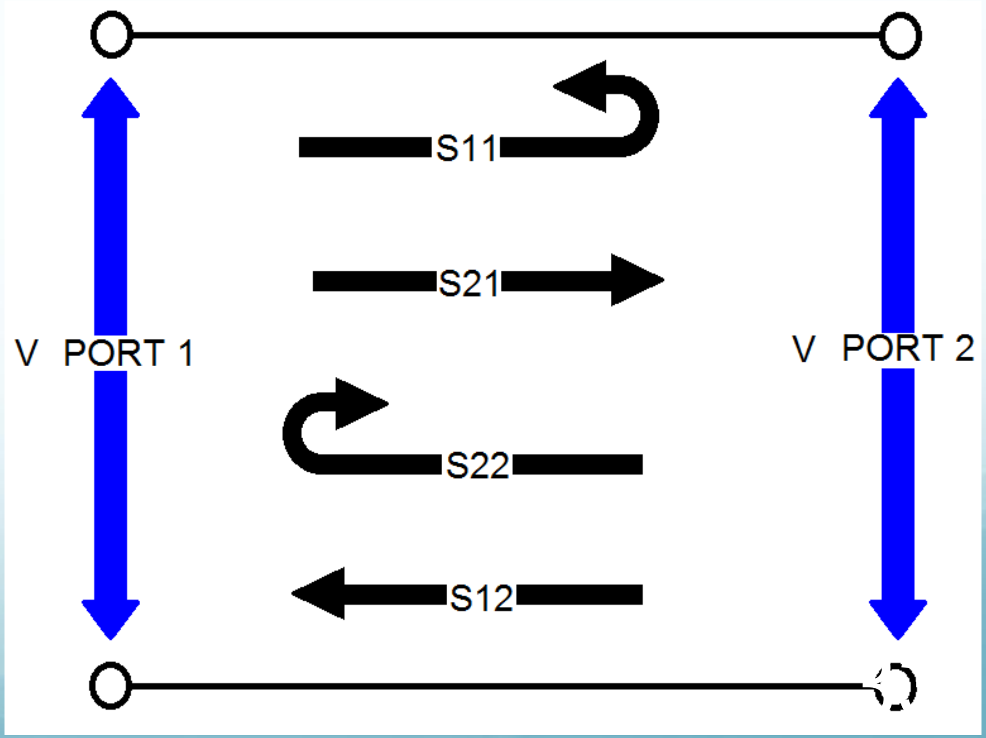
The number of quasi-particle has steep dependence on temperature at low T

Concepts in microwave measurements

Kinetic Inductance detectors work on the principle of the complex impedance of microwave resonant circuits. Therefore it is worth introducing the typical microwave tools we use to characterize and readout KIDs.

Microwave circuits are usually analyzed in terms of their scattering parameters or S-Parameters.





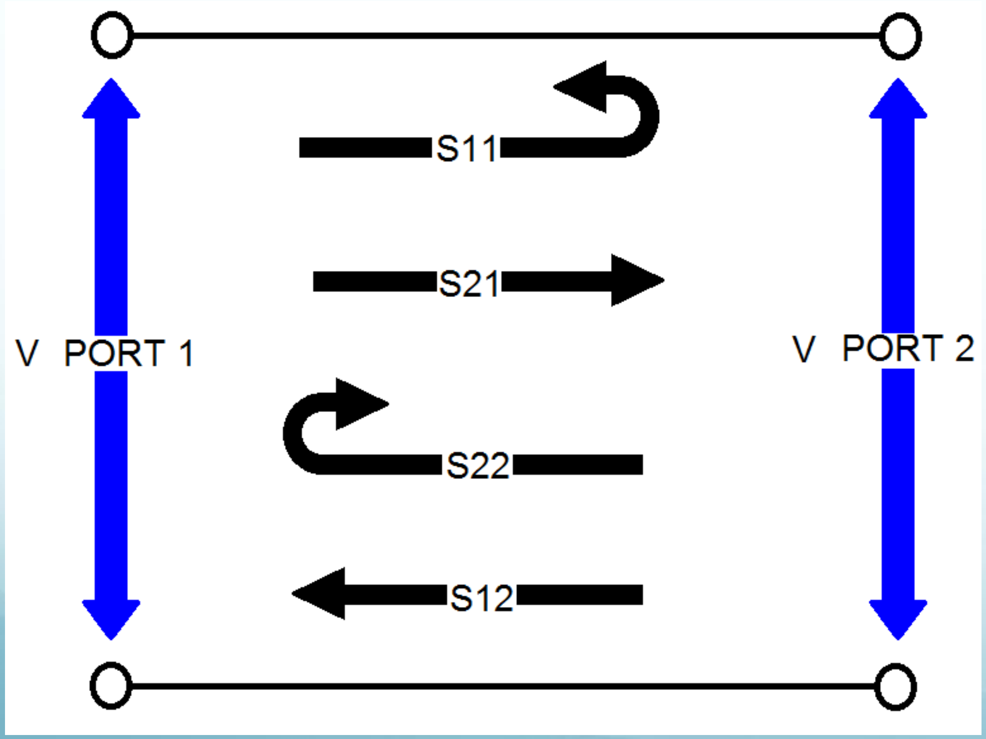
S-Parameters

$$S11 = \frac{V_1^-}{V_1^+}$$

$$S21 = \frac{V_2^-}{V_1^+}$$

$$S12 = \frac{V_1^-}{V_2^+}$$

$$S22 = \frac{V_2^-}{V_2^+}$$

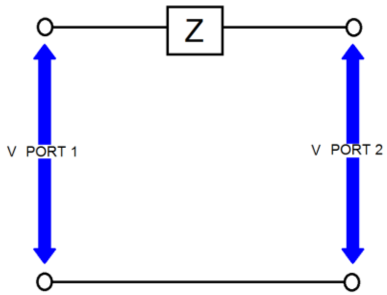


Here V_n+is the amplitude of a voltage wave incident on port n and V_n- is the amplitude of a voltage wave reflected from port n

ABCD Matrices

The ABCD matrix is a powerful tool used to analyze microwave circuits. A single circuit element in various configurations can be characterized by an equivalent ABCD matrix. Some typical circuit configurations are shown below.

1

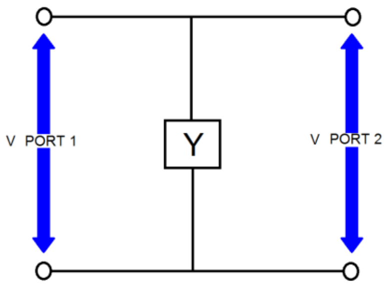


$A = 1 \quad B = Z$

$C = 0 \quad D = 1$

A series impedance Z (Z=R+jX)

2

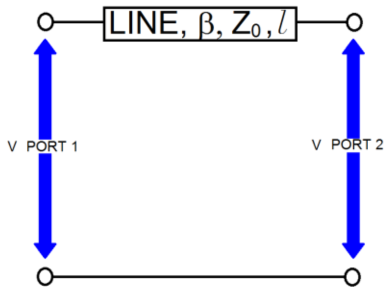


$A = 1 \quad B = 0$

$C = Y = 1/Z \quad D = 1$

A shunt impedance Z (Y=1/Z)

3



$A = \cos(\beta l)$

$B = jZ_0 \sin(\beta l)$

$C = jY_0 \sin(\beta l)$

$D = \cos(\beta l)$

$Y_0 = 1/Z_0$

A transmission line section

Calculating S-parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

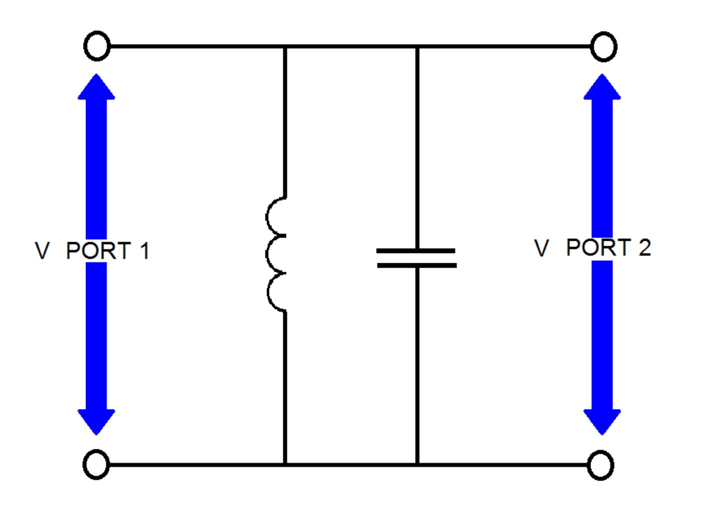
$$S11 = \frac{A + BZ_0 - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S12 = \frac{2(AD - BC)}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S21 = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S22 = \frac{-A + \frac{B}{Z_0} - CZ_0 + D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

A worked example



L=1x10^{-9} H = 1nH

C=1X10^{-12} F = 1pF

X_L=jωL ➔ Y = 1/jωL

X_C=1/jωC ➔ Y=jωC

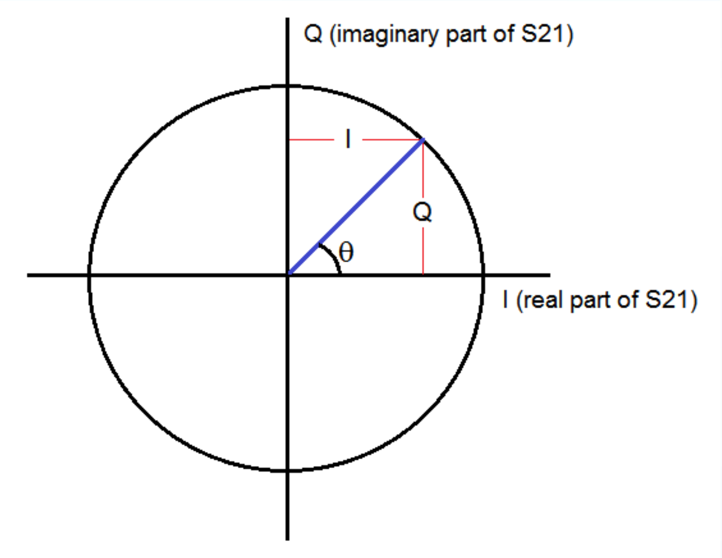
$$\begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ Y_L & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_c & 1 \end{bmatrix} = \begin{bmatrix} 1\times1 + 0\times Y_C & 1\times0 + 0\times1 \\ Y_L\times1 + 1\times Y_C & Y_L\times0 + 1\times1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_L + Y_C & 1 \end{bmatrix}$$

$$S21 = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D} = \frac{2}{2 + Z_0(Y_L + Y_C)}$$

A worked example

$$S21 = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D} = \frac{2}{2 + Z_0(Y_L + Y_C)}$$

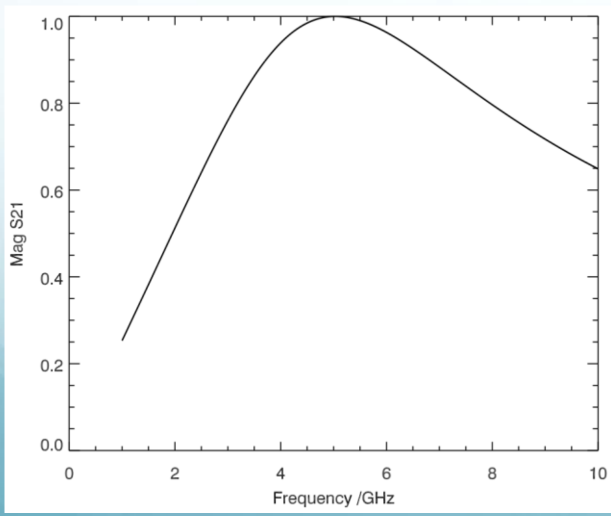


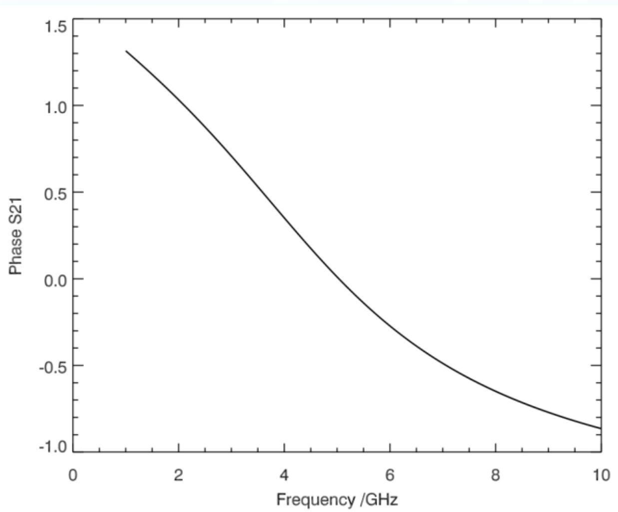
$$S21_{MAG} = \sqrt{S21^2}$$

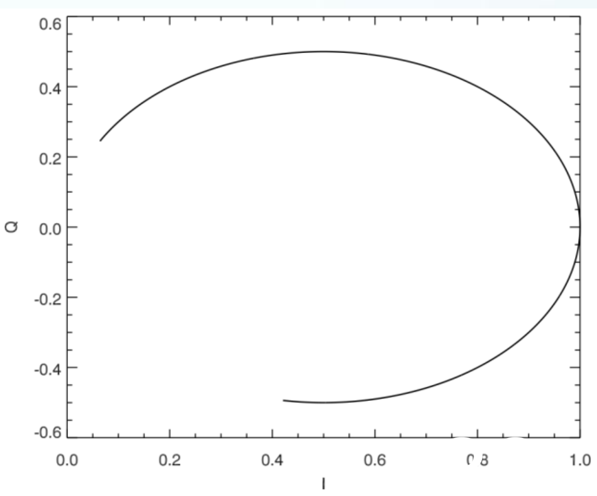
$$I = Real\ (S21)$$

$$Q = Imaginary\ (S21)$$

$$Phase\ S21 = \tan^{-1}\frac{Q}{I}$$







Resonator Theory

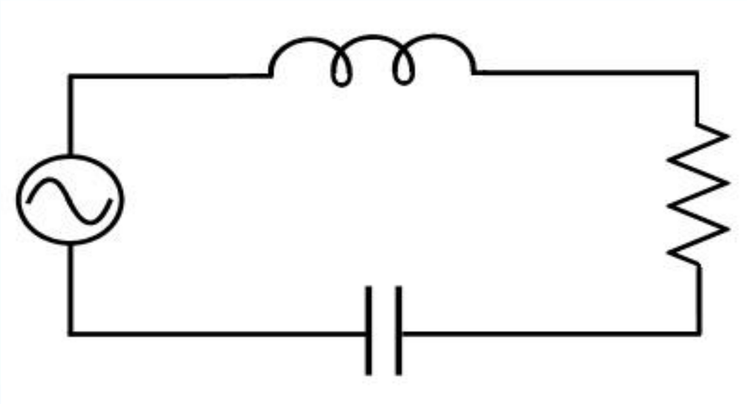
To accurately measure the variation in complex of superconductors, micro-resonant structures are made. Here we will examine the series LRC circuit which applies to the LEKID architecture.

Average energy stored in the capacitor

$$W_e = \frac{1}{4} C|V|^2$$

Average energy stored in the inductor

$$W_m = \frac{1}{4} L|I|^2$$



At resonance (ω_0) the energy stored in both the inductive and capacitive elements are equal hence we can define ω_0 by:

$$W_m = W_e = \frac{1}{4} L|I|^2 = \frac{1}{4} C|V|^2 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Quality factor Q

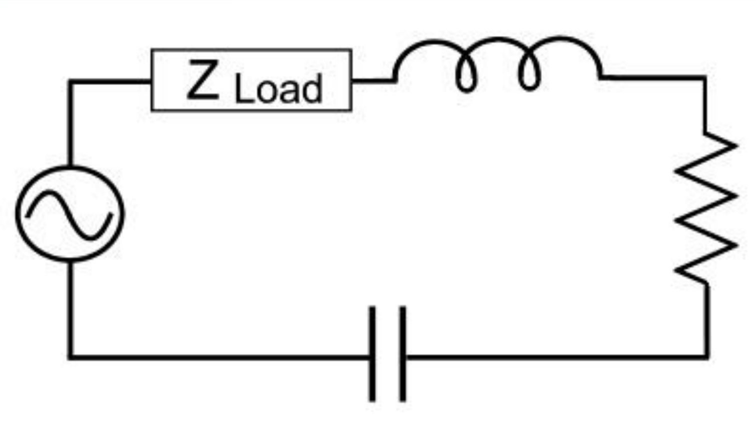
$$Q = \frac{\text{Average energy strored}}{\text{Average Power dissipated}}\omega_0 = \frac{W_m + W_e}{P_L}\omega_0$$

$$Q_u = \frac{2W_m}{P_L}\omega_0 = \omega_0 \frac{\frac{1}{2}L|I|^2}{\frac{1}{2}|I|^2 R} = \omega_0 \frac{L}{R}$$

The quality factor calculated here is know and the un-loaded quality factor Q_u. Here only the components of the resonator effect Q.

Loaded Quality Factor (Q_L)

In reality to drive a resonator we need to couple it to a supply. The impedance of the supply adds to the overall loss of the resonator lowering the overall Q.



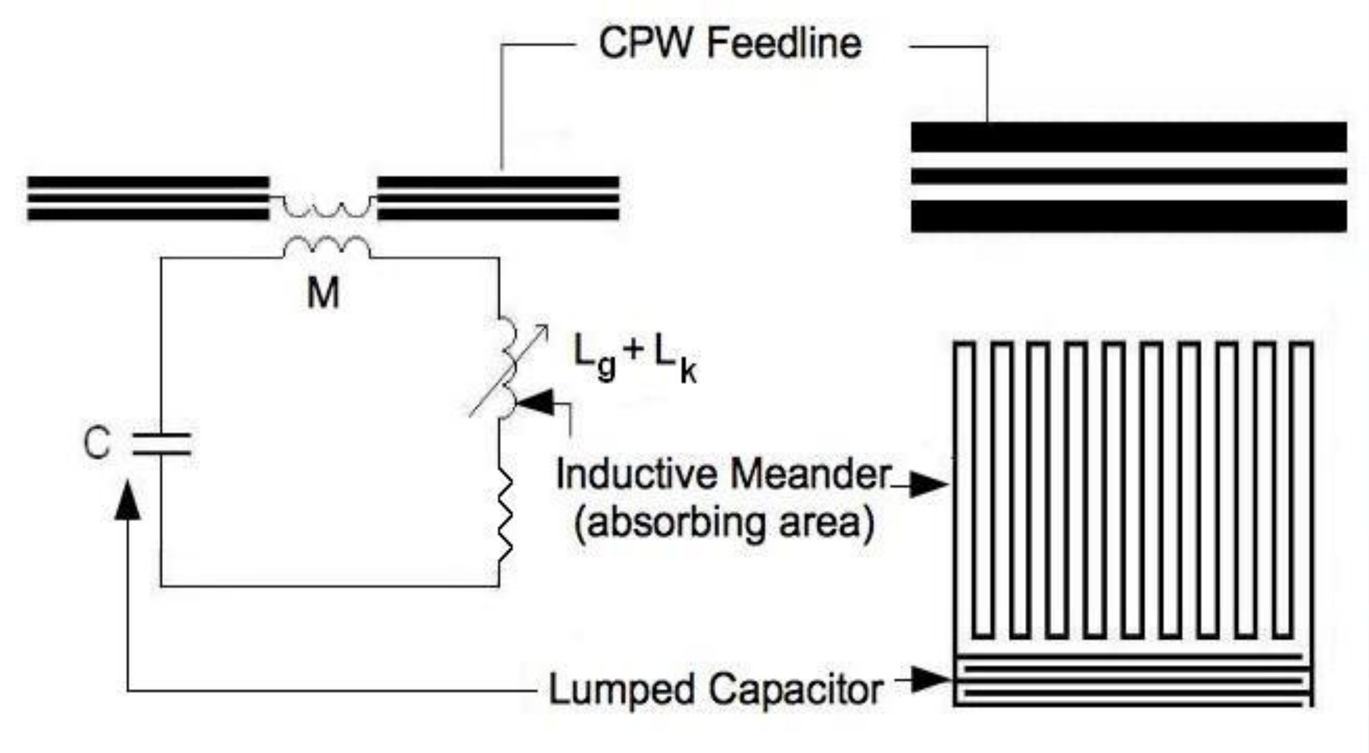
$$Q_L = \omega_0 \frac{L}{R + R_L}$$

We often attribute the reduction in Q by defining an external Q, Q_e.

$$Q_e = \omega_0 \frac{L}{R_L}$$

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_u}$$

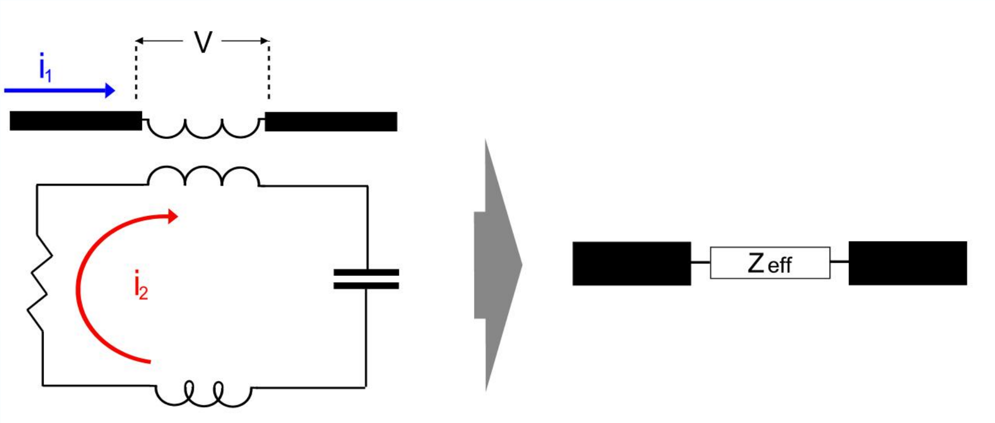
The Lumped Element Resonator



$$\omega_0 = \frac{1}{\sqrt{(L + L_k)C}}$$

S-parameters of the Lumped Element Resonator

We can study the S-parameters of the Lumped Element Resonator by describing it as a series complex impedance as follows:



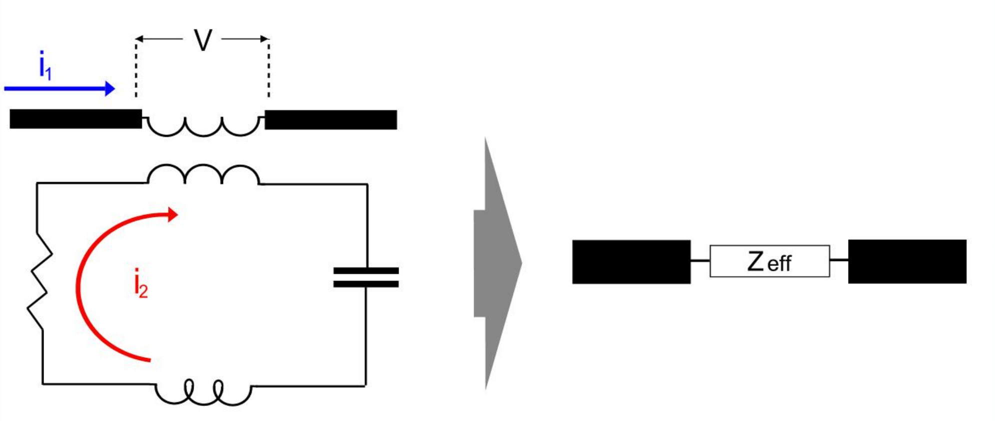
$$V_{line} = jMi_2$$

$$j\omega M i_1 + i_2 Z_{res} = 0 \rightarrow i_2 = \frac{j\omega M i_1}{Z_{res}}$$

$$V_{line} = j\omega M \frac{-j\omega M i_1}{Z_{res}} = \frac{\omega^2 M^2 i_1}{Z_{res}}$$

$$Z_{eff} = \frac{V}{i_1} = \frac{\omega^2 M^2}{Z_{res}}$$

S-parameters of the Lumped Element Resonator



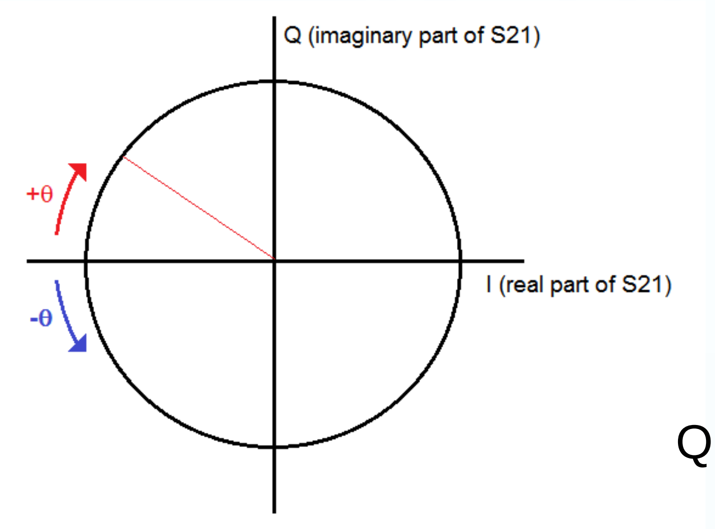
$$Z_{eff} = \frac{V}{i_1} = \frac{\omega^2 M^2}{Z_{res}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_{eff} \\ 0 & 1 \end{bmatrix}$$

$$Z_{res} = j\omega L + \frac{1}{j\omega C} + R$$

$$\boxed{S21 = \frac{2}{2 + \frac{Z_{eff}}{Z_0}} = \frac{1}{1 + \frac{\omega^2 M^2}{2 Z_{res} Z_0}}}$$

Effects on coupling on S21

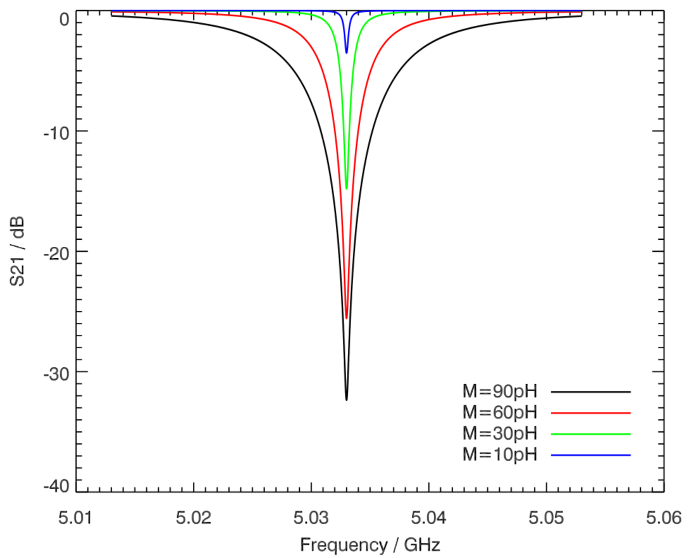


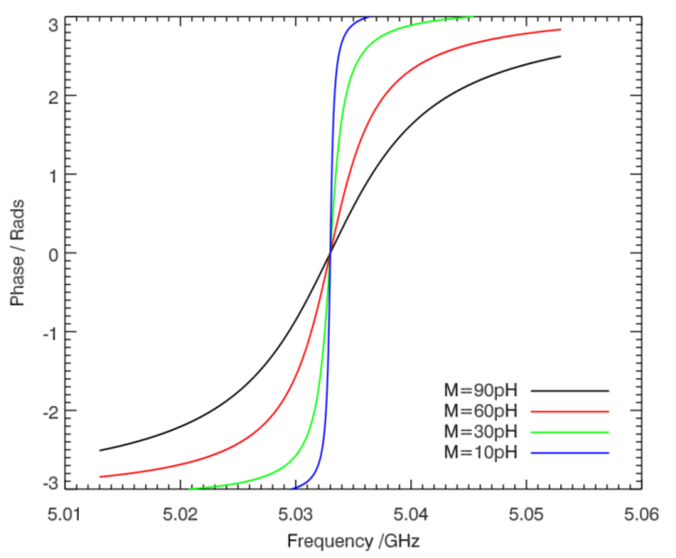
$$S21_{dB} = 10\log_{10} S21_{mag}$$

M=90pH QL ≈ 380

M=60pH QL ≈ 830

Q_L is proportional to $1/M^2$





The Lumped Element Resonator as a detectors (LEKID)

$$S21 = \frac{2}{2 + \frac{Z_{eff}}{Z_0}} = \frac{1}{1 + \frac{\omega^2 M^2}{2Z_{res}Z_0}}$$

$$Z_{res} = j\omega L + \frac{1}{j\omega C} + R$$

Variable through L_k

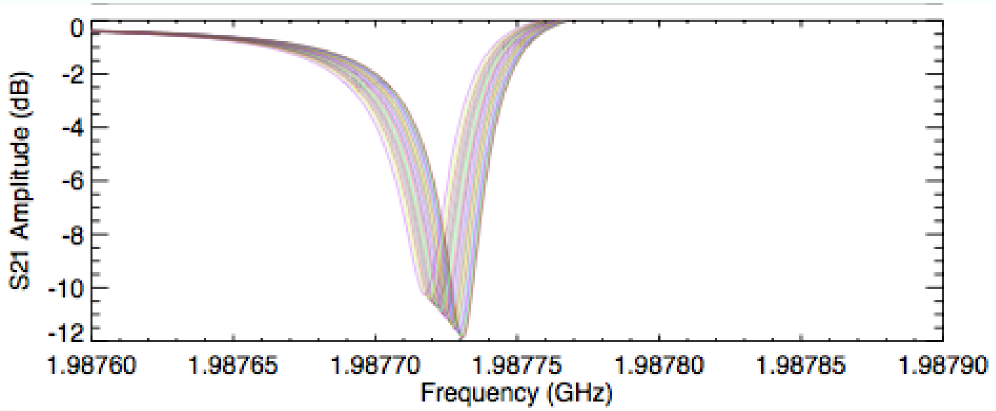
Constant

Variable

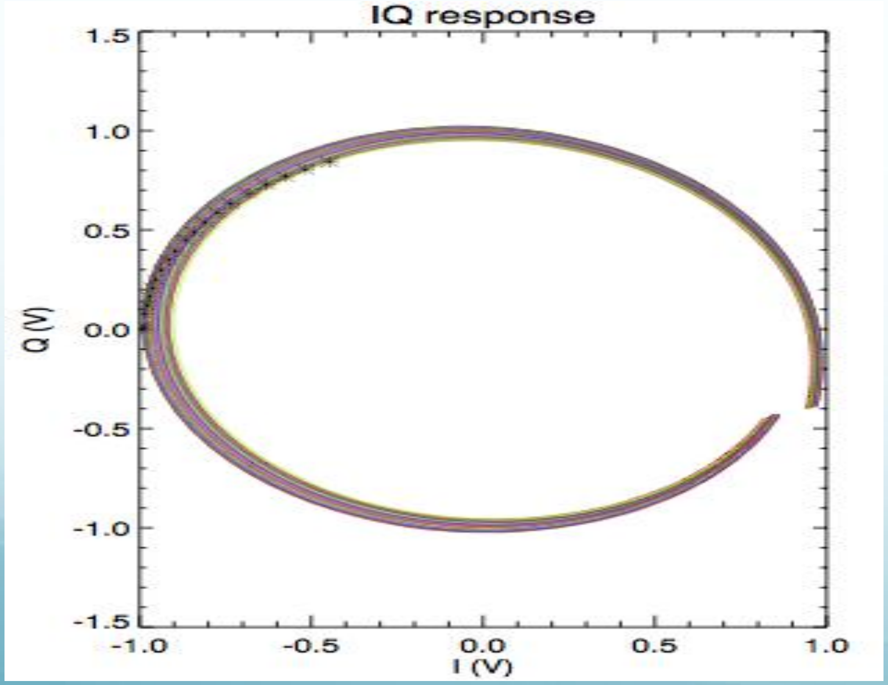
$$R = L_k \omega \frac{\sigma_1}{\sigma_2}$$

$$L_{int} = L_k + L_m = \frac{\mu_0 \lambda_L}{2} \coth\left(\frac{t}{2\lambda_L}\right)$$

Response of a LEKID to light

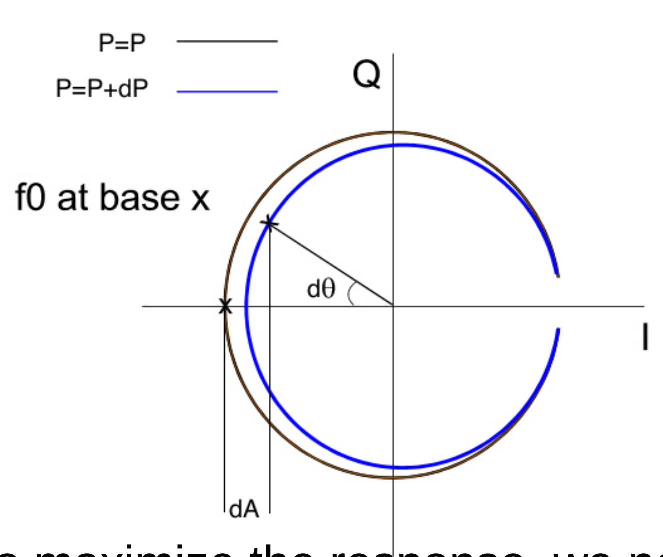


As Cooper pairs are broken L_k increases as too does R. The result is that the resonance feature shifts to lower frequencies and becomes shallower



In the IQ plane the resonant feature traces out a circle. The points marked * represent the point ω_0 under zero optical loading

How to maximize response.



Any KID device can be read out using either phase (dθ) or amplitude (dA) response. He we will look at which properties we need to address in order to maximize phase response.

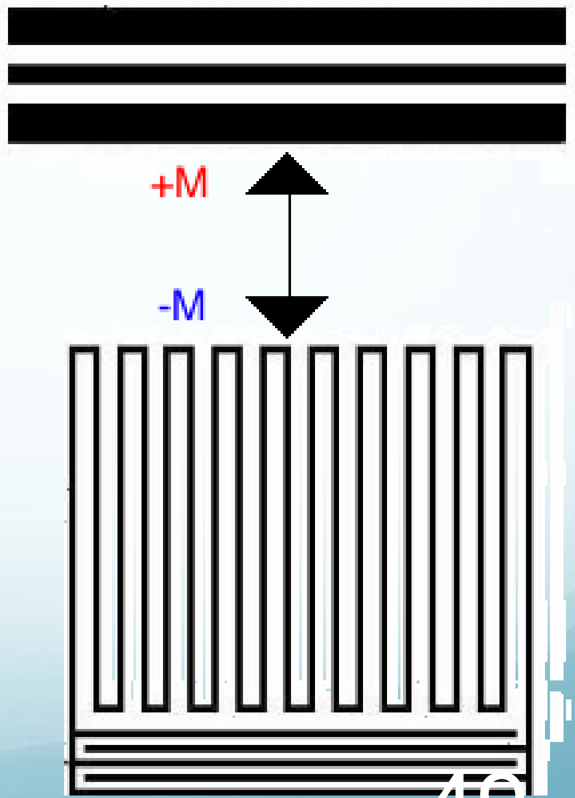
To maximize the response, we need to maximize each of the following:

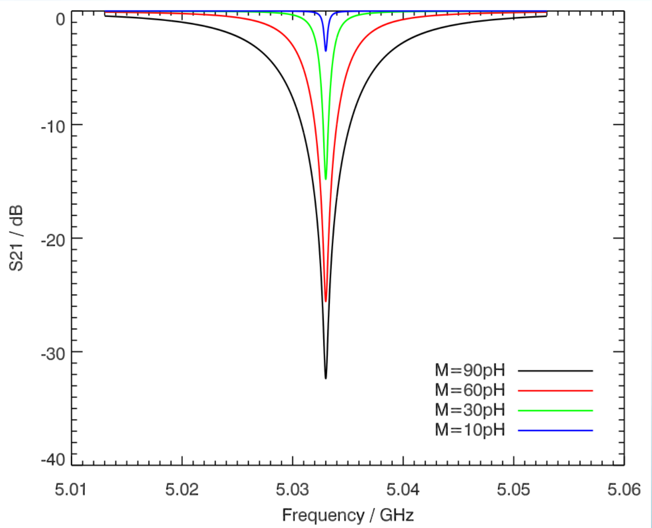
1) $d\theta/d\omega_0$ Make the change in phase for a given shift in resonant frequency as large as possible
2) $d\omega_0/dL_{tot}$ Make the change I resonant frequency as large as possible for given change in inductance.
3) $dL_{tot}/d\sigma_2$ make the change in inductance as large as possible for a give change in complex conductivity.
4) $d\sigma_2/dT$ make the change in complex conductivity as large as possible for a change in **effective temperature.**
5) dT/dN_{qp} make the change in effective temperature with quasi-particle number as large as possible

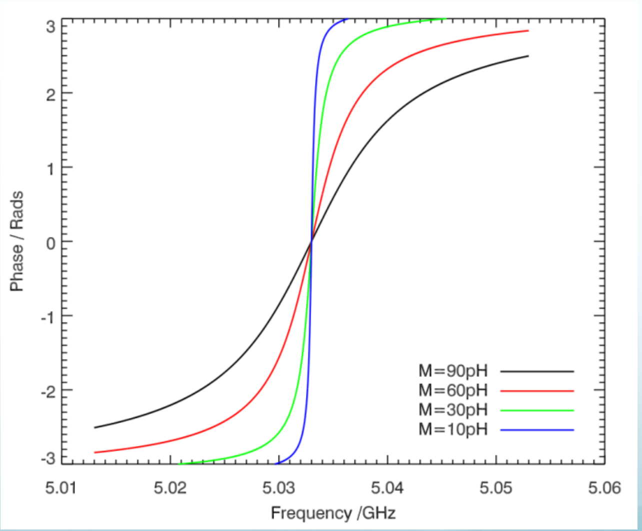
How to maximize the response of a LEKID

$d\theta/d\omega_0$ Make the change in phase for a given shift in resonant frequency as large as possible.

As shown earlier $d\theta/d\omega_0$ can be maximized by increasing Q_L which is proportional to $1/M^2$. M can be varied by moving the LEKID further from the feed-line or by changing the meander properties







How to maximize the response of a LEKID

$d\omega_0/dL_{tot}$ Make the change I resonant frequency as large as possible for given change in inductance.

$L_{tot}=(L_g+L_k)$ where L_g is the geometric inductance of the meander which does not change with pair breaking events. In order to maximize the change in resonant frequency with change in L_K, we need L_K to be large.

Defining the ratio of L_k to L_{tot} as

$$\alpha = \frac{L_k}{L_{tot}}$$

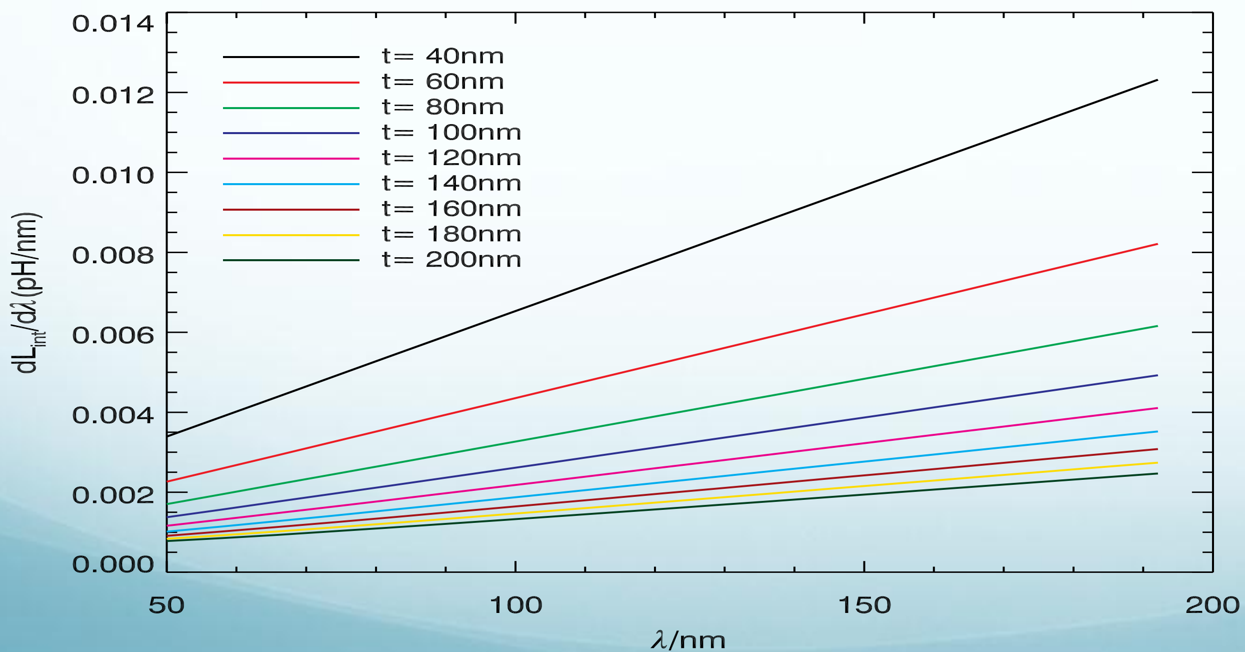
$$\frac{d\omega_0}{dL_{tot}} = -\frac{1}{2L_{tot}^{3/2}C^{1/2}} = \frac{\omega_0}{2L_{tot}} = \frac{\omega_0\alpha}{2L_k}$$

Need to maximize α by working with high kinetic inductance materials

How to maximize the response of a LEKID

$$\frac{dL_{tot}}{d\sigma_2} = \frac{-\mu_0}{8}\sqrt{2}\left[\frac{2\sqrt{\gamma}\coth\left(\frac{t}{2}\sqrt{\gamma}\right) - \gamma t + \gamma\coth\left(\frac{t}{2}\sqrt{\gamma}\right)^2}{\sqrt{\frac{\mu_0}{\sigma_2\omega}}\sigma_2\omega\sqrt{\gamma}}\right]$$

$$\sigma_2\omega_0\mu_0 = \gamma$$

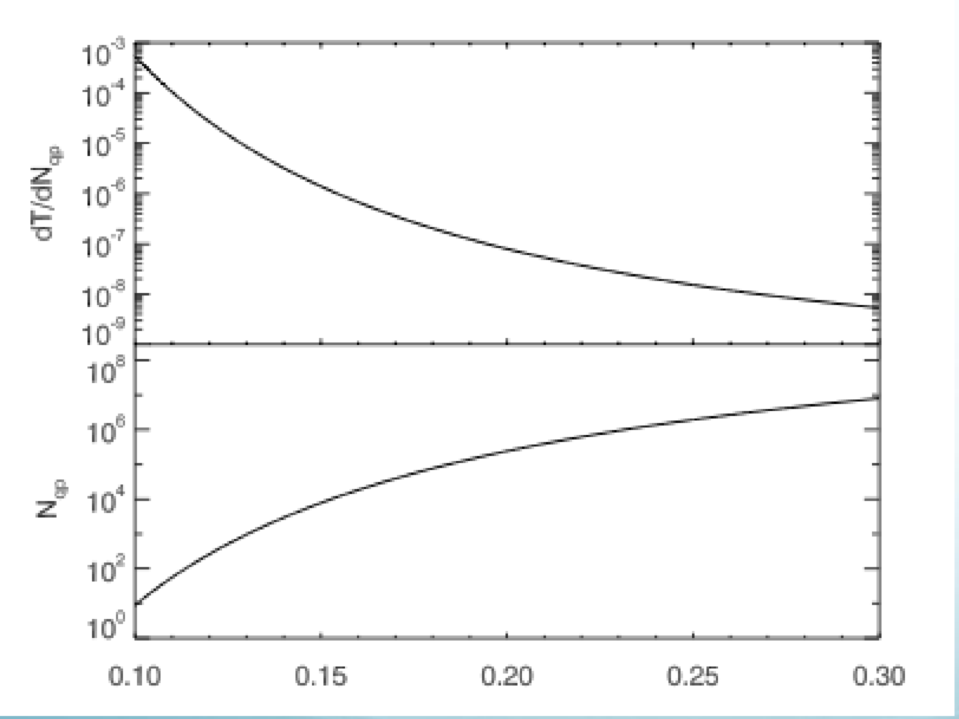


Working with thinner films increases $dL_k/d\sigma_2$

How to maximize the response of a LEKID

dT/dN_{qp} make the change in effective temperature with quasi-particle number as large as possible

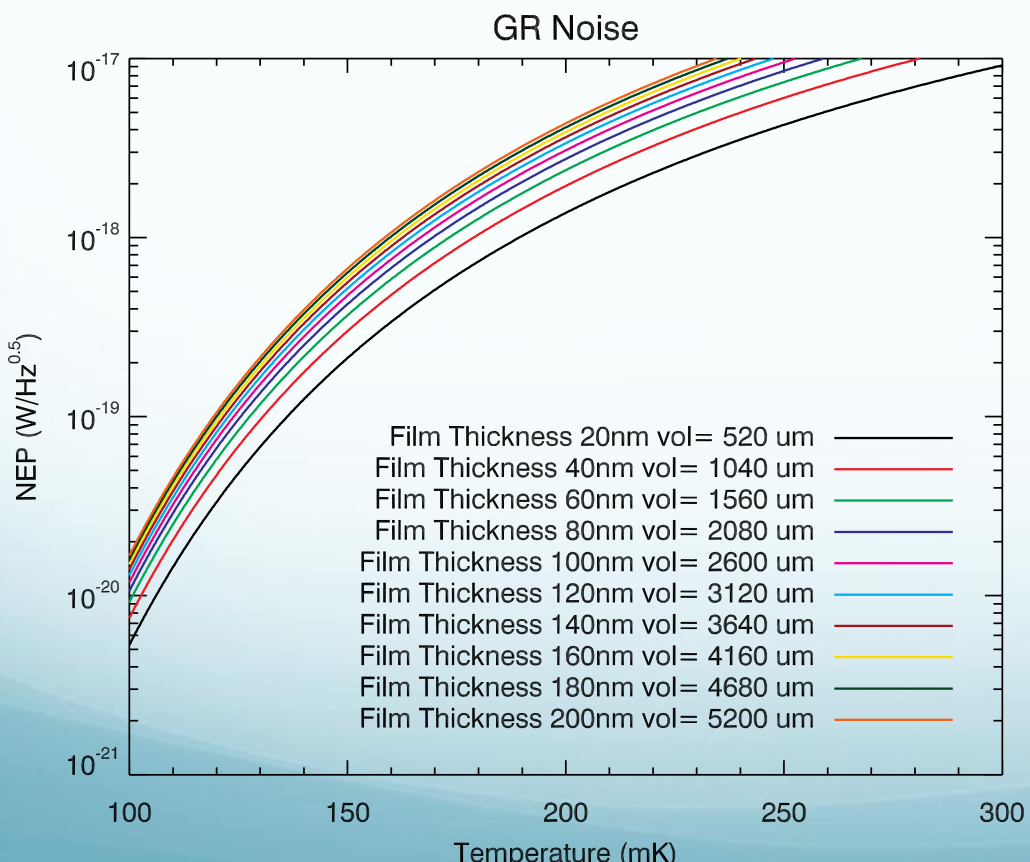
$$n_{qp} = 2N(0)\sqrt{2\pi k_B \Delta(0))}\, e^{\frac{-\Delta(0)}{k_B T}}$$



Work at low T and make film volume as small as possible

Fundamental noise in KID devices

The fundamental noise limit in any KID devices arises from generation and recombination of quasi-particles. By working at low temperatures this noise is reduced by minimizing the number of quasi-particles in the and increasing the quasi-particle lifetime.



$$NEP_{gr} = 2\Delta\sqrt{N_{qp}/\tau_{qp}}$$

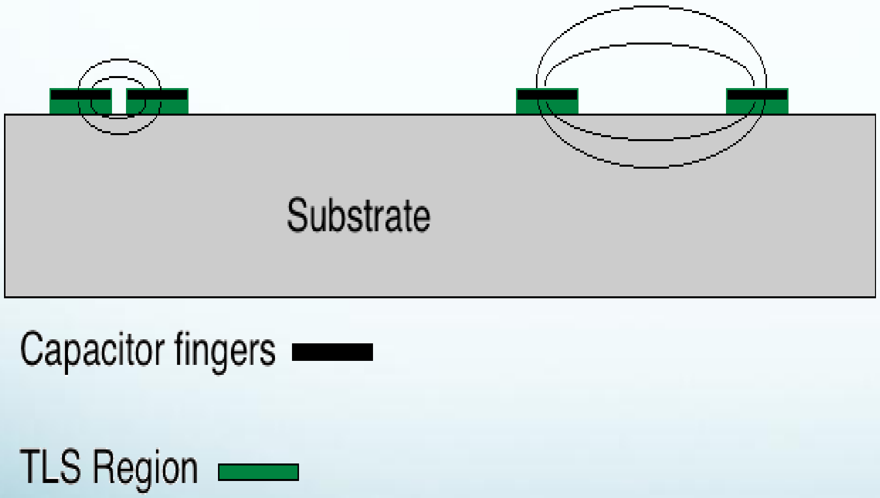
Lowest GR noise

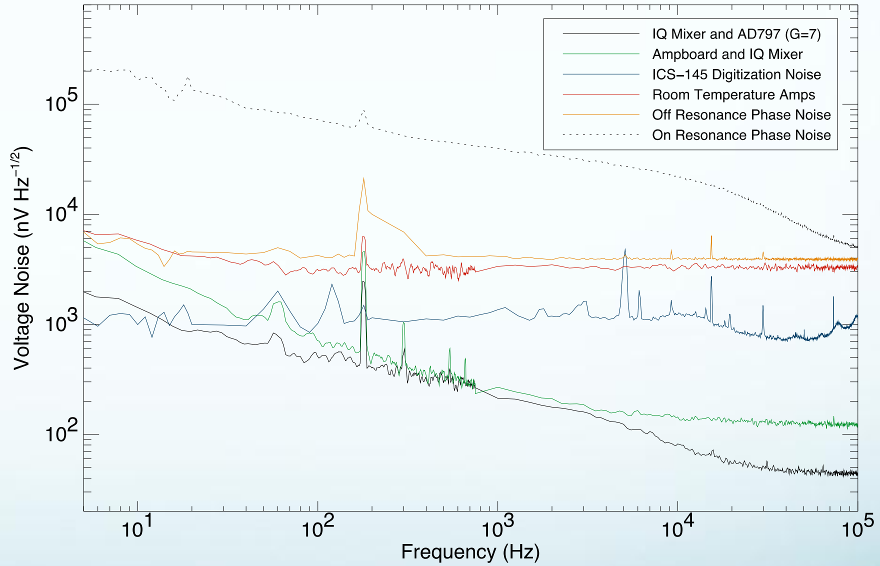
Low T

Low film volume

Excess noise in KIDs devices

Early KID devices demonstrated an excess noise in the phase direction in the IQ trajectory. This noise was attributed to the microwave field of the resonator exciting two level systems (TLS) on the surface of the superconductor. This causes a fluctuation in the dielectric constant of the oxide on the surface of the film causing the capacitance to fluctuate. This in turn causes random changes in ω0. This can be mitigated by choosing a capacitor geometry such that the electric field lines mainly exist outside of the TLS region

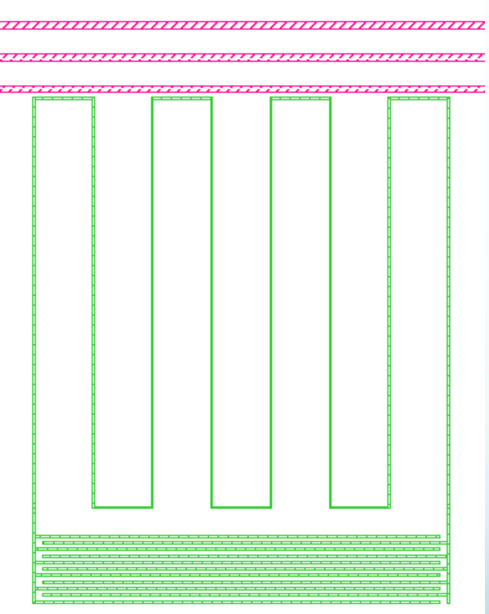




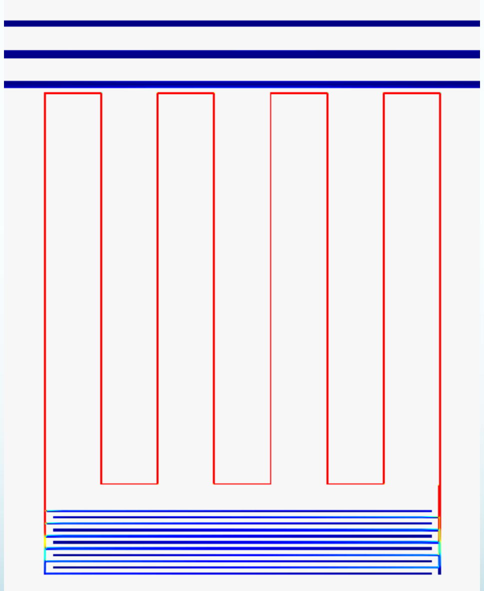
Sonnet Simulations of a LEKID

2D Microwave simulating software are extensively used in LEKID design. They can be used to

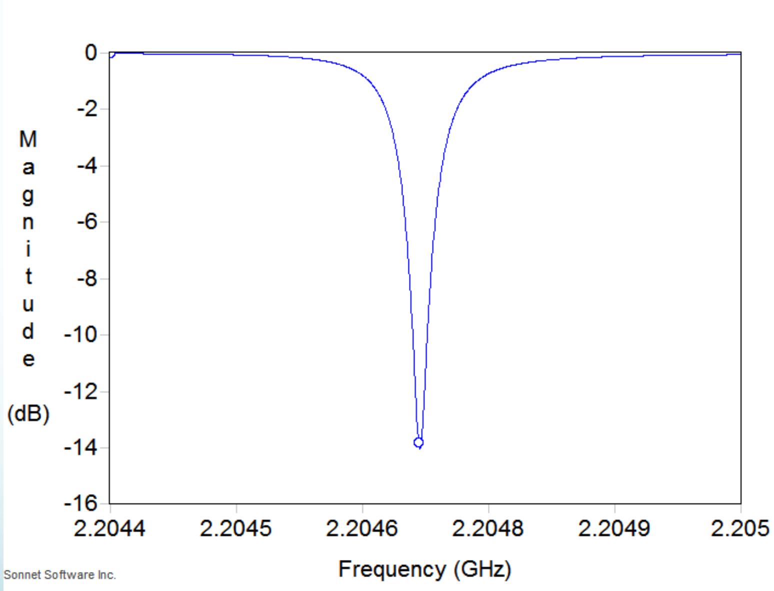
- Find ω_0 of a given resonator geometry
- Find L_g of a give geometry
- Look at current distributions
- Find Q_L and hence M



Layout



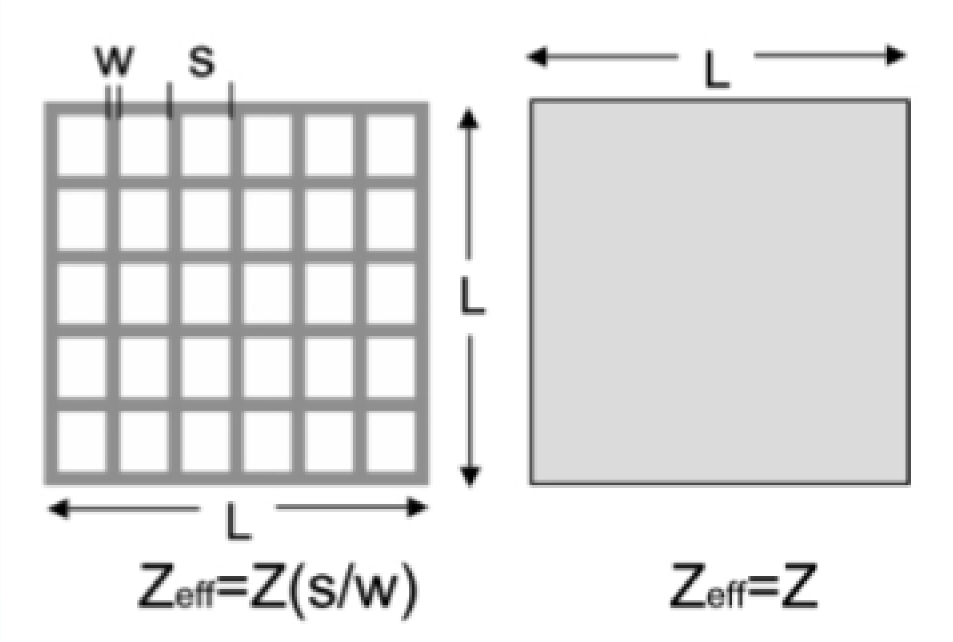
Current Density



S21

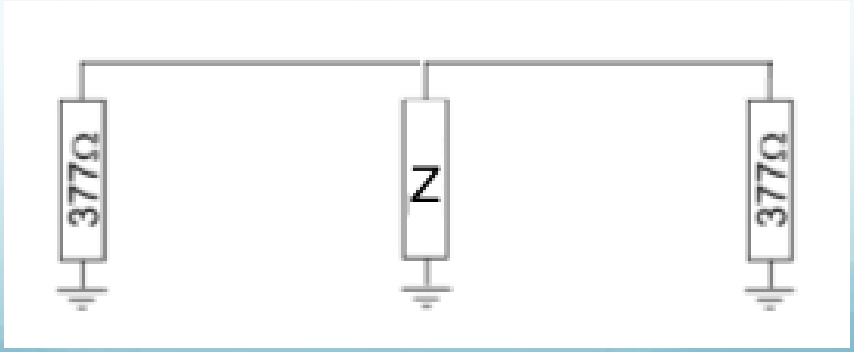
Optical Coupling to a LEKID

Partially filled absorbers

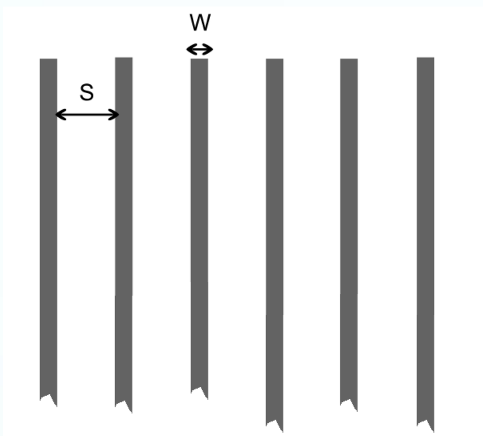


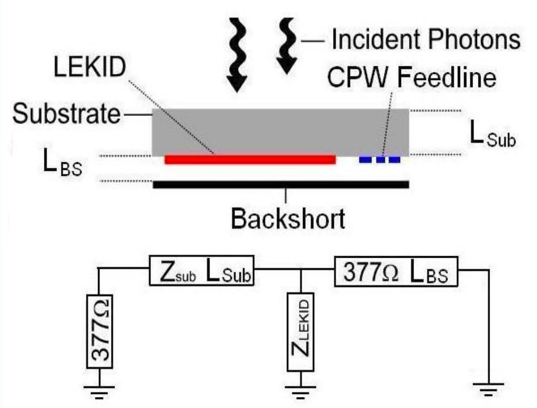
Z = Square impedance of the film = ρ1/t where t is the film thickness

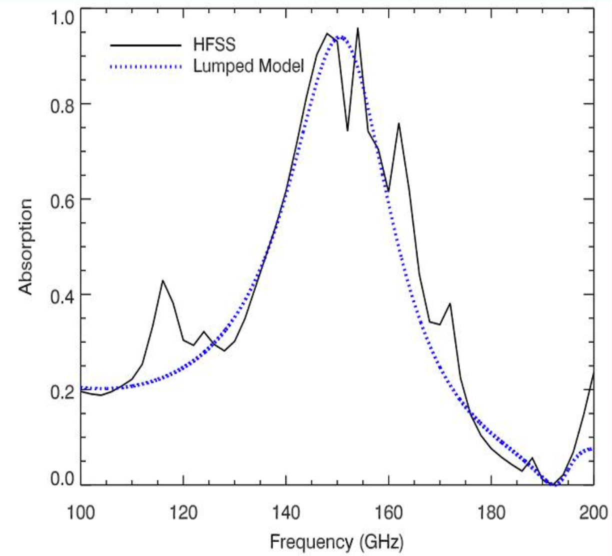
Z is modified



Optical coupling to a single polarization LEKID meander







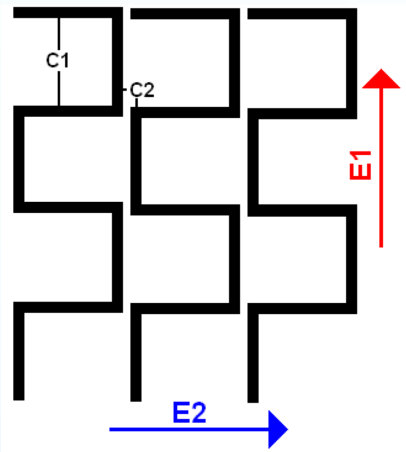
$$L_{eff} = \frac{s}{2\pi c}\ln\left[\operatorname{cosec}\left(\frac{\pi s}{W}\right)\right]377$$

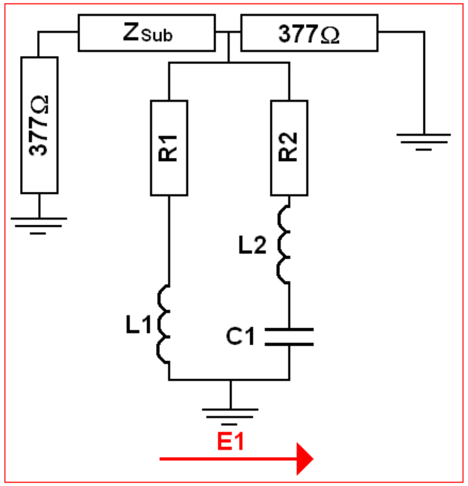
$$R_{eff} = R_{Sheet}\frac{s}{w}$$

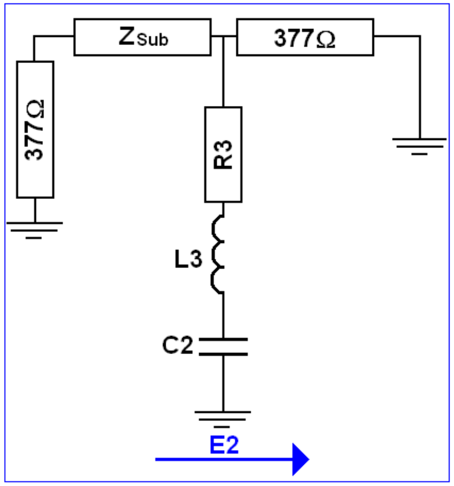
$$Z_{LEKID} = R_{eff} + j\omega L_{eff}$$

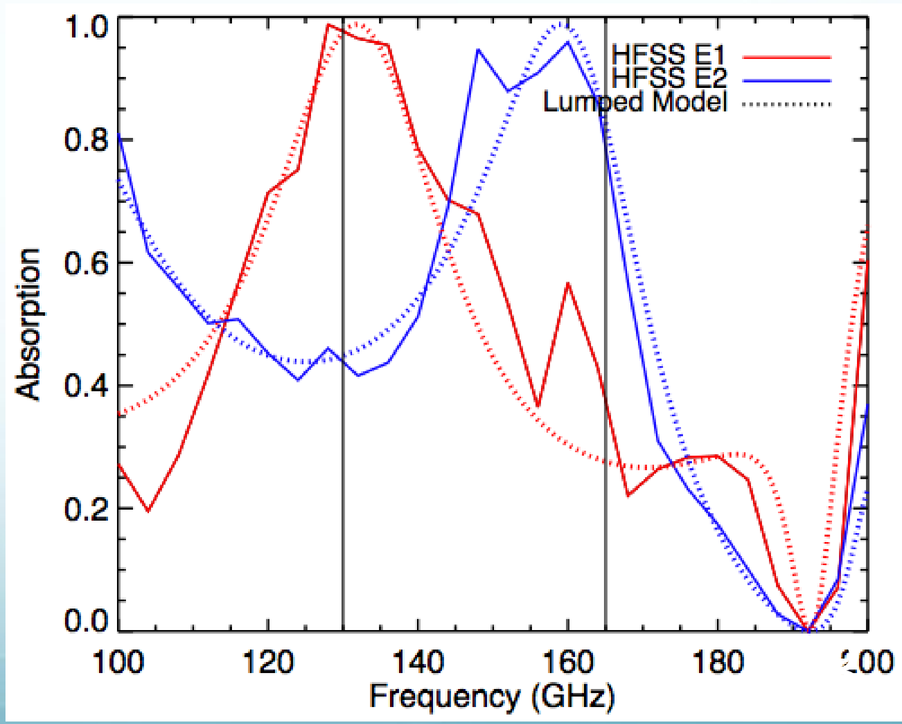
Optical coupling to a dual polarization LEKID meander

Using a "double meander" it is possible to couple to both polarizations.





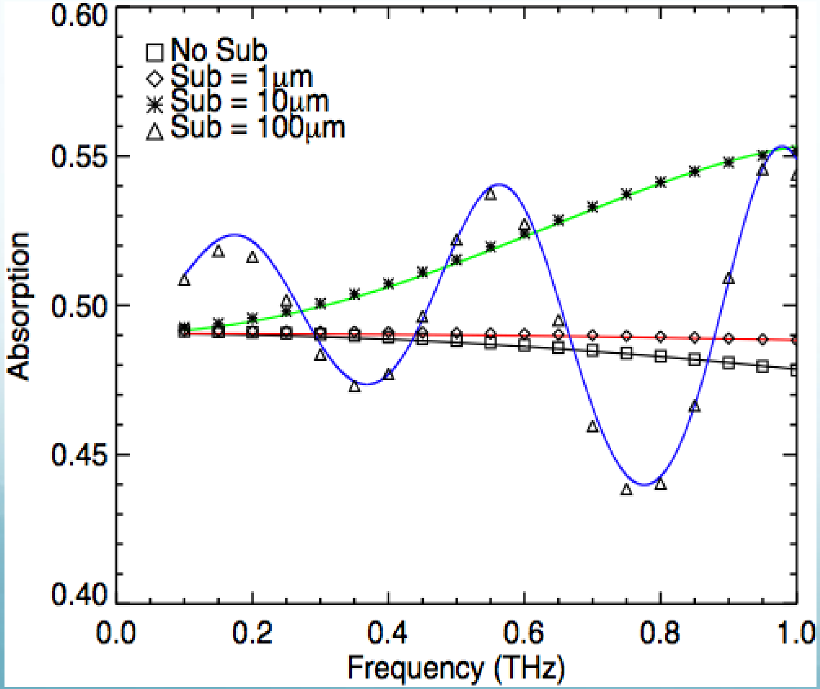


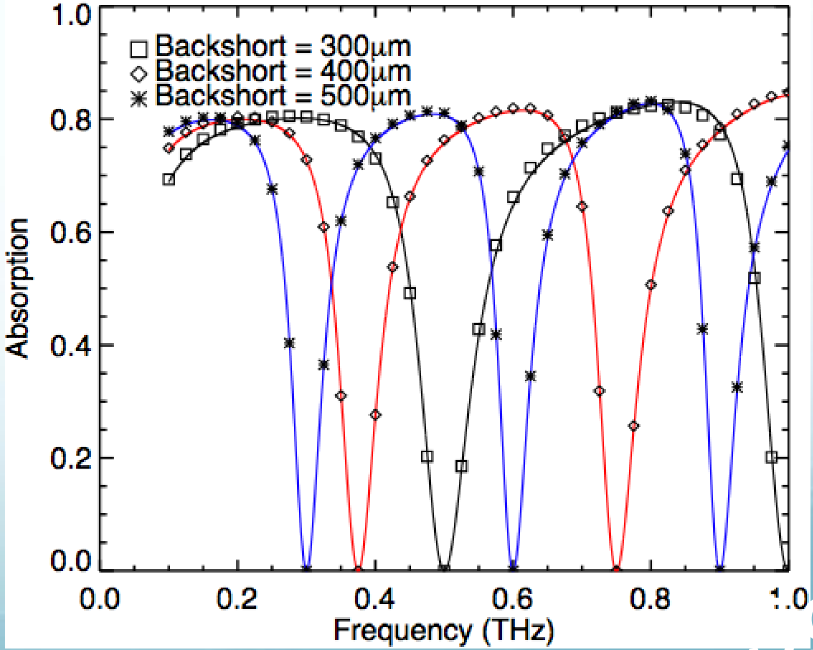


Optical coupling to LEKIDs

New high sheet impedance materials such as TiN have the potential to improve optical coupling of LEKIDs into the THz and beyond

- Aluminium typically has around 1Ω/sq for a 40nm film
- Have to space meander lines far apart to increase R_{eff}, this increases the effective sheet inductance making the sheet transparent at higher frequencies.
- TiN has a sheet impedance of around 10-30Ω/sq
- Can make low inductance sheet by placing meander lines close together while maintaining a high sheet resistance





Practical LEKID devices

The ultimate LEKID device would have high dynamic range, low intrinsic noise and high response. To achieve this we need the following:

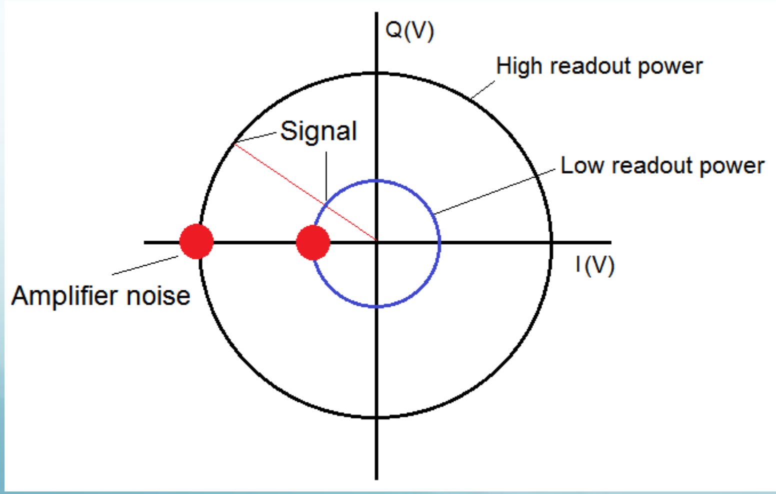
- High Q_L ➔ large phase response
- Small volume ➔ Large dT/dn_{qp} and low GR noise
- Low T ➔ Low GR noise and large dT/dn_{qp}
- Thin films ➔ High α and large $dL_k/d\sigma_2$
- Long Quasi-particle lifetimes ➔ large dN_{qp} / dP
- Low or non-existing phase noise (TLS noise)

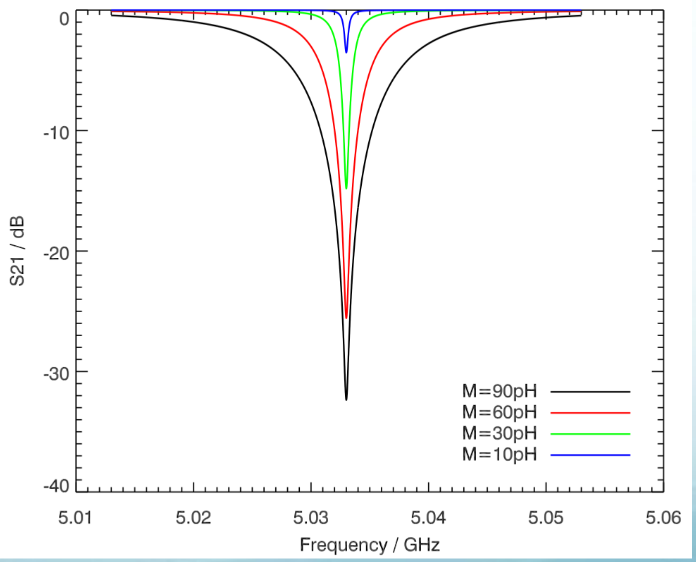
In practice these criteria are difficult to meet especially under a loading fro m say a ground based telescope.

Optimizing a LEKID

Maximizing QL has the following drawbacks:

- Current in the LEKID increases with Q_L - need to reduce readout power to stop the LEKID from being driven normal, more constraints on Low noise Amplifier.
- TLS is higher in high Q_L devices
- May loose resonant feature under high loading
- Can't make Q_L higher than Q_u.





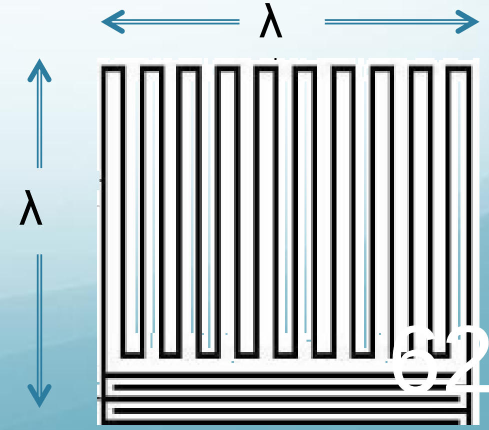
Optimizing a LEKID

Reducing the film volume and thinning the film has the following drawbacks

The LEKID works as a direct absorber so the dimensions of the LEKID need to be suitable to achieve good optical coupling (Line spacing and overall detecting area).

Film volumes can be reduced by thinning which also increases $dL_k/d\sigma_2$. However this does reduce power handling of a LEKID by increasing current density I the meander section.

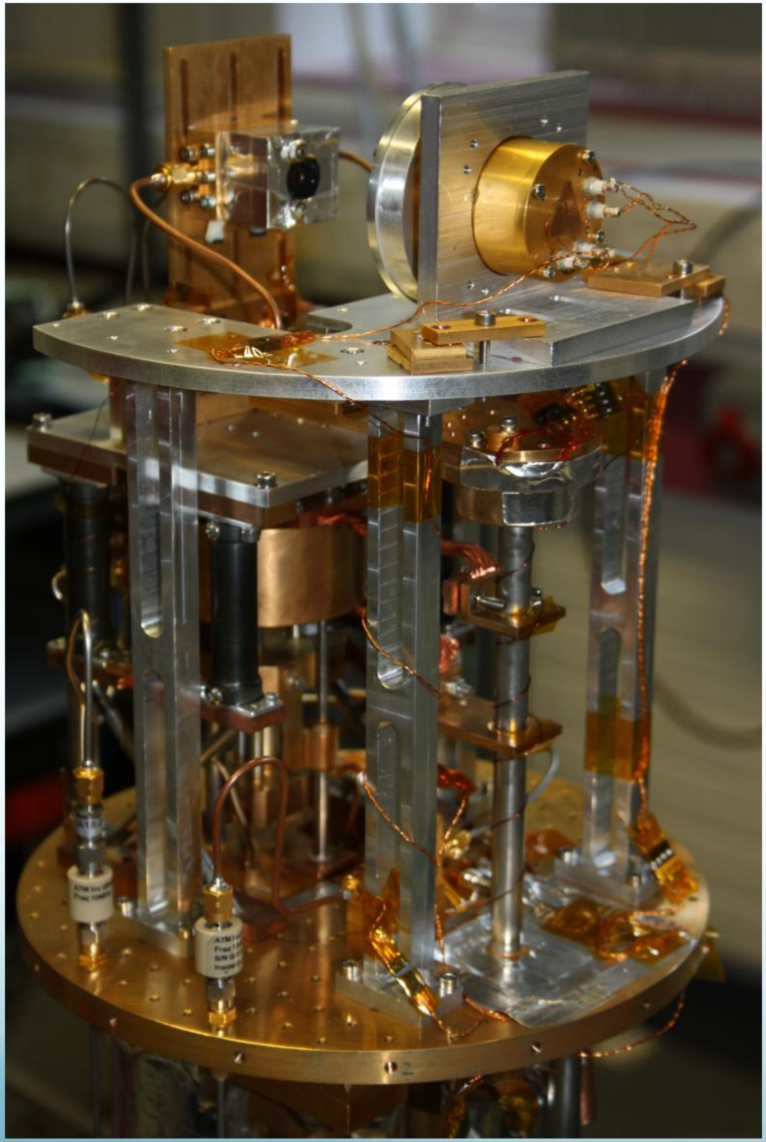
Reducing the film volume also increases the quasi-particle density under a given load. This in turn reduces increases the **effective temperature** of the detector reducing Q_L and $\tau_{qp.}$

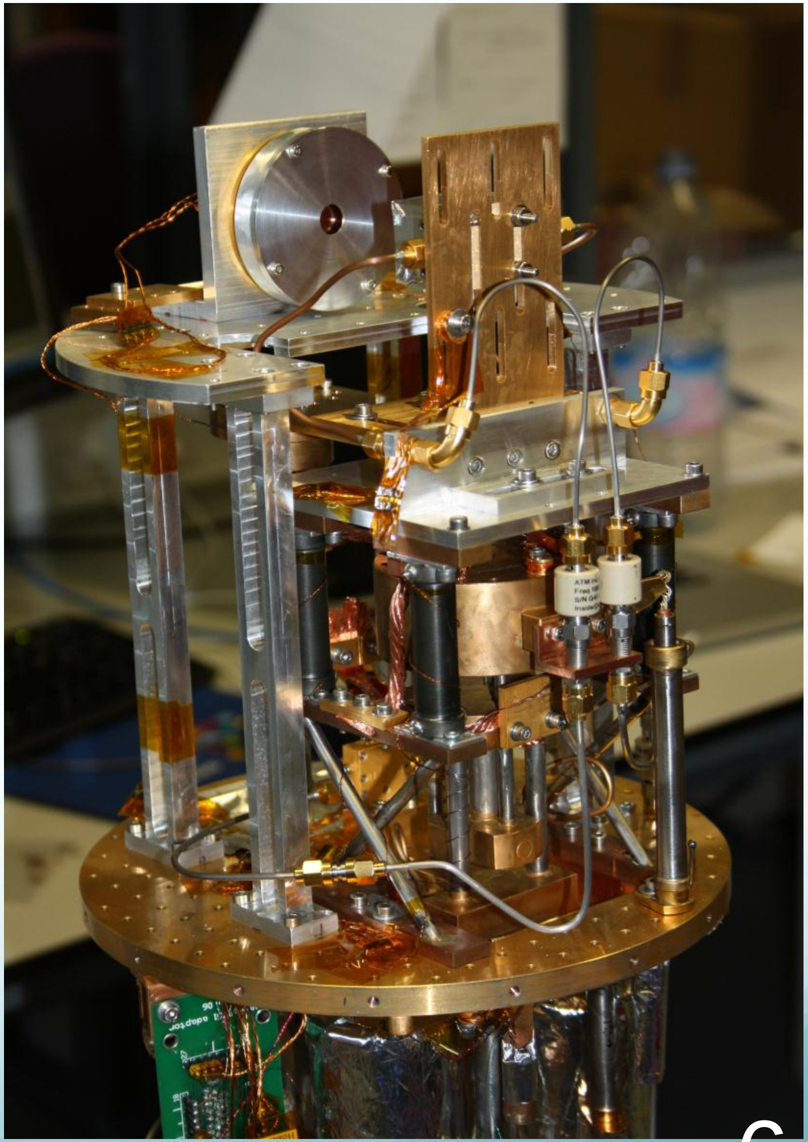


Optimizing a LEKID

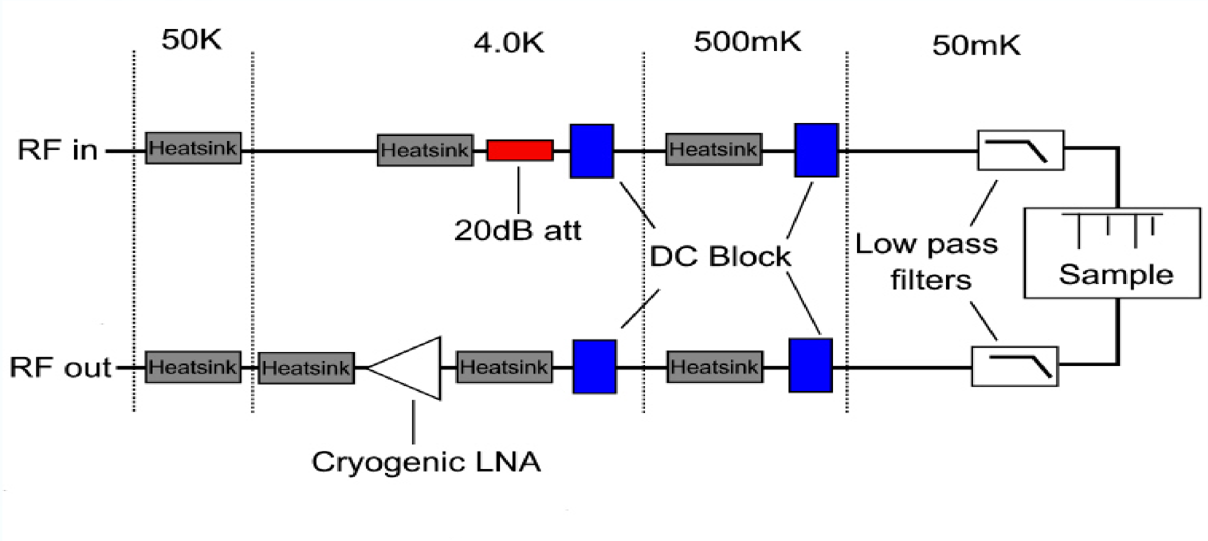
- In reality the loading conditions need to be considered
- Phase noise is usually avoidable by correct design of the capacitor section
- GR noise is never the limiting factor in practical applications
- Amplifier and readout noise should be as low as possible
- Stray light creates quasi-particles and reduces responsivity so should be reduced to a minimum!

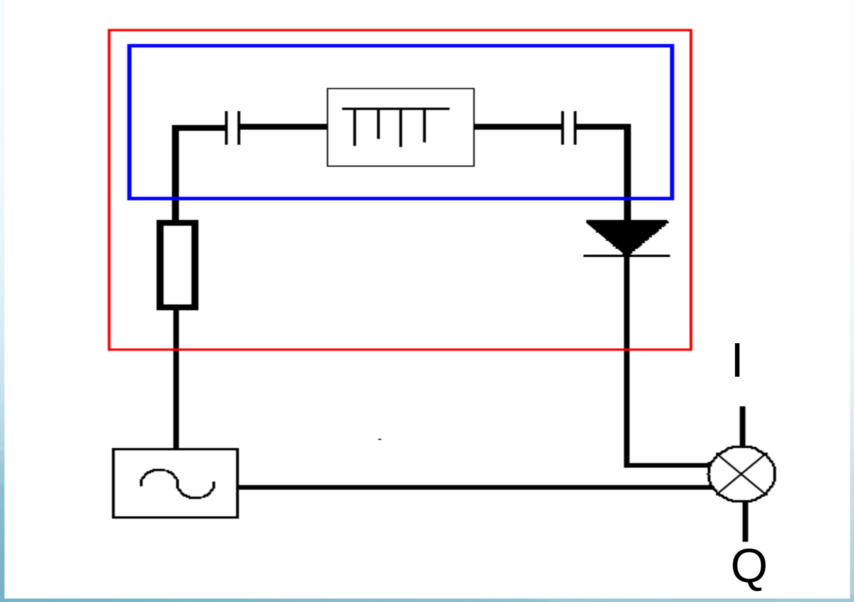
Measuring LEKID performance





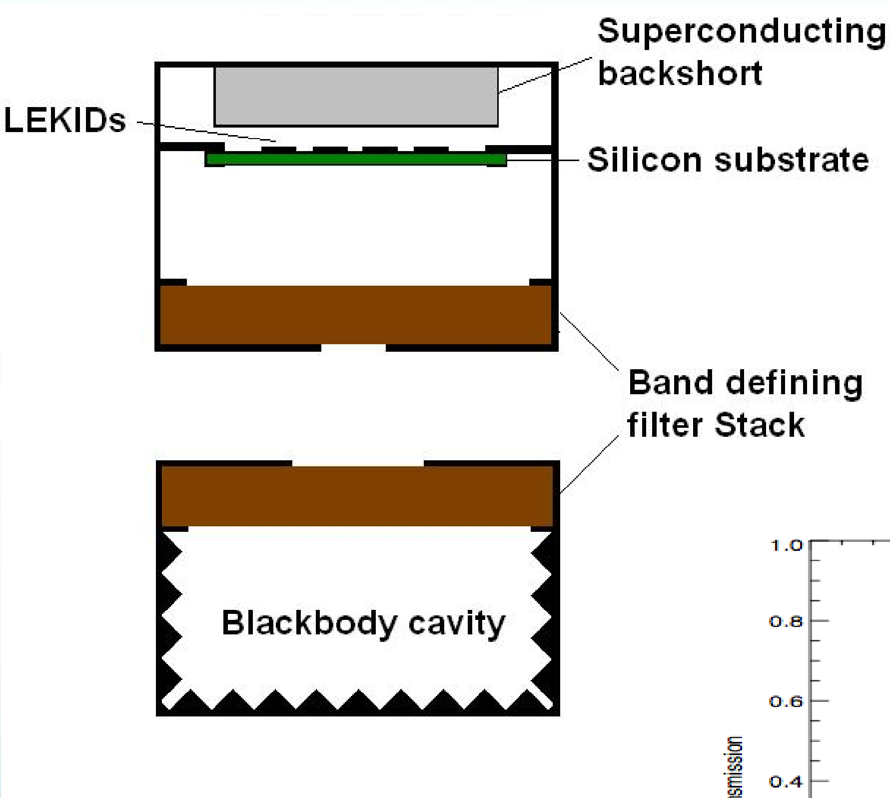
Readout electronics

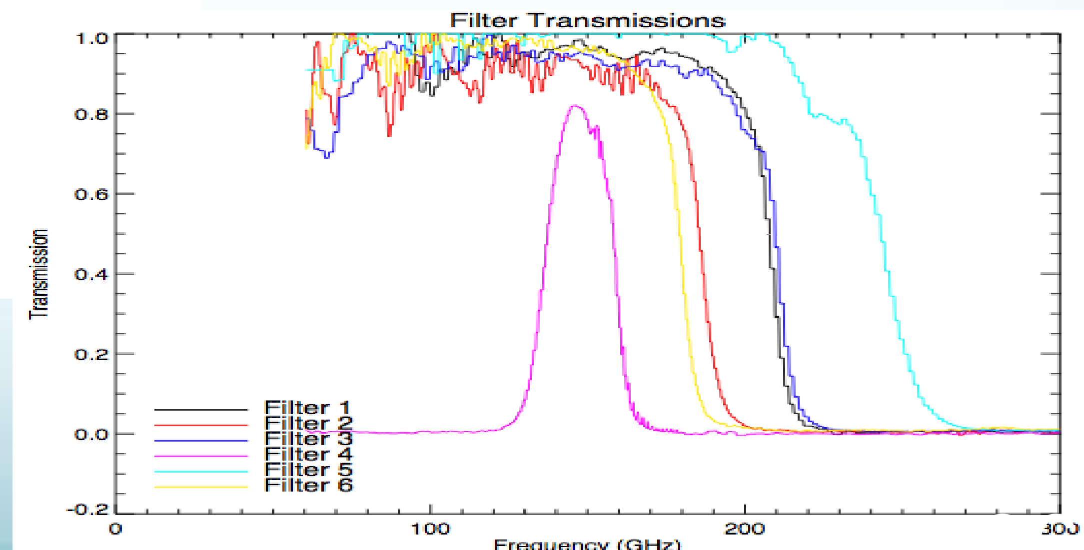




Homodyne mixing gives I and Q from the RF signal

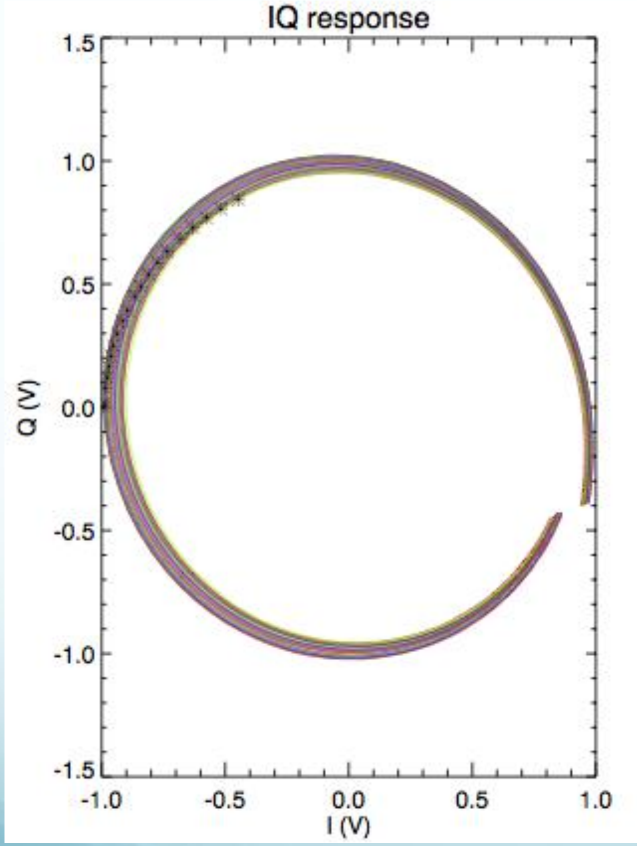
Measuring LEKID performance

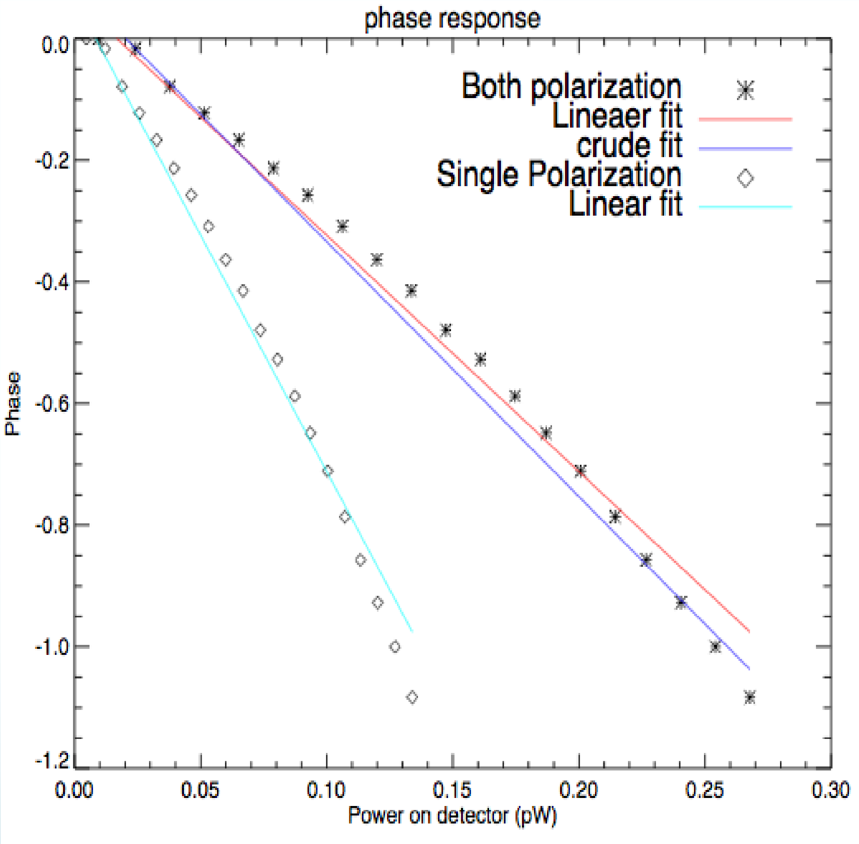




Measuring LEKID performance

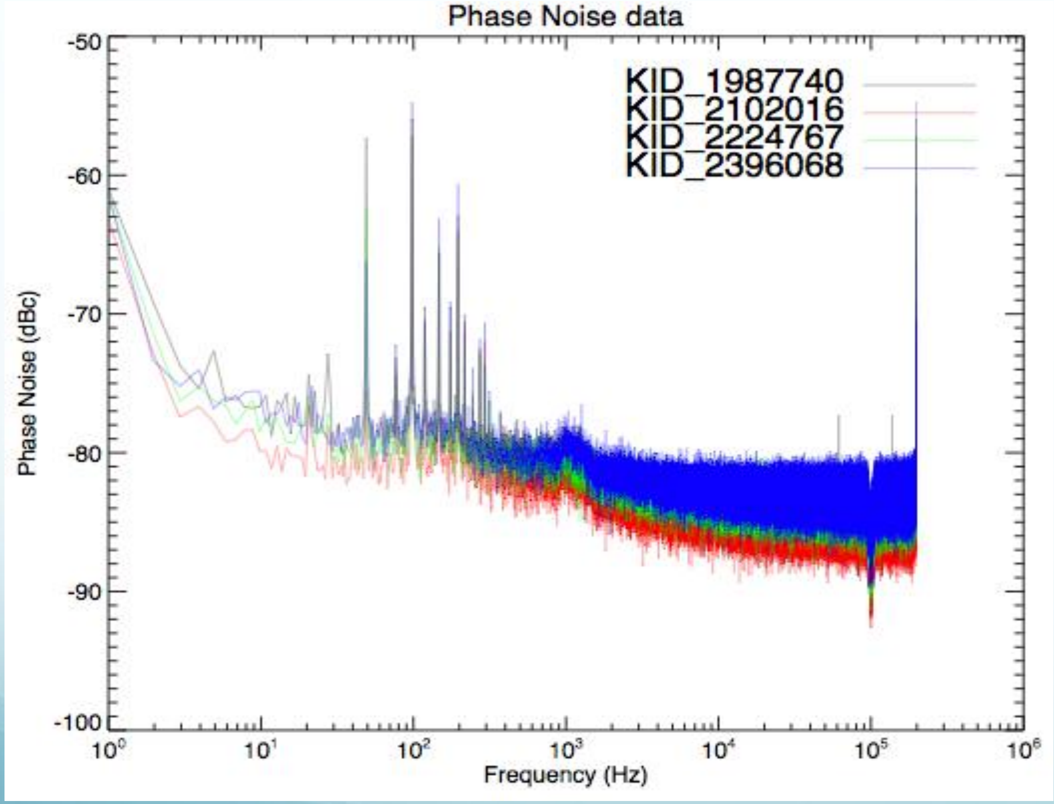
The phase response of the LEKID is measured directly as a function of the blackbody temperature





Measuring LEKID performance

The phase noise is measured by taking the power spectral density of the phase time stream θ=Atan(Q/I)

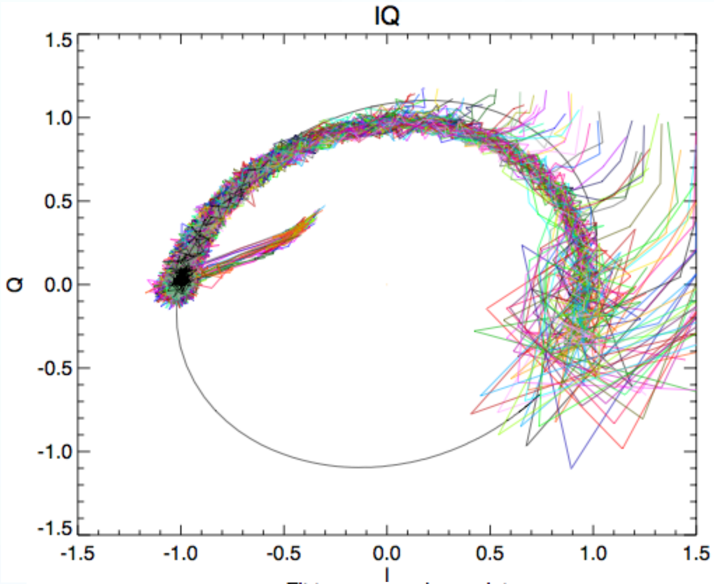


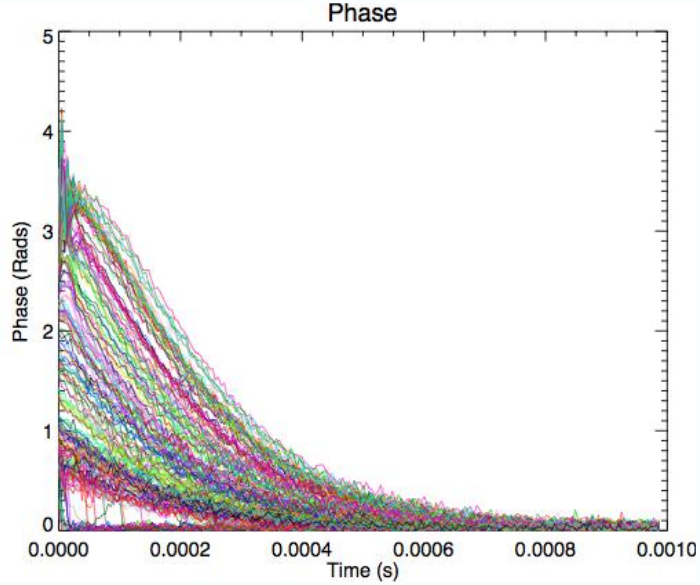
Phase noise is usually quoted in the microwave standard units of dBc.

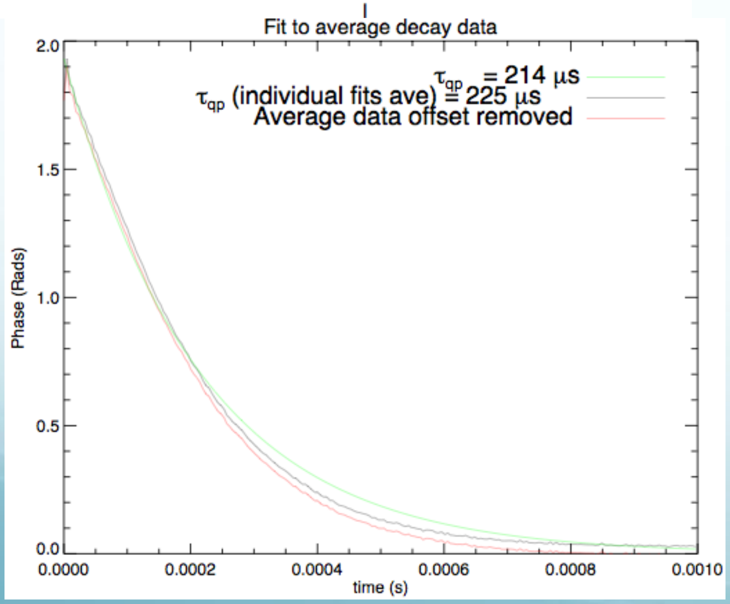
$$\mathrm{dBc} = 10\log_{10}(S_\theta)$$

Where S_θ is the phase noise power spectrum (rads^2/Hz)

Measuring LEKID performance







T_{qp} can be measured using cosmic ray hits or a pulse from an LED. The decay curve is then fit to to acquire the lifetime

Measuring LEKID performance

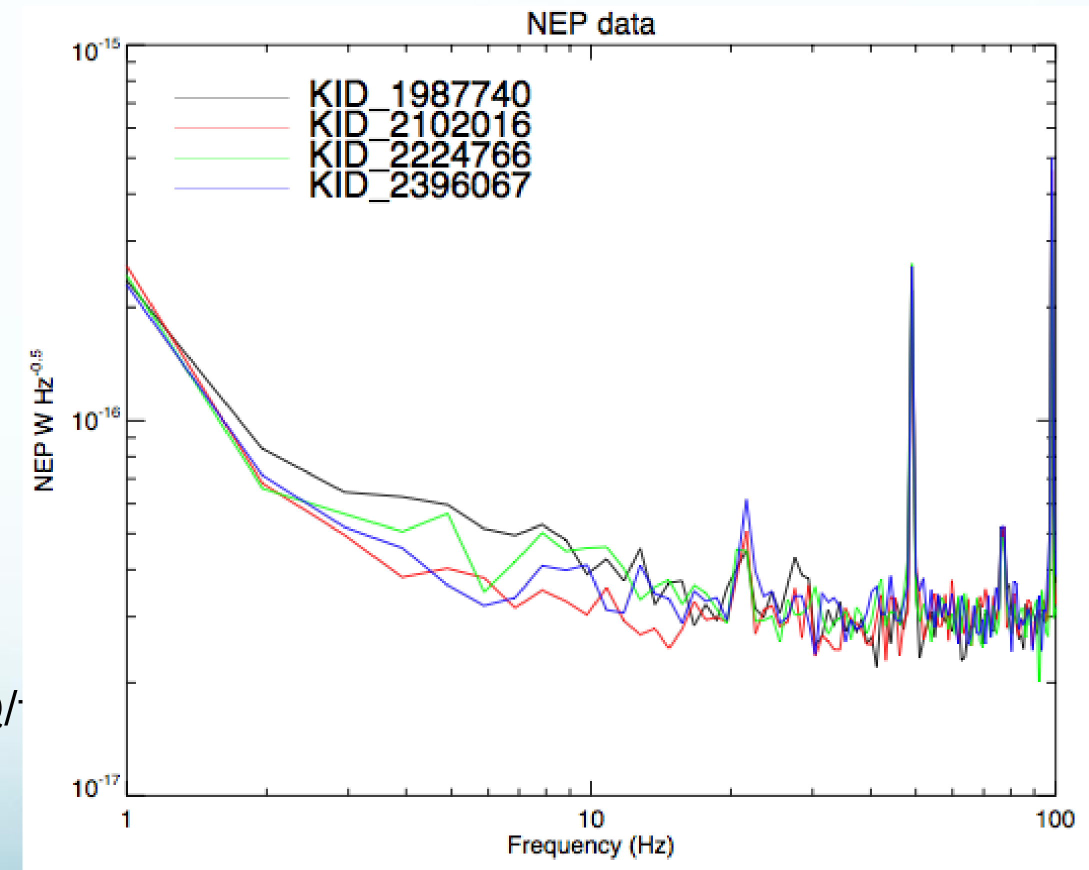
$$NEP^2 = S_x \left[\frac{d\theta}{dP}\right]^{-2} \left(1+\omega^2\tau_{qp}^2\right)\left(1+\omega^2\tau_{RD}^2\right)$$

Where:

S_x is the phase noise power spectral density ($rads^2$/Hz)

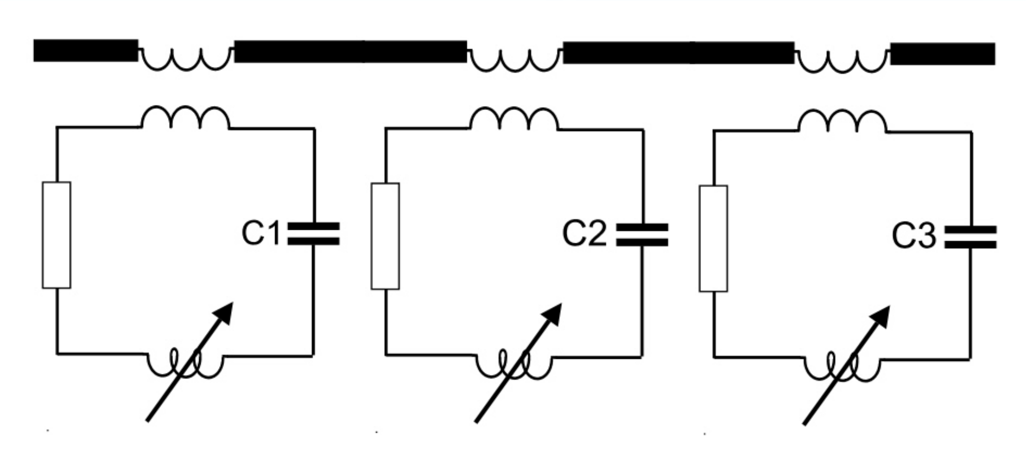
τ_{qp} is the quasi-particle lifetime

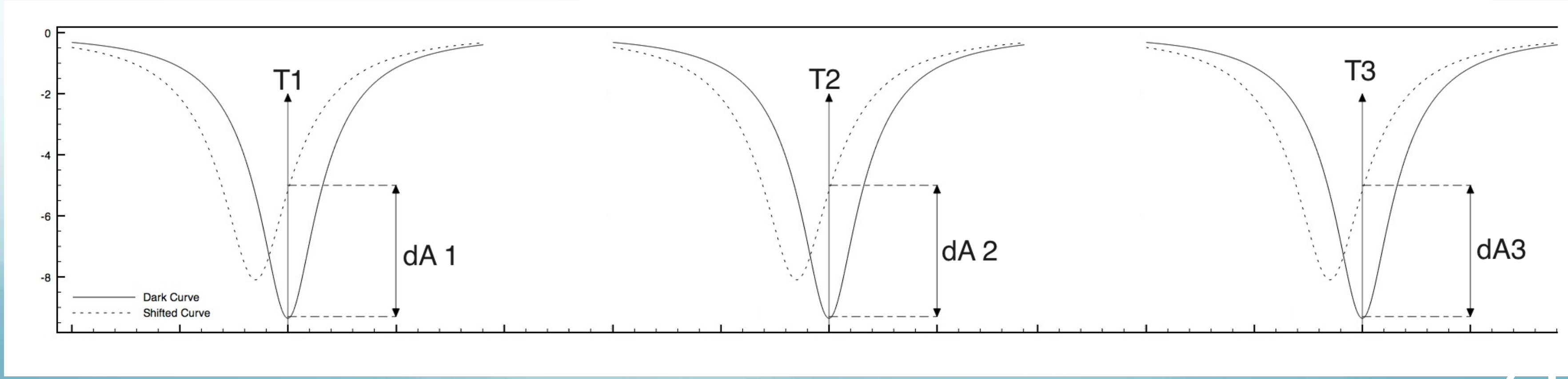
τ_{RD} is the resonator ring-down time =Q/



Natural multiplexing in KID devices

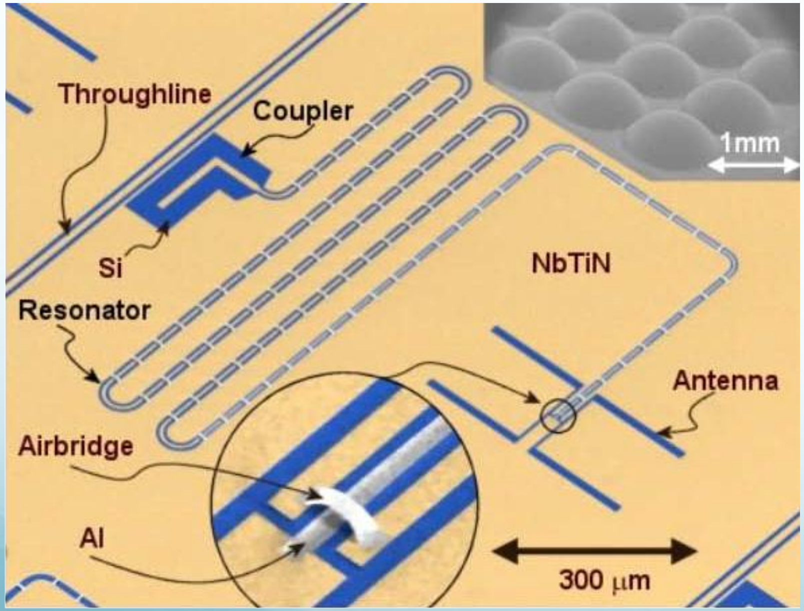
Any KID forms a high Q resonant circuit and therefore will only respond to a microwave tone close to ω_0. We can therefore couple many KIDs to a single transmission line each with a different value of ω0 and probe them all simultaneously with a set of microwave tones each set to the resonant frequency of a single KID.

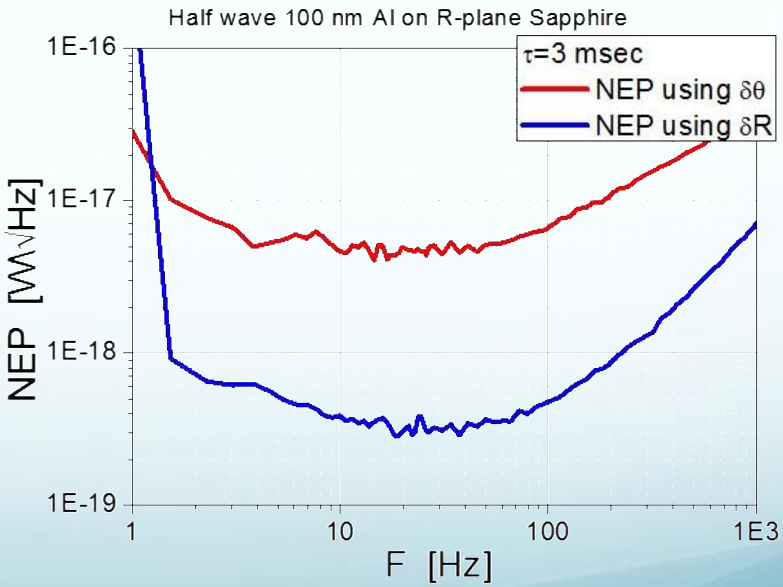




Antenna Coupled Distributed KIDs

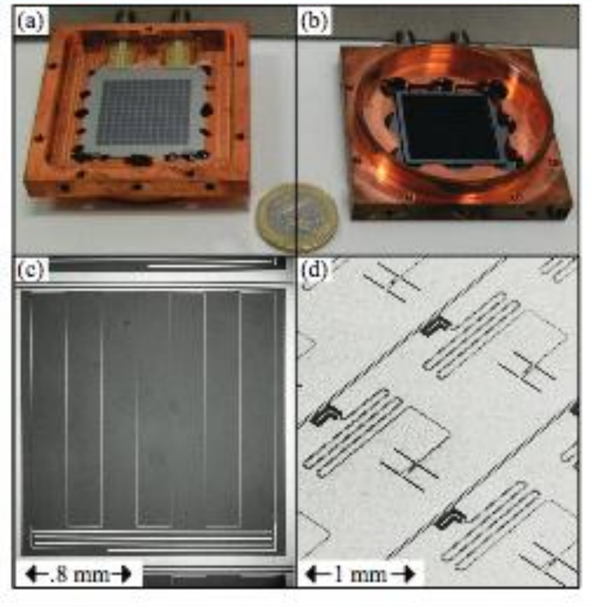
Antenna coupled distributed KIDs (SRON)



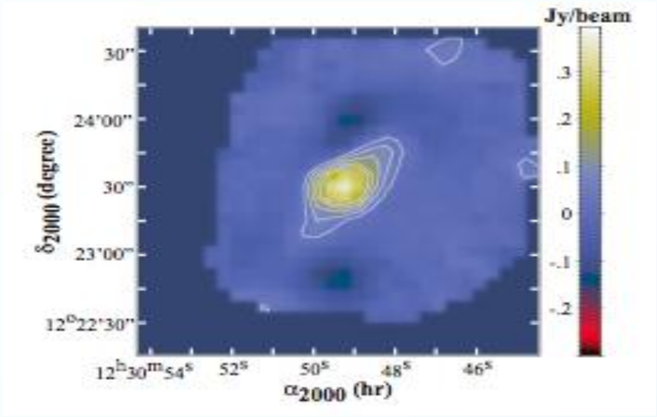


Current KID instruments

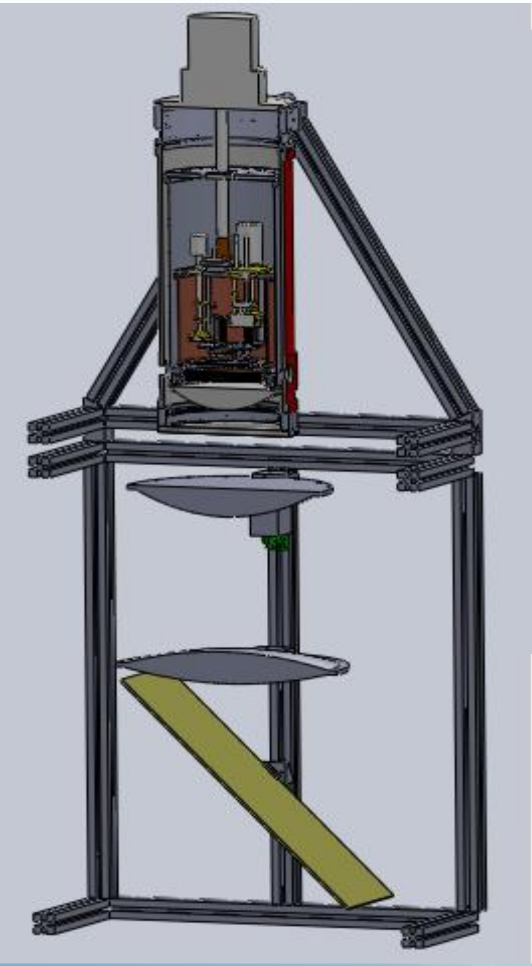
NIKA A dual band (150GHz and 220 GHz) mm astronomical camera on the IRAM telescope

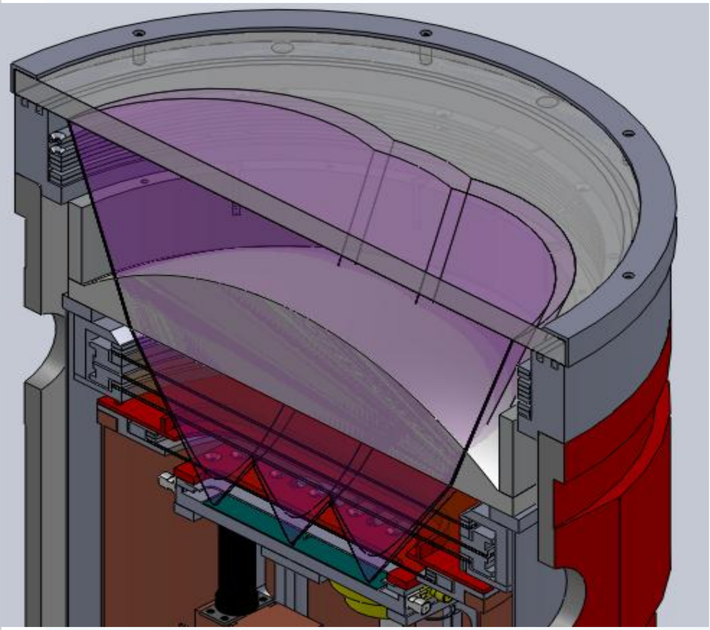


M87



A THz imager for high background applications KIDcam





Conclusion

Kinetic Inductance Detectors have been proven to be a very promising technology to provide sensitive highly multiplexed large arrays of detectors in the submm and FIR. In this lecture we have covered the basic principles of the LEKID a type of kinetic inductance detector.

Only the basic principles have been covered here more detail can be found in the following literature

- Lumped Element Kinetic Inductance Detectors , Doyle PhD thesis http://www.astro.cardiff.ac.uk/~spxsmd/
- Corse notes to follow
- A.C Rose-Innes and H.E. Rhoderick “Introduction to Superconductivity”
- T Van Duzer and C.W Turner, “Principles od Superconducting Devices and Circuits”
- D.M. Pozar, “Microwave Engineering”