A simple facility for the teaching of plasma dynamics and plasma nuclear fusion

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A small plasma focus (3.3 kJ) is designed from the viewpoint of simplicity, reliability, and cost effectiveness to act as a source of pulsed high-density plasmas. The simplicity of the device and associated diagnostics coupled with its rich variety of plasma phenomena makes this device ideal for the teaching of plasma nuclear fusion particularly for developing countries where such facilities are at present rarely available. Six sets of the device have been constructed and tested in various gases with better than 95% reliability and reproducibility in various plasma phenomena including neutron production of $0.5–1.0\times10^8$ per discharge when operated in 3-Torr deuterium. The design principles, procedures, and parameters are discussed and test results shown.

I. INTRODUCTION

Plasma physics has attained sufficient importance in its present applications and also in its potential application associated with plasma fusion energy that there have been considerable recent efforts (e.g., the Plasma Physics Colleges organized by the International Centre for Theoretical Physics, Trieste) to provide plasma physics education at an international level aimed towards stimulating the study of plasma physics even in developing countries.[1,2] Published literature has also played a role in these efforts.[3–13] This worthwhile educational effort encounters considerable difficulties at the experimental level particularly from the equipment point of view. There is not even a clear idea as to which experiment to develop, although low-cost, cost effectiveness, simplicity, and educational value combined with a large variety of plasma phenomena amenable to study by simple diagnostics should be among the factors to be considered in relation to devices for developing countries. Thus the glow discharge, the electromagnetic shock tube, and the linear Z pinch may be considered as the most likely candidates for this exercise, particularly since these are "classical" devices with well-known technology. However, these three devices, even the linear Z pinch in the low-cost scaled-down form necessary for this educational exercise, do not produce plasma conditions anywhere near necessary for measurable nuclear fusion. We have therefore considered the plasma focus as another educational device for this purpose.

The plasma focus is an excellent device for teaching plasma dynamics and thermodynamics besides being a rich source for a variety of plasma phenomena including soft x rays and plasma nuclear fusion. It is certainly superior to both the electromagnetic shock tube and the linear Z pinch in its range of plasma parameters, combining as it does the essential features of both devices in such a properly sequenced manner that the features of the electromagnetic shock tube and the pinch, and many more, may be produced in one single simple low-cost device, if properly designed.

However, plasma focus facilities are not commonly available and there has not been any systematic investigation on the design of such a facility in terms of simplicity, cost effectiveness, and ease of duplication. In the course of the United Nations University Training Programme on Plasma and Laser Technology it became necessary to design and operate such a facility so that several sets could be easily duplicated for transfer to various countries for teaching and educational purposes. What evolved from the programme is the UNU/ICTP PFF (United Nations University/International Centre for Theoretical Physics Plasma Fusion Facility) which is a 3.3-kJ plasma focus system powered by a single 15-kV, 30-μF Maxwell capacitor switched by a simple parallel-plate swinging cascade air gap. Low-cost features include the use of a step-up transformer from a standard television set to trigger the spark gap. Thus the only item that needs to be purchased (apart from metal sheets, miscellaneous materials, and components) to build the device is the Maxwell capacitor. A small rotary pump is sufficient for the vacuum and a 50-MHz oscilloscope is sufficient for the diagnostics.

The system produces remarkably consistent focusing action in several gases including air, argon, hydrogen, helium, carbon dioxide, and deuterium. A consistent neutron yield of $0.5–1.0\times10^8$ neutrons per discharge is obtained at 3 Torr of deuterium operating the focus at 15 kV and 180 kA current.

In this article we discuss the design principles and procedures and the design parameters and performance when subjected to computation. The experimental arrangements are then described together with simple diagnostics measuring plasma current, voltage, and magnetic field which give information on plasma dynamics. A simple arrangement for measuring neutron yield is also presented. The results of test measurements are discussed.

II. THEORY AND DESIGN

The focus design is based on a dynamic model[14,15] (see Fig. 1) that considers the focus dynamics in two separate phases—the axial rundown (shock tube) phase that crucially delays the radial focus, or pinch phase, until the plasma current has reached its peak value. The design point is therefore to have

$$t_r = t_{a\,\exp}, \quad (1)$$

where

$$t_r = (2\pi/4)t_0, \quad (2)$$

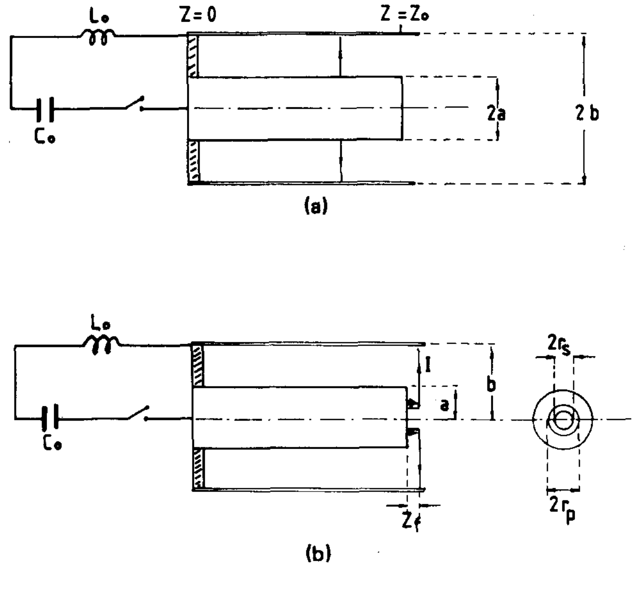

Fig. 1. (a) The axial run-down (shock tube) phase of the plasma focus. (b) The radial focus (pinch) phase of the plasma focus.

with

$$t_0 = \sqrt{L_0 C_0}, \tag{3}$$

and

$$t_{a\,\exp} \sim 2t_a, \tag{4}$$

where

$$t_a = \left(\frac{4\pi^2(b^2 - a^2)}{\mu \ln(b/a)}\right)^{1/2} \frac{z_0 \rho_0^{1/2}}{I_0}. \tag{5}$$

Here t_r is the current rise time and $t_{a\,\exp}$ is the transit time of the plasma layer for the axial phase. The quantities t_0 and t_a are characteristic times of the axial phase according to the model.[15] Here L_0 is the inductance of the capacitor C_0 together with all connections up to the plasma section of the focus tube, "a" and "b" are, respectively, the inner and outer radii of the focus tube, z_0 its length, ρ_0 the ambient gas density, μ the permeability of the plasma (same as the permeability of free space), and $I_0 = V_0/(L_0/C_0)^{1/2}$, where V_0 is the initial charge on the capacitor.

The second design point involves the characteristic "pinching" time of the plasma focus phase. This may be shown to be[14]

$$t_p = \frac{4\pi}{\mu^{1/2}(\gamma+1)^{1/2}} a^2 \frac{\rho_0^{1/2}}{I_0}, \tag{6}$$

where γ is the specific heat ratio of the plasma. From this expression of t_p it may be shown that the ratio of the characteristic axial transit time to characteristic focus time is

$$t_a/t_p \simeq cF,$$

where F is the aspect ratio z_0/a, and $c = b/a$.

A crucial factor in the operation of the Mathers plasma focus is that the axial phase occurs over a relatively long period $t_{a\,\exp}$ enabling the build up of capacitor current. The pinch phase then occurs over a relatively short period t_p. During this time t_p, approximately 10%–20% of the initially stored energy is transferred to the pinch plasma in approximately 2% of the current rise time. This results in a power enhancement factor during the pinching phase that is crucial to the proper operation of the plasma focus. It is important then that the ratio t_a/t_p be of the order of 30–50 for the Mathers focus.

The third point to be considered in the design is that there are limits[14] of speed and pressure in the operation of the plasma focus. In deuterium, for good focusing and consistent neutron yield, the axial speed just before focusing should be between the limits 6 and 10 cm/μs; the lower limit being the minimum speed required for a good snowplowing action in the axial phase and the higher limit being imposed by restriking[14,16] of the discharge at the backwall or in the shock tube section. The limits of test gas pressure appear to be between 0.5 and 20 Torr for deuterium; the lower limit apparently governed by restriking; the upper limit by current filamentation.[17]

The design of a plasma focus may take as a starting point the availability, or choice, of a capacitor bank. For the present exercise from the point of view of economy and cost effectiveness, a single Maxwell capacitor rated at $C_0 = 30$ μF, $V_0 = 15$ kV with an equivalent series inductance, ESL, of less than 40 nH was selected. A parallel-plate geometry was selected for the capacitor connections and the switch, with coaxial cables being used to connect to the plasma focus input flanges. The value of L_0 was estimated at 110 nH. Having fixed C_0 and L_0, Eqs. (3) and (2) give a value of t_r of 2.9 μs and $I_0 = 248$ kA. The time-matching condition of Eq. (1) fixes $t_{a\,\exp}$ at 2.9 μs. The value of z_0 was then chosen at 16 cm to give an average axial speed of 5.5 cm/μs or a peak axial speed of $\sim$9 cm/μs just before the focus phase. This axial speed is expected to be suitable for a good focusing action in deuterium.

The value of I_0 is considerably smaller than most operational plasma focus machines that typically have I_0 of the order of 500 kA or more. Observing from Eq. (5) that the axial speed is $\sim I_0/[(b^2 - a^2)\rho_0]^{1/2}$ and from Eq. (6) that the radial speed is $\sim I_0/(a\rho_0^{1/2})$, it is noted that a reduction in I_0 may be compensated in the first instance by a reduction in a and $(b^2 - a^2)^{1/2}$ in order to maintain the axial speed within the speed limit indicated earlier. Thus we design for $a = 9.5$ mm and $b = 32$ mm which are smaller than typical values of most operational plasma focus devices. Moreover, the value of $b/a \sim 3.4$, in this case, is near optimum.[18] It is also noted that the value $t_a/t_p \sim 40$ for this design.

Having fixed the values of I_0, z_0, b and a, and t_r it remains to fix the value of ρ_0 from Eq. (5). This gives $\rho_0 = 0.21 \times 10^{-3}$ kg^{-3}. This is the density of deuterium at 0.9 Torr, which is within the pressure limits for deuterium focus operation as mentioned earlier.

Summarizing the design parameters we have $C_0 = 30\times10^{-6}$ F, $L_0 = 110\times10^{-9}$ H, $V_0 = 15\times10^3$ V, $a = 0.95\times10^{-2}$ m, $b = 3.2\times10^{-2}$ m, $z_0 = 0.16$ m, and $\rho_0 = 0.21\times10^{-3}$ kg^{-3} (0.9 Torr D_2), giving $I_0 = 2.48\times10^5$ A, $c = 3.37$, $F = 16.84$, $t_0 = 1.82\times10^{-6}$ s, $t_r = 2.86\times10^{-6}$ s, $t_a = 1.45\times10^{-6}$ s, $t_{a\,\exp} = 2.9\times10^{-6}$ s, $t_p = 35.8\times10^{-9}$ s, and $t_a/t_p = 40.4$.

The above design parameters have been subjected to a computation using a dynamic model[14,15] in which the axial trajectory is computed using a snow-plow model and the radial dynamics is traced using a generalized slug model that considers a pinching plasma of increasing length with the plasma layer lying between a shock front at position r_s and magnetic piston at position r_p (see Fig. 1). This model has the advantage of giving a realistic final pinch radius ratio. Using the design parameters for the present device, the scaling parameters for the generalized slug model are

$$\alpha = t_0/t_a = 1.26, \quad \text{and} \ \beta = L_a/L_0 = 0.36,$$

where L_a = maximum inductance of axial phase $= z_0(\mu/2\pi)\ln c$. Also

$$\alpha_1 = t_a/t_p = 40.4, \quad \beta_1 = \beta/\ln c = 0.294.$$

The other parameters used for the model are $c = 3.37$, $F = 16.84$, and $\gamma = \frac{5}{3}$ (for fully ionized deuterium).

The computation indicates a strong focus with a large focusing voltage spike. The parameter α was varied between 0.7 and 1.5 (corresponding to pressure range of 0.5–2 Torr) and the computation repeated at each α. Good focusing was indicated over this range of pressure. These computation results add confidence to the design of the plasma focus. However, it has been found that machine effects such as current and mass shedding, reduced channel size due to boundary effects, and current restrike that are not included in the dynamic model may alter the actual performance of the plasma focus. It is therefore to be expected that in actual operation the focus may need to be tuned[18] by a variation of the five parameters V_0, ρ_0, z_0, a, and b. If the design is not too far from optimum, operation over a range of ρ_0 may be sufficient to establish a regime of good focus. This is the procedure adopted in the present experiment.

III. EXPERIMENTAL ARRANGEMENT

A simple parallel-plate spark gap with a swinging cascade configuration (see Fig. 2) was developed giving a low inductance at minimum cost. The ratio of the gap is 3:2 ($4\frac{1}{2}$–3 mm). The gap is triggered via an isolating capacitor from an 800-V SCR unit via a TV transformer that was found to have a step-up ratio of 17 times and a rise time of 1 μs. The isolating capacitor is a 1-m length of UR67 coaxial cable. The parallel-plate spark gap is made from $\frac{1}{4}$-in.-thick copper plates and proved maintenance free for 200 discharges between 13–15 kV before it was cleaned. The triggering jitter was found to be within $\pm$ 50 ns. The circuit is shown in Fig. 2.

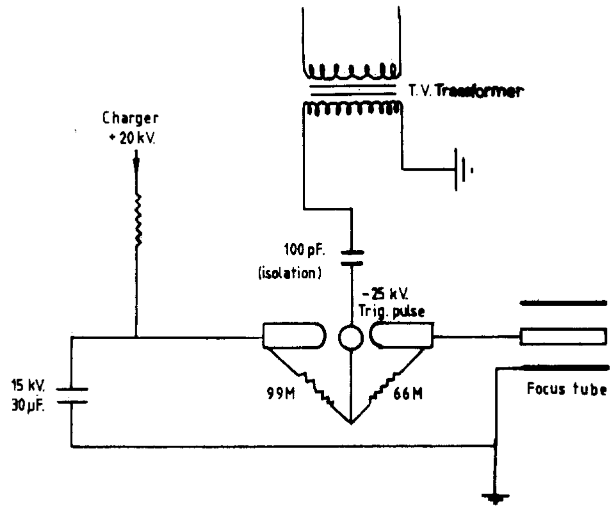

Fig. 2. Circuit for the swinging-cascade spark gap and focus tube.

The arrangement for the capacitor, the connecting plates, the spark gap, and the output coaxial cables is shown in Fig. 3(a). To keep the inductance low, the Earth plate of the capacitor (labeled no. 10) is extended nearly up to the anode and insulation is provided by a nylon cap (no. 5) around the anode stud. The cap dips into a pool of oil (no. 4) that is prevented from splashing out by means of an O-ring (no. 8). Mylar sheets (no. 11), 2 in. wider all around than the conducting plates, sandwiched by polyethylene sheets (no. 12) complete the insulation between the HV plate (no. 14) and the Earth plate as shown in Fig. 3(a). The Earth plate (no. 10) runs unbroken to the output position where the Earths of the coaxial cables connect. On top of the insulating sheets the HV plate is connected to the spark gap. Between the spark gap electrodes and all along it is placed a $\frac{1}{4}$-in.-diam copper tubing (no. 17) that acts as the trigger electrode. The output plate of the spark gap is connected to the focus tube by means of 16 coaxial cables (no. 18) used in parallel.

Essential to the structure of the focus tube is the backwall [see Fig. 3(b)] insulator. The glass insulator (no. 30) plays an important role in the symmetrical formation of the current sheet and has to be properly mounted to avoid being broken by vibrations. In the present design this glass insulator is mounted in a rubber holder (no. 24) which when compressed tightly and symmetrically by the brass flange (no. 28) grips the glass insulator. The rubber holder also acts as a vacuum and high-voltage seal. Figure 3(b) also shows the anode collector plate (no. 20) and the cathode collector plates (no. 25) onto which the coaxial cables connect.

The plasma chamber (no. 32) consists of a 30-cm length of $6\frac{1}{2}$-in.-diam mild steel tubing that is chromed. Vacuum is provided by a single-stage rotary pump reaching an ultimate base pressure $\sim$0.01 Torr. The system was adjusted for a leakage rate of less than 2 μ/min and pressure is read with a mechanical diaphragm gauge. Operating at a test pressure of 1 Torr and with a delay of less than 5 min between gas filling and focus operation, the air impurity in the system is about 2%. This level of vacuum proved to be sufficient for operating with good focus in various gases and good neutron yield when operated in deuterium.

To measure the relative strength of the plasma focus action, a Rogowski coil (no. 22) with an integration time constant of 200 μs displayed on a 50-MHz CRO is used to measure the current flowing into the anode. A resistive voltage divider (not shown in figure) with 15-ns response time is strapped across the anode collector plate (no. 20) and the cathode collector plate (no. 25) to measure the voltage across the focus tube. In a plasma focus device, the axial drive phase is characterized by a smooth near-sinusoidal rising current and a corresponding smooth waveform with a voltage value[14] that is proportional to the axial drive speed as the rate of change of current reduces to zero at peak current. As the focus occurs, the strong electromechanical action draws energy from the magnetic field pumping the energy into the compressing plasma. This mechanism is indicated in the distinctive current dip and voltage spike displayed by the current and voltage waveforms. In general, the stronger the focus, the more severely the plasma is compressed and the bigger the current and voltage spike.

To measure the magnetic field a 10-turn 1-mm coil jacketed in a 3-mm glass tubing (no. 33) is inserted into the focus tube and orientated to measure the azimuthal magnetic field. The passage of the current sheath driving the

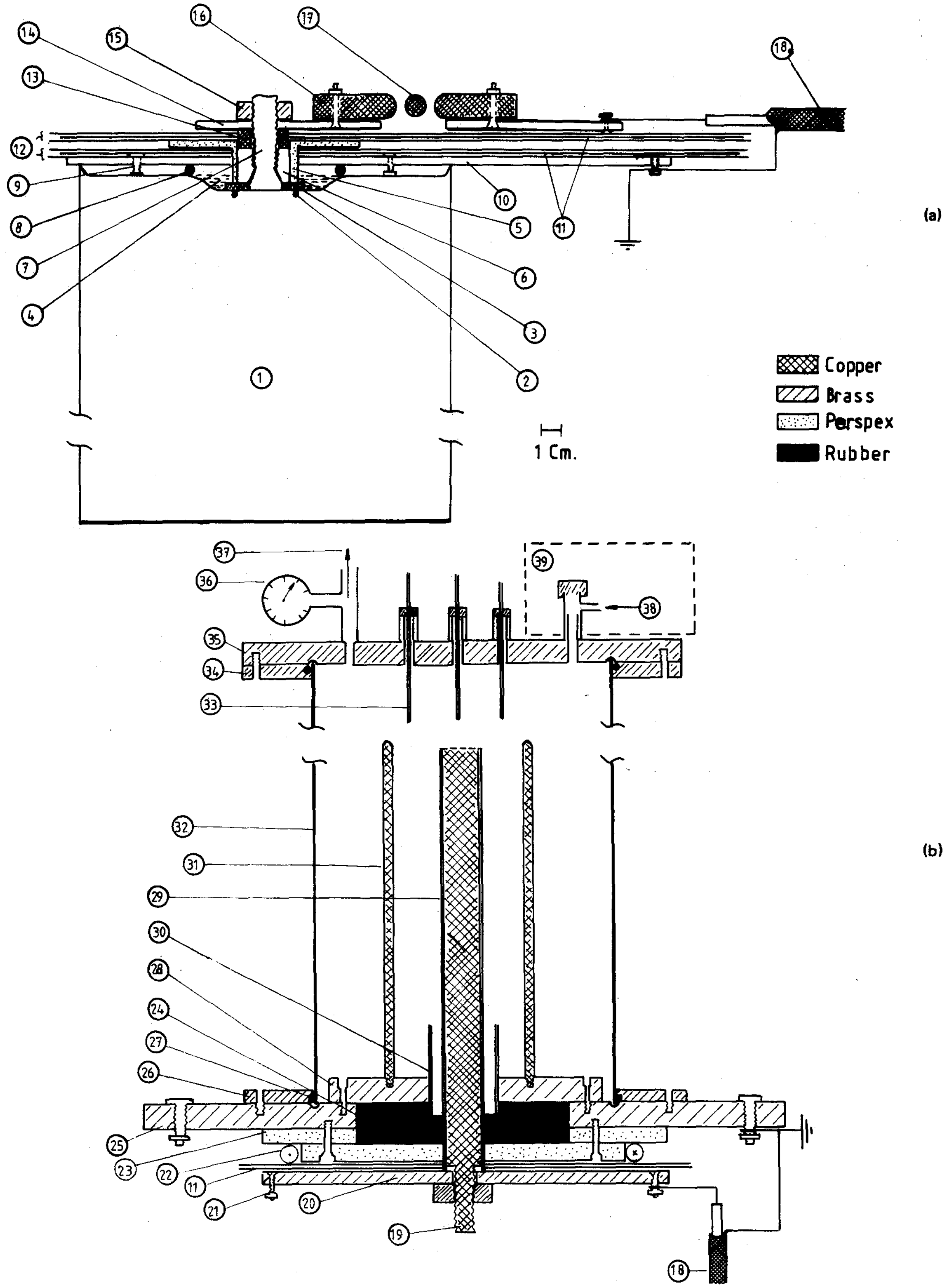

Fig. 3. (a) The capacitor connecting plates, the spark gap, and output coaxial cables. 1 = 15-kV, 30-μF capacitor; 2 = capacitor O-ring seal; 3 = washer; 4 = oil; 5 = nylon cap; 6 = steel nut; 7 = capacitor output seal; 8 = O-ring seal; 9 = Earth stud; 10 = Earth plate; 11 = 5-mil Mylar film; 12 = polyethelene film; 13 = copper ring HV connector; 14 = capacitor high-voltage (HV) output plates; 15 = lock nut for HV plate; 16 = HV electrode for swinging cascade spark gap; 17 = trigger electrode; and 18 = output coaxial cables (16 in parallel). (b) The plasma focus tube. 18 = input coaxial cables (16 in parallel); 19 = stud of anode; 20 = anode collector plate; 21 = connecting points for coaxial cable HV lead; 22 = Rogowski coil; 23 = perspex spacer; 24 = rubber holder; 25 = cathode collector plate; 26 = mild steel flange; 27 = O-ring seal; 28 = focus cathode support plates; 29 = focus anode; 30 = glass insulator; 31 = focus cathode (6 rods); 32 = mild steel focus chamber; 33 = movable magnetic probe in glass jacket; 34 = flange; 35 = back flange; 36 = diaphragm gauge; 37 = outlet to vacuum pump; 38 = inlet for test gas; 39 = wax container with indium foil and PM-scintillator activation counter.

plasma layer may be measured as a sharp rise in magnetic field as the sheath sweeps past the probe. This measurement may be used to confirm the dynamics required to ensure a good focus.

An indium foil activation system is used to count fusion neutrons from the plasma. This system consists of an indium foil covering an NE 102 scintillator sitting on the photocathode of a 2-in. photomultiplier tube. The assembly is placed in a paraffin wax enclosure so as to thermalize the fusion neutrons. The detector is placed on the end flange of the plasma focus tube (no. 39). The PM tube is connected to a counter via a discriminator and a preamplifier and has a calibration constant of 5×10^4 neutrons per count, the counts being taken for a 30-s period immediately after the focus is fired.

IV. RESULTS

The system was tested between 13 and 15 kV in various gases including air, argon, hydrogen, and deuterium. The strength of the focusing action is gauged from the current dip and voltage spike. Figure 4(a) shows an oscillogram of the current and voltage waveforms of the plasma focus in 0.5 Torr of air, with focusing action about 1 μs after peak current. Figure 4(b) shows a deuterium focus, at 13 kV, 2.5 Torr with focusing action occurring at peak current. The deuterium focus shows signs of a secondary focus occurring some 0.4 μs after the first voltage spike. The occurrence of definite clean dynamics in the axial region preceding the focus region is confirmed by magnetic probe measurements. Figure 4(c) shows the output of a magnetic probe (lower trace) placed at $z = 10.2$ cm (i.e., in the axial drive region 10.2 cm from the backwall) in a discharge of 15 kV, 3.5 Torr of deuterium. From this oscillogram and in comparison with the current oscillogram (upper trace) it is found that the current sheath arrives 0.6 μs before focusing occurs off the end of the anode at $z = 16$ cm giving a speed of 9.7 cm/μs (corresponding, from shock theory, to a temperature $\sim 2 \times 10^5$ K) over this section ($z = 10.2$–16 cm) of the axial drive region. From the rise time (10%–90%) of the magnetic signal and the speed this gives a current sheath thickness of 2 cm. The thickness and speed of this current sheath is typical of that in a good plasma focus system. The current dip during focusing is also seen as a dip in the magnetic probe output that shows two other current dips occurring at 0.2 and 0.6 μs after the first dip. These confirm the occurrence of multiple focusing in deuterium in the device.

In air good focus was obtained at 13 and 15 kV in a narrow pressure range of 0.5–1.1 Torr. In argon the pressure range for good focusing is greater at 0.3–3 Torr. At 15 kV very strong focusing action was obtained at 0.8 Torr. In helium the range of focusing is from 0.7 to 3.5 Torr while in carbon dioxide focusing is observed below 1 Torr. In hydrogen the pressure range for focusing is 1.1–6 Torr. However, it is noticed that the focusing action, although definite, is not as intense, in terms of a focusing voltage spike, as in argon. The strongest focus in hydrogen occurs at 3.3–4.3 Torr. In deuterium strong focus is observed at 1–5 Torr with best focusing at 2.5–3 Torr.

In deuterium when operated at 15 kV and optimum pressure conditions of 3 Torr consistent counts of 1000–2000 are obtained using the PM-scintillator counter. This corresponds to 0.5–1×10^8 neutrons per shot.

The system shows remarkably consistent and reproducible operation. Six systems were assembled one after the other and tested over a period of 2 months. Each system was assembled and tested over a period of 1 week averaging between 100–200 shots in the various gases. Once a system

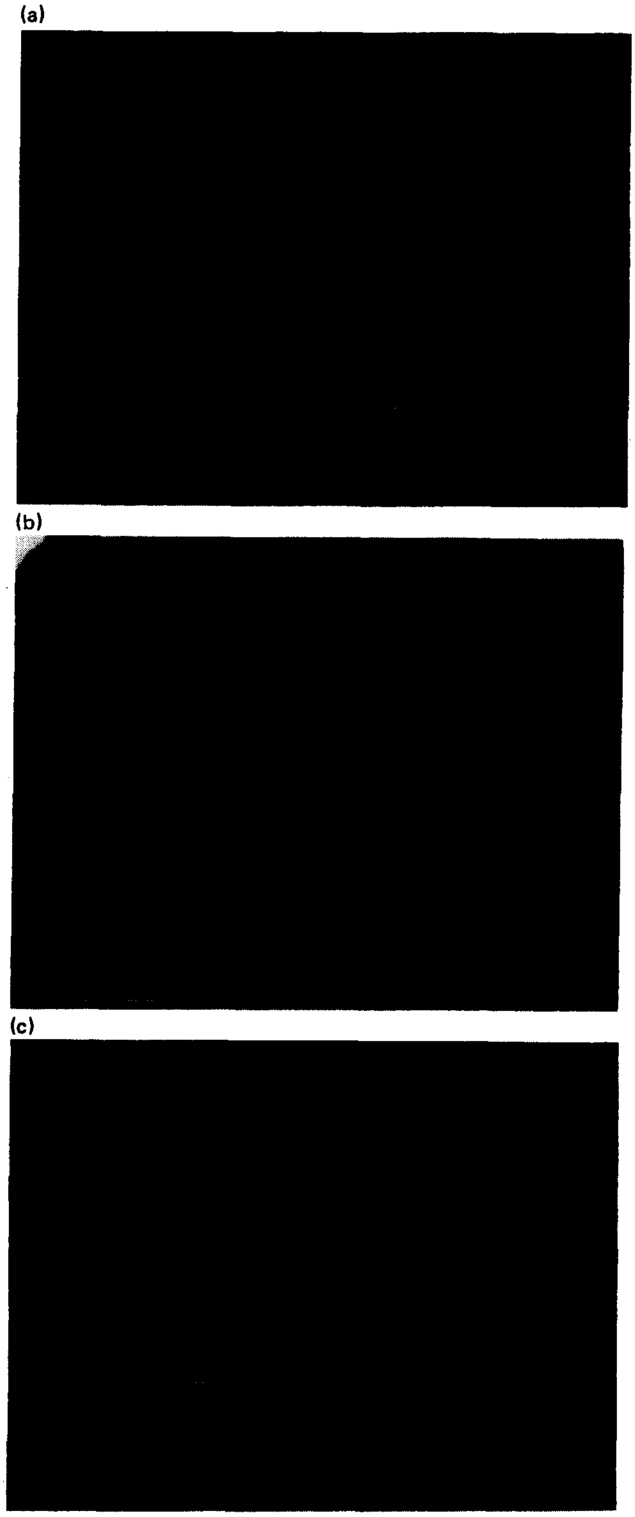

Fig. 4. (a) Current (upper trace) and voltage (lower trace) of plasma focus in air: 13 kV, 0.5 Torr; Top trace: 73 kA/cm; Bottom trace: 2 kV/cm; Time scale (horizontal): 1 μs/cm. (b) Current and voltage trace of plasma focus in deuterium: 13 kV, 2.5 Torr; Top trace: 75 kA/cm; Bottom trace: 4 kV/cm; Time scale (horizontal): 1 μs/cm. (c) Current and magnetic trace of plasma focus in deuterium: 15 kV, 3.5 Torr; Magnetic probe placed at $z = 10.2$ cm; Top trace: 73 kA/cm; Bottom trace: 0.6 T/cm.

has been established to be operating normally, that is, without undue leakage and after an initial period of out-gasing involving some three to five discharges, proper focusing is achieved for better than 95% of the discharges, apart from those discharges deliberately operated outside the established suitable pressure range for the gas used.

V. DISCUSSION

It is of interest to estimate the range of temperatures and densities available in this device. This is best discussed separately for each of the two phases of operation, namely, the axial drive phase and the radial collapse phase. In deuterium in typical operating conditions in the axial drive phase between $Z = 8$ and $Z = 16$ cm a steady plasma temperature of 2×10^5 K may be estimated (as shown above from the measured shock speed) in a slug length of 4 cm. For operating at an ambient density of 3 Torr of deuterium, complete ionization is achieved at this temperature giving an ion density and an electron density each of about 10^{18} per cm^3 since a shock mass density ratio of 4 may be expected for a strong fully ionized deuterium shock. In the radial focus phase, soft x-ray techniques[19,20] have been used to estimate temperatures in a similar small plasma focus to be 0.7–3 keV while interferometric techniques[21] have been used to measure peak electron densities in the maximum compressed pinch column of $1–5 \times 10^{19}$ per cm^3. These experimental results agree with computations based on the dynamic theory[15] already discussed in Sec. II.

When heavier gases such as argon are used as the test gas the dynamic theory predicts enhanced compression due to the specific heat ratio[22] being reduced below 5/3. In the case of argon, higher temperature (about 4 keV) and electron densities (10^{20} per cm^3), may then be predicted by the dynamic theory applied to the pinch phase. There is also experimental evidence[23] to back up these predictions.

At such high temperatures, thermodynamic computations show that even argon becomes almost fully ionized[22] and experimental work shows that worthwhile spectroscopic studies may be made of the focus using, e.g., argon as a test gas. Peacock[23] *et al.* have used a 2-m Rowland circle grating spectrograph and a de Broglie spectrometer to obtain spectrograms of argon and neon focus discharges. The results are sufficiently reproducible for identification of transitions in H-like and He-like Ne and Ar ions. If line profile scans are to be made then it is usual to obtain a profile over a number of discharges. The question of reproducibility then becomes important. In the present setup the reproducibility has been studied by using two simultaneous monitoring criteria. First, the neutron yield of each discharge is monitored and those that are beyond 10% of the average may be rejected. Second, the voltage spike is monitored for its time position and its shape (rate of rise, single spike, or multiple spikes). The time position of its peak (single peak spike) may be used as a reference point to fix the time position of the photomultiplier output. A discharge with a voltage spike shape that does not conform with the average shape may also be rejected. A close study of the data shows that when the system is properly set up and adjusted, at least 80% of the discharges are sufficiently reproducible for scanning applications.

It may also be of interest to inquire about the types of experiments that may be done on a plasma focus machine. First, as already mentioned in detail earlier, the axial drive phase may be used to study plasma dynamics and energetics.[19] The use of simple voltage, current, and magnetic probes, together with a coupled circuit-dynamic analysis enables one to obtain the dynamics and energetics of the system and also shock plasma temperature and densities. These may be confirmed with spectroscopic and interferometric measurements to check the validity of the dynamic model used.

In the radial pinch phase, the pinching action is more severe than the Z or theta pinch for two reasons. The first is that the use of the axial drive phase delays the focus pinch so that it occurs at peak current and enables a smaller radius pinch to occur at higher ambient density. Second, the smaller radius pinch is an elongating pinch that also contributes to an enhancement of pinch compression due to the smaller resulting pinch radius ratio.[24] Thus even a small plasma focus achieves sufficiently intense plasma conditions to produce consistent nuclear fusion and may be used as the lowest-cost device for demonstrating nuclear fusion from a plasma. Even a simple 3-kJ device such as the one presently discussed here may be used as a starting point for studying neutronics.[25] For example, measurement of the half-life of ^{116}In has been carried out[26] using the UNU/ICTP PFF as the neutron source. It is known that the deuterium focus produces a deuteron beam[21] of several hundred keV. The effect of this beam on targets may also be studied for the enhancement of neutron yield.[25] The corresponding electron beam, accelerated in the opposite direction into the anode, has relativistic speed[27] and may be taken out of the system by using a hollow electrode. Thus the focus may also be used as a REB source. These two effects, i.e., consistent neutron and REB production, are not available in the Z pinch and theta pinch[23] because of insufficient densities (10^{17} per cm^3) and temperatures (several hundred eV). This demonstrates the enhanced intensity of the plasma focus device.

The REB may also be used to sputter anode material downstream of the focus.[21,28] By using different materials as inserts in the anode face, different materials may be sputtered.

Other experiments and applications may be listed.[21] In the experience of the UNU Training Programme[29] the study of plasma dynamics particularly in the axial drive phase and the demonstration and study of plasma nuclear fusion are in themselves sufficiently interesting and of sufficient scope for a good beginning to be made in the field of experimental plasma physics.

VI. CONCLUSION

A simple cost-effective plasma focus device based on a 3.3-kJ single capacitor and a maintenance-free parallel-plate spark gap has been designed and operated as a reproducible and reliable neutron source (10^8 neutrons per discharge in deuterium), as well as a reliable source of focused plasmas of air, argon, hydrogen, helium, and carbon dioxide. In a period of 3 months six sets of the equipment were constructed and tested in the course of a UNU Training Program. These sets are now being reassembled in various countries as postgraduate teaching facilities. The remarkably reproducible results suggest that this device may be readily built as a very cost-effective facility for the teaching of plasma dynamics, plasma phenomena, and plasma nuclear fusion. This could be a valuable contribution, particularly to the developing countries where dense plasma facilities are hardly available.

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A demonstration calculation of self-consistent stellar models with convective cores

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A quick procedure for calculating self-consistent stellar models is described. The procedure involves the matching of radiative stellar envelope solutions to convective core solutions. The core is assumed to have the adiabatic temperature gradient. This procedure was developed for demonstrating stellar model calculations in upper year undergraduate astrophysics courses.

I. INTRODUCTION

In upper year undergraduate astrophysics courses it is common to introduce stellar model calculations using the standard methods used in, e.g., Novotny.[1] Chemically homogeneous spherically symmetric stellar models only are considered and represent main sequence stars. The four stellar differential equations are the equation continuity of mass,

$$\frac{dM}{dr} = 4\pi r^2 \rho, \tag{1}$$

with M the mass inside radius r and ρ the density at r; the equation of hydrostatic equilibrium,

$$\frac{dP}{dr} = \frac{-GM_\rho}{r^2}, \tag{2}$$