Electrooculography: A new approach

Developing a Human-Computer-Interaction method with a low budget

MATHEMATICAL MODEL FOR BIOMEDICAL SYSTEMS

&

THEIR ENGINEERING ANALOGY

Luis Cruz

http://www.ees.intelsath.com/

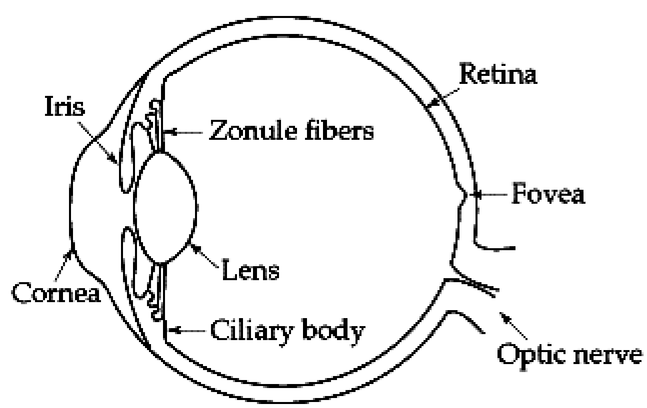
Introduction

People with physical disabilities face a lot of problems in communication with their fellow human beings. In this report, the design of an eye-controlled software focusing on a Human-Computer-Interface (HCI) based on Electro-oculography (EOG) is presented. Computers can be used by persons with disabilities for communication, environmental control, source of information and entertainment, however, a type of HCI must be used in order to give the user the ability to manipulate data.

There are a quite a few eye movement tracking systems for cursor control. But some of them are equipped with sophisticated systems using complicated designs and using a very high-priced hardware. The goal of this project is to provide a reliable system as a very affordable solution for these people.

Electro-oculography (EOG) is a technology that consists of placing electrodes on the person’s forehead around the eyes to record eye movements. The voltage that exists between the eyes is a very small electrical potential that can be detected using electrodes. People with certain disabilities may use these systems in order to have certain communication.

The voltage difference is measured between the cornea and the retina. The resting potential ranges from 0.4mV to 1mV. However, the voltage difference when there's an eye movement can be as small as just some microvolts. One electrode is more positive or negative with respect the ground electrode, therefore, the recorded signal is either negative or positive.



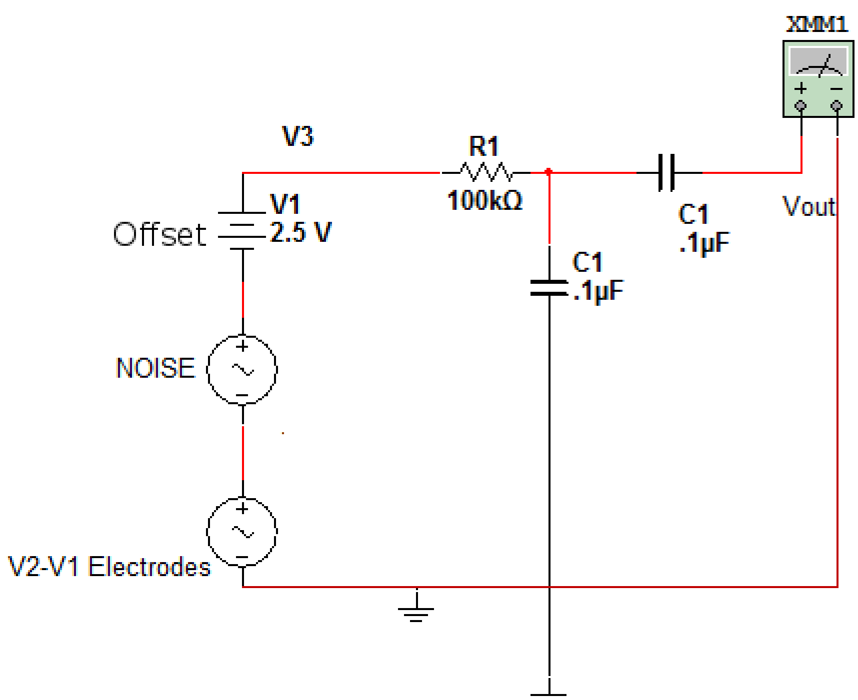
After the development of the bioamplifier, digital electronics and software engineering become some handy fields when it comes to the development of the Human-Computer-Interaction softwares. I hereby present the mathematical model I've developed for my system, with emphasis on many other biomedical systems in general as well.

Mathematical Model

$$V_{out} =$$

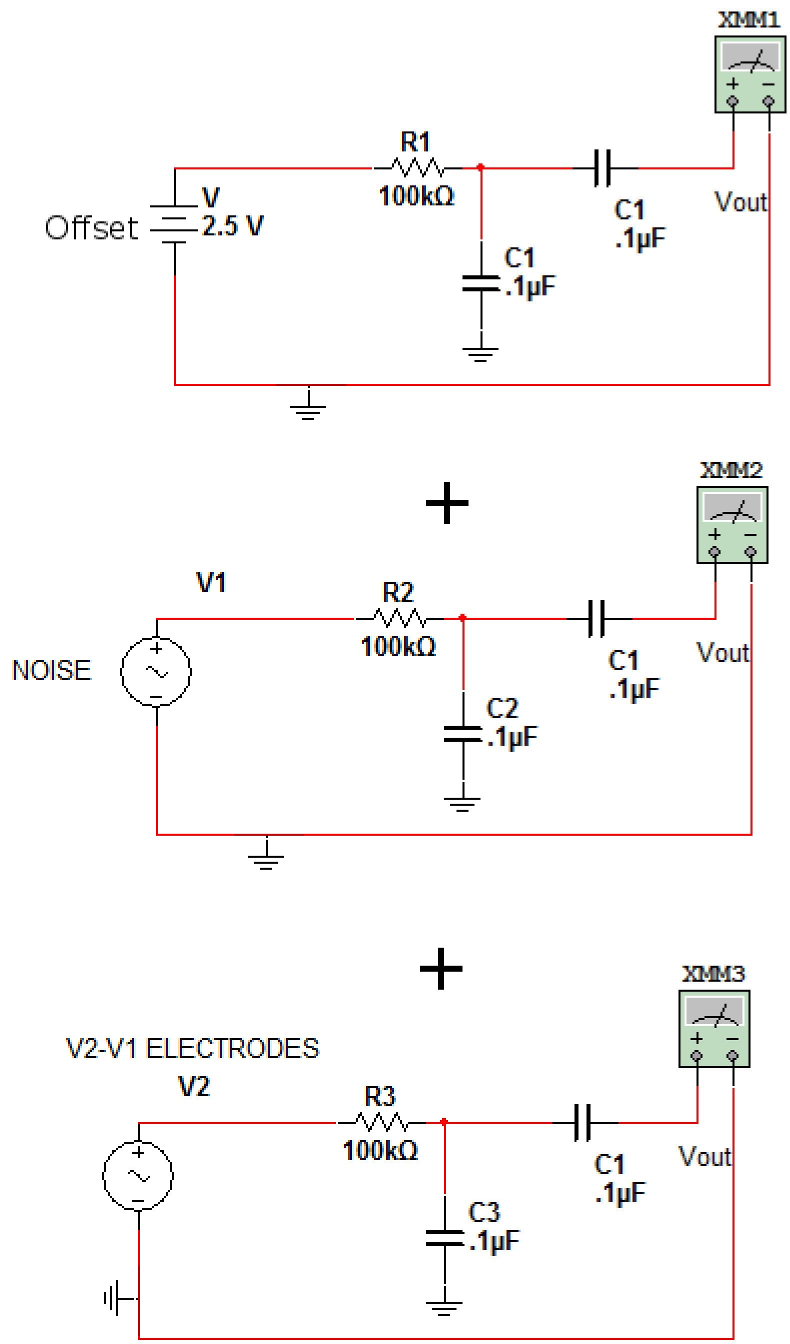
$$\frac{dV}{dt}\left[g(v2-v1) + |A_{cm}|(v2-v1)/2 + (2\mathrm{Log}_2 f*2*\pi*R*C)^{-1} *1/n* (V_N - g(v2-v1) + |A_{cm}|(v2-v1)/2\right] + R2/(R1+R2) * V$$

Proof



The system relies on three different factors: Offset, noise, and the differential voltage.

In electronics engineering these factors affecting the output signal should be summed in order to calculate the output voltage.



The output signal on the biomedical system should be a sum of the offset, noise, and the differential potential.

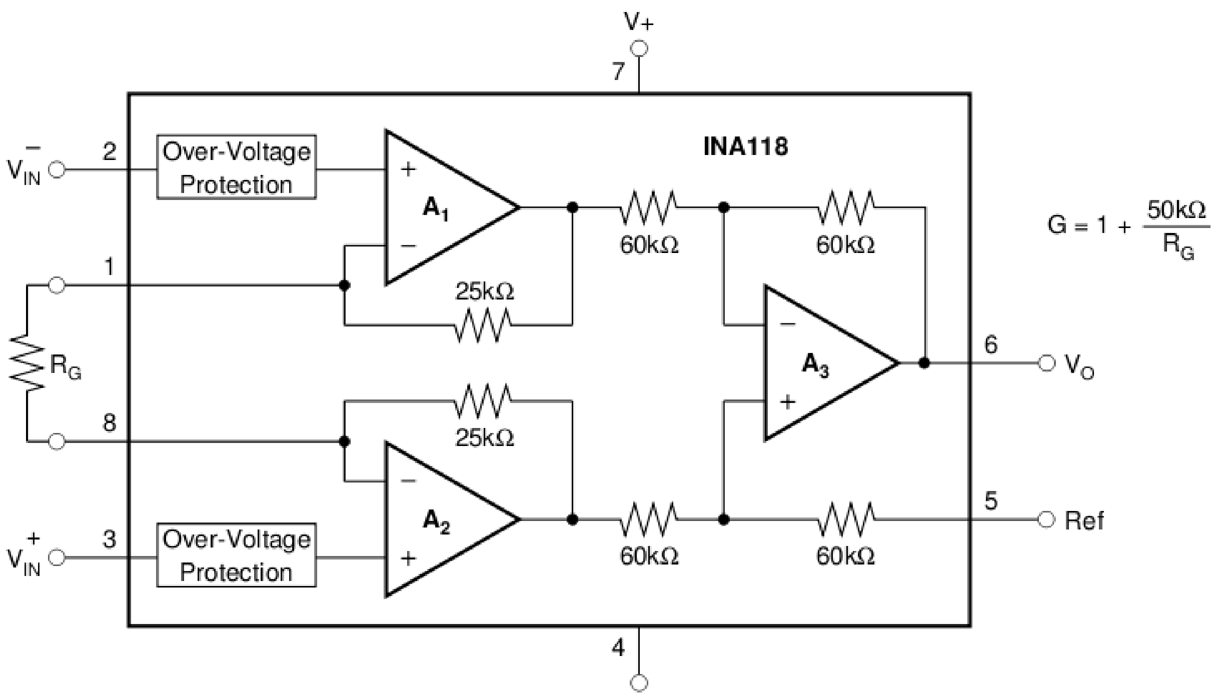
Vout = Offset + Noise + ΔV

The differential amplifier: INA118

The instrumentation amplifier INA118 is one of the best options in biomedical systems. The INA118 is a low power, general purpose instrumentation amplifier offering excellent accuracy. Its versatile 3-op amp design and small size make it ideal for a wide range of applications.

$$Gain = g,\ g = 50k/Rg + 1$$

The INA118 has a high Common Mode Rejection Ratio, 110dB at G = 1000.



Output voltage: g(V2-V1) + 1/2|Acm|(V2-V1)
Where Acm is the CMRR 'negative' gain.

INA118 CMR gain ~ 316227.732 ∵ 20Log |Acm| = 110dB, Acm = 10^5.5 ~ 316227.73

Noise Reduction
Noise reductions is the process of removing noise from a signal. Or removing unwanted perturbation of a wanted signal, usually noise comes at a high frequency. When amplifying very low signals, noise is more likely to be present.
Low-pass filters are commonly used to attenuate high frequency noise. A low pass filter is a filter that passes low-frequency signals but attenuates signals with frequencies higher than the cutoff frequency. The cutoff frequency is the boundary in a system's frequency response at which the energy flowing through the systems begins to be reduced (attenuated).

To calculate the cutoff frequency on a filter the following formula was used:

1÷[(2*pi*R1+R2...+Rx*C1+C2...Cx)^(1/n)]

Where *R* is the resistance used in the filter in ohms, *C* is the capacitance in Farads and *n* is the order of the filter, or the number of reactive components used in the filter (the number of capacitors). *2π* is the change from Radians to a full cycle, the cutoff frequency is thus represented in Hertz.

E.g A high order filter is recommended to get a more stable and clean signal. The cutoff frequency of a second order low pass filter is calculated:

*1÷[√(2*pi*R*C)]*

On this project a second order filter was used, however, another technique was used. Two passive low pass filters were put in cascade, both with a cutoff frequency of around 16 Hz:

*1÷[(2*pi*100kΩ*0.1µf)]*
*1÷[(2*3.1416*100000Ω*0.0000001F)] ~ 15.9Hz*

A passive low pass filter gives an attenuation of 6dB/octave. That means that whenever the frequency doubles, the voltage should be reduced by 2(this amount changes according to the order), or attenuated by half.
Therefore the noise amplitude should be divided by the number of octaves times 2.

|G| = 2, or G = dB/8ve , Octaves = (2Log F/Fc) / Log 2

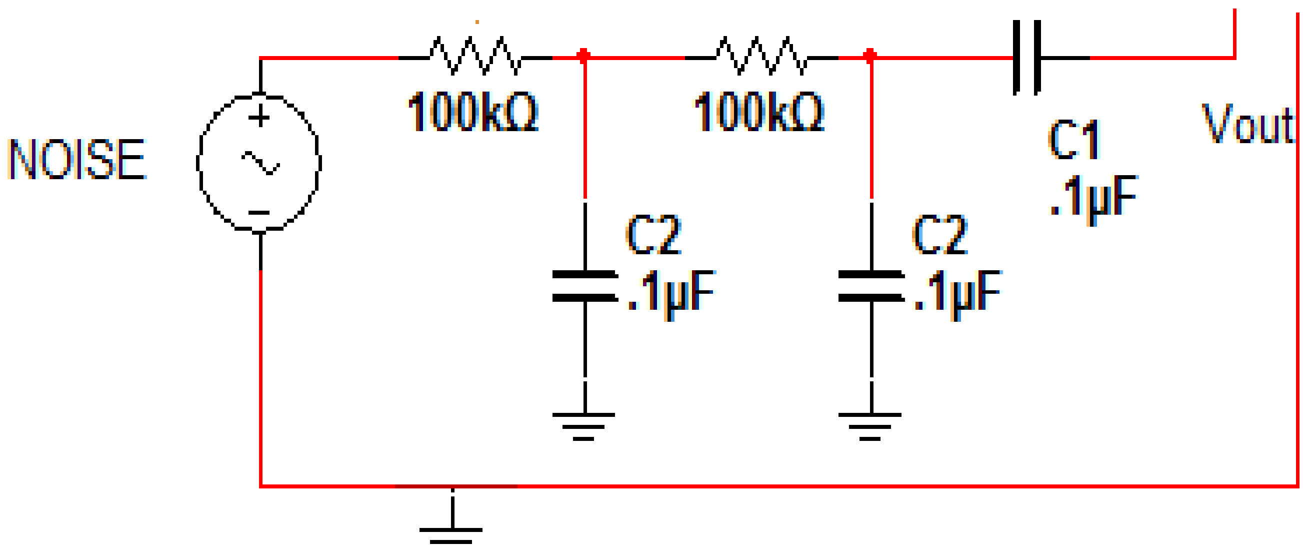
Where F is the current frequency, and Fc is the cutoff frequency.

Attenuation:

Vnoise ÷ *(2n * octaves)* ∴ Vn ÷ *(2n * log2 f/[1/2pi*R*C])*
→ *Vn * (2n * Log2 f*2*pi*R*C)^ -1*

Where n is the number of poles, or the order number of the filter.

Second order filter: two passive low pass filters in cascade.



Removing DC Offset

There's a resting potential between the eyes, this “constant” voltage varies depending on several factors such as light, eyes' size, skin conductivity etc.
After amplifying the differential voltage using the Operational amplifiers that resting potential is unwanted in the EOG signal. On this project the system should be able to read the fast eye horizontal movements and it's therefore, measured as a signal with a slope. To remove the unwanted DC offset and just be able to read the wanted signal wave, simply a small capacitor is added.

The current (measured in Amperes) that passes through a capacitor is defined by:

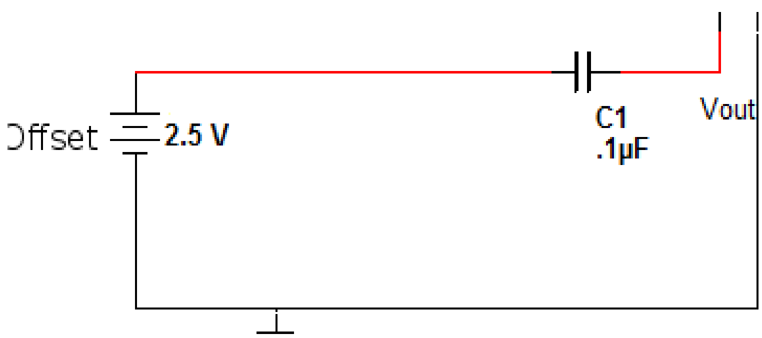
$$\mathbf{I = C * dv/dt}$$

The current is defined as the capacitance times the rate of change of the voltage that passes through a capacitor, hence when the derivative of the constant potential is zero (the derivative of a constant value is = 0) the voltage after the capacitor is going to be equal to zero.

$$I = C * dv/dt$$

$$dv/dt = 0 \Rightarrow I = 0 \therefore V = 0 \because V = IR \rightarrow V = 0*R \rightarrow V = 0$$

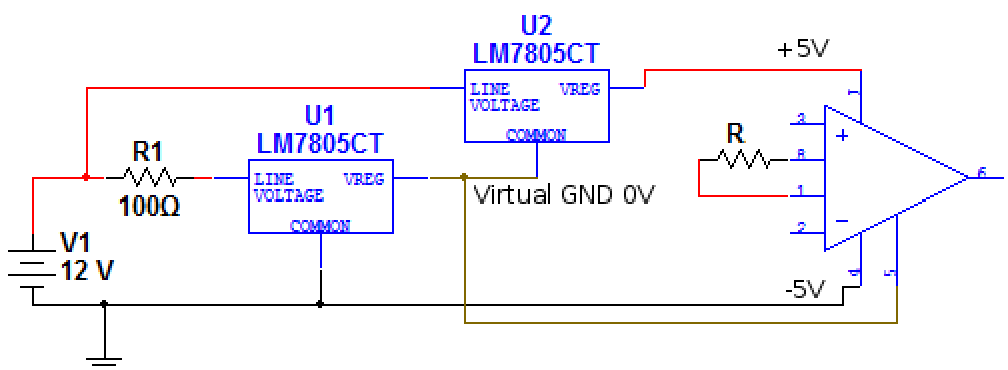
Output Voltage = 0



Operational Amplifiers offset issue

The operational amplifiers need an offset so it can work correctly, specially on this case while working with negative and positive voltages. Adding an simple offset would make the reading unstable, because of the fact that the resting potential in the electrodes is not constant but depends on several environmental factors.

Op amps usually work with dual polarity supply, that is positive and negative voltage with respect to virtual ground. To lower costs and complexity, just a couple of 7805 voltage regulators were used for a dual power supply.



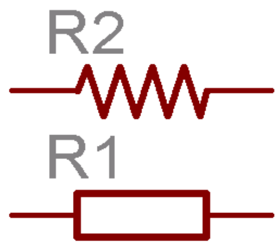
Adding a controlled DC offset

The output voltage is between ± 5V, the microcontroller, however, can only read voltages between 0 and +5V. Therefore, it's important to add a constant DC offset. This can be easily accomplished using an extra op-amp, in order to keep the circuit simple though, just a couple of high impedance resistors are used to create a voltage divider circuit. A voltage divider is just a simple linear circuit that produces an output voltage (Vout) that is a fraction of its input voltage (Vin).

Resistors dividers in series of high impedance can be used to have a constant DC offset on the output, the following formula is used to calculate the output voltage:

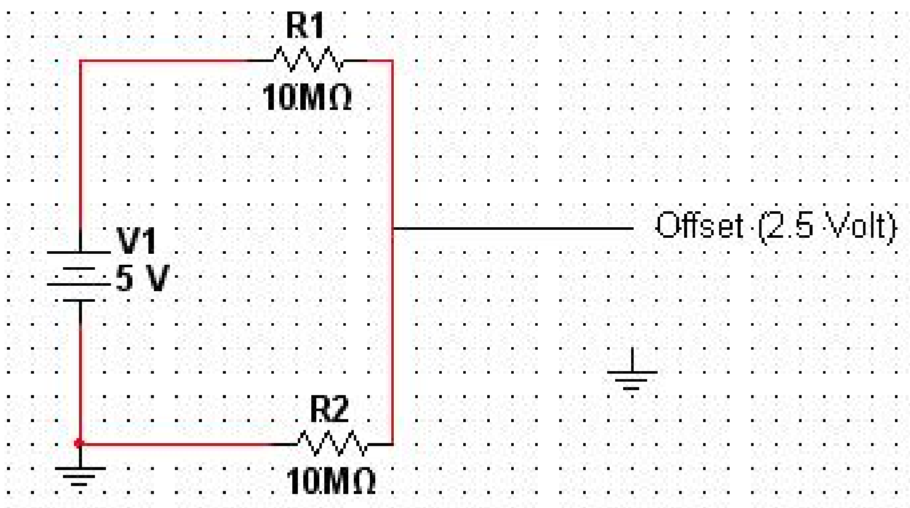
Vout = [R2 ÷ *(R1+R2)] * Vin*

*10MΩ ÷ (10MΩ + 10MΩ) * 5V = 2.5V*



R1 = R2 => R/(2R) * Vin = Vin/2

∴ Vin = 5 => Vout = 2.5V



The microcontroller

In digital electronics, a microcontroller is a small computer built into a single integrated circuit.
A very special function of a microcontroller is the use of Analog-to-Digital Converters (ADC) and Digital-to-Analog converters (DAC), that is the conversion from analog signals to digital information and vice versa.

The output voltage from the EOG circuit is then read by the microcontroller using a 10-bits ADC, the output data returned from the microcontroller according to Vout is therefore less than 1024.

The microcontrollers used for this project: Atmega328P. It can be programmed using C language or assembly.

```
volatile int center = 0;

volatile int centerTimer;//decides wheter or not the timer should be triggered according to the
volatile int32_t centerThe_time; //time at center

ISR(ADC_vect)   //ADC interrupt
{

     //Center: Due to the capacitor ability to remove the DC and the resistors dividers, the cc
     // I = C * dv/dt, the derivative of a constant is cero; therefore if I(current) equals cer
      if( adc_read() >= 501 && adc_read() <= 513 ){ center = 1; left = 0; right = 0; }

      if( center == 1 )
      {
          //set pins to high, +5V, do not send any info to other micro
          PORTB |= (1<<PB4);
          PORTB |= (1<<PB5);
          centerTimer = 1;
      }


   //Detect eye movements if the timer is running. if timer is running the derivative of the si
   if( centerThe_time >= 50 ){
      //left: Slope should be positive at ~ +50 in adc units, then it will decrease, negative v
      if( adc_read() >= 535 ){ left = 1; center = 0; right = 0; }

      //right: Slope should be negative at ~ -50 in adc units. positive val shall be ignored.
      if( adc_read() <= 460 ){ right = 1; left = 0; center = 0; }


      if( right == 1 )
```

By using complex concepts about low-level programming, I'm able to use interrupts and timers to make the data reading more reliable.

The HCI softwares

The communication between the PC computer and the microcontroller is done via the serial port, RS232 protocol. A python script is used to accomplish this task.

```
#Author: Luis Cruz

while True:
 import serial
 serial = serial.Serial("/dev/ttyUSB0", 115200, timeout=1)
#uses the serial port
 data = serial.read()

#right
 if data == "0":
  f = open("eog.txt", "w")
  f.write("0")
  f.close()

#center
 if data == "1":
  f = open("eog.txt", "w")
  f.write("1")
  f.close()

#left
```

C++ application

The python software decodes the serial port data, and then the information is sent to the C++ application so the user can interact with the graphical application on the computer. The Graphical user interface provides the user the ability to be able to choose letters from some boxes and form complete words. This is done just with simple horizontal eye movements, the letter is chosen by the user, then automatically after a 4 second delay the letter is then written “pasted” in the notepad.
Everything written by the user is then recorded in a text file that the user can then check.

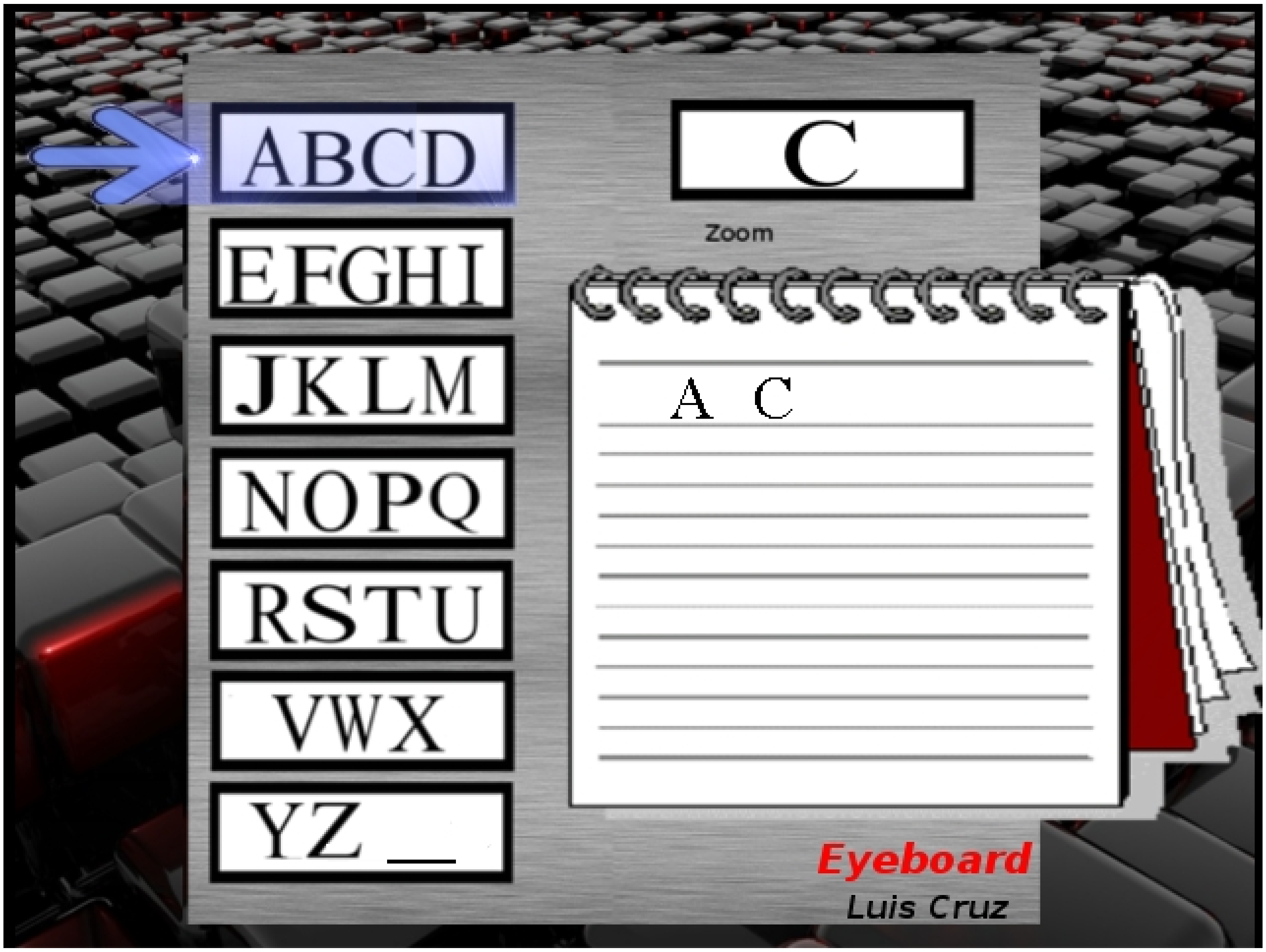
```
65, 44); //characters
   drawSprite(currentLetterNotepad2, screen, NULL, NULL, notepadPos[2], 182,
65, 44); //characters
   drawSprite(currentLetterNotepad3, screen, NULL, NULL, notepadPos[3], 182,
65, 44); //characters
   drawSprite(currentLetterNotepad4, screen, NULL, NULL, notepadPos[4], 182,
65, 44); //characters
   drawSprite(currentLetterNotepad5, screen, NULL, NULL, notepadPos[5], 182,
65, 44); //characters
   drawSprite(currentLetterNotepad6, screen, NULL, NULL, notepadPos[6], 182,
65, 44); //characters
}


//keeps tracks of time
      thisTime = SDL_GetTicks();
      deltaTime = (float)(thisTime - lastTime) / 1000;
      lastTime = thisTime;

      speedX += SPEEDX * deltaTime;

    if( speedX >= 100 ){
     if( z3 == 0 ){
```

The software's GUI



The box can be chosen using left eye movements, and while inside the box a letter can be chosen with right eye movements.

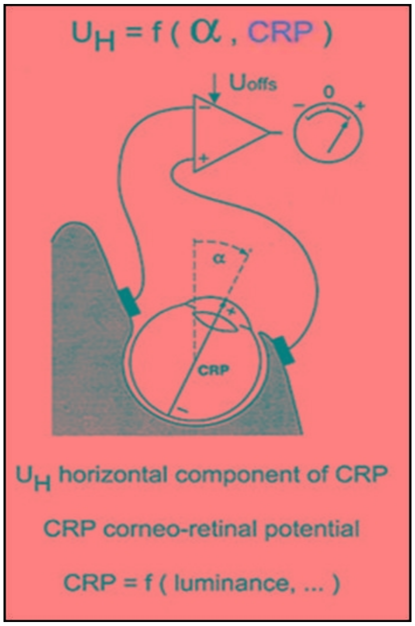
Further information
Engineering Concepts, Computer Science, Digital Electronics & more!

Most information on the following section was used as study references to develop some aspects of the project.
Some information was taken out from the following websites (Bibliography):

http://www.electronics-tutorials.ws/
http://zone.ni.com/devzone/cda/tut/p/id/11958
http://en.wikipedia.org/wiki/Microcontroller

The Electrooculogram

The human eye is polarized, with the front of the eye being positive and the back of the eye being negative. This is caused by a concentration of negatively charged nerves in the retina on the back of the eye. As the eye moves the negative pole moves relative to the face and this change in the dipole potential can be measured on the skin in micro volts. To translate this voltage into a position, two sets of electrodes are used to measure the differential voltage in the vertical and horizontal direction, on this project, however, just horizontal movements are recorded. The figure below indicates how the electrodes are placed on the face. The red and black leads measure movement in the horizontal direction and the white and brown leads measure movement in the vertical direction. The green sensor is placed behind the ear or on the ear lobe to provide a ground reference.

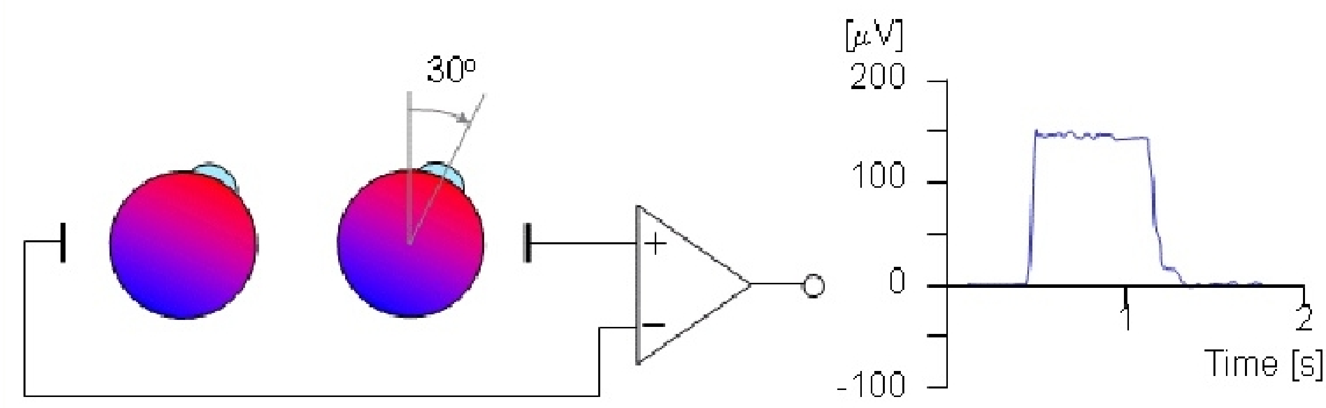


Using these leads we can translate each change in voltage into a change in the eye's position. If the eye looks to the right for instance, the positive pole of the eye will be nearest the right side sensor causing it to read a positive voltage while the back of the eye will be nearest the left side sensor causing it to read a negative voltage. From the voltage potential between these two sensors the left/right position of the eye can be determined. The same technique is used to determine the vertical position.

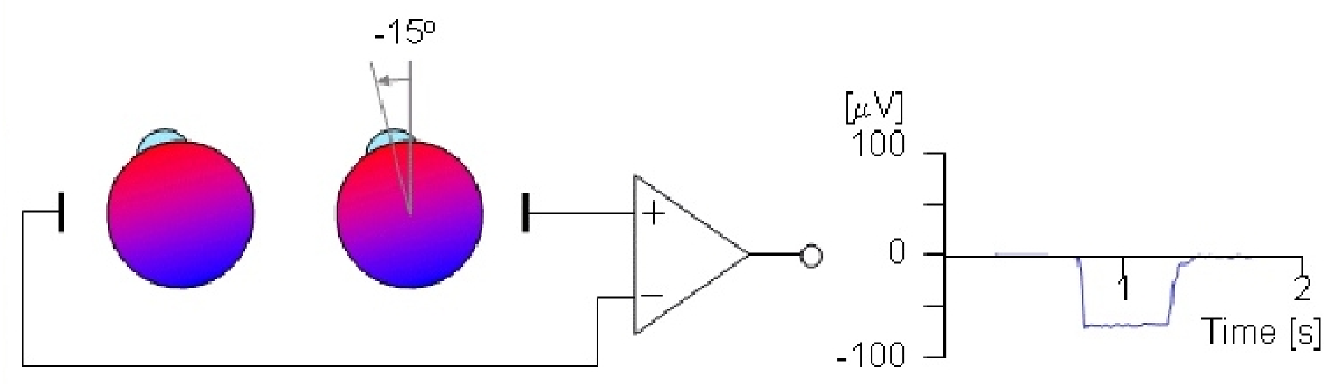
The eye has a standing electrical potential between front and back, sometimes called the corneo-fundal potential. The potential is mainly derived from the retinal pigment epithelium (RPE), and it changes in response to retinal illumination .

The potential decreases for 8–10 min in darkness. Subsequent retinal illumination causes an initial fall in the standing potential over 60–75 s (the fast oscillation (FO)), followed by a slow rise for 7–14 min (the light response). These phenomena arise from ion permeability changes across the basal RPE membrane.

The clinical electro-oculogram (EOG) makes an indirect measurement of the minimum amplitude of the standing potential in the dark and then again at its
peak after the light rise. This is usually expressed as a ratio of 'light peak to dark through' and reffered to as the Arden ratio.



Eyes moving 30º to the right

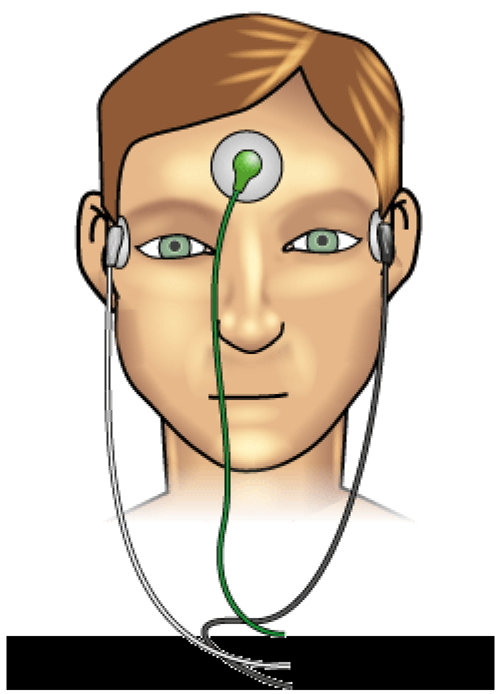


Eyes moving 15º to the left

The EOG on this project is used as a way to interact with the computer. However, there are some other uses as well.

Other Applications

• Sleep and dream research
• Eye tracking for marketing purposes
• Reading ability and visual fatigue
• Retinal dysfunction
– Some disorders of the retina exhibit abnormal or absent changes in the corneal-retinal potential during dark and light adaptation processes.
• Vestibular and balance dysfunction
– Nystagmus: characteristic slow-phase/fast-phase eye movement measurable by EOG
– prevent drunk drivers from taking to the road.



The Operational Amplifier

INTRODUCTION

The operational amplifier is an extremely efficient and versatile device. Its applications span the broad electronic industry filling requirements for signal conditioning, special transfer functions, analog instrumentation, analog computation, and special systems design. The analog assets of simplicity and precision characterize circuits utilizing operational amplifiers.

Computation Control Instrumentation

Originally, the term, "Operational Amplifier," was used in the computing field to describe amplifiers that performed various mathematical operations. It was found that the application of negative feedback around a high gain DC amplifier would produce a circuit with a precise gain characteristic that depended only on the feedback used. By the proper selection of feedback components, operational amplifier circuits could be used to add, subtract, average, integrate, and differentiate.

As practical operational amplifier techniques became more widely known, it was apparent that these feedback techniques could be useful in many control and instrumentation applications.

Today, the general use of operational amplifiers has been extended to include such applications as DC Amplifiers, AC Amplifiers, Comparators, Servo Valve Drivers, Deflection Yoke Drivers, Low Distortion Oscillators, AC to DC Converters, Multivibrators, and a host of others.

What the operational amplifier can do is limited only by the imagination and ingenuity of the user.

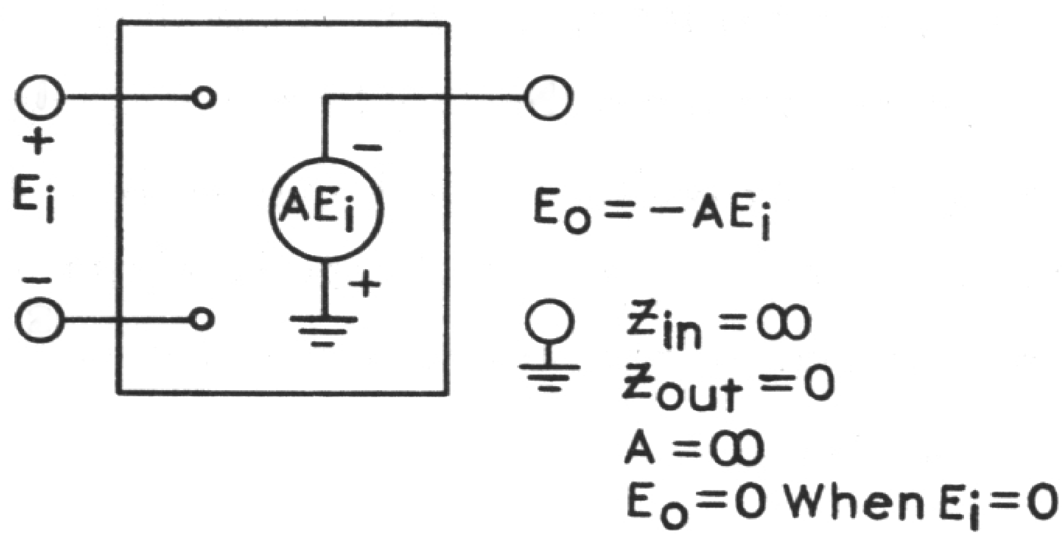
With a good working knowledge of their characteristics, the user will be able to exploit more fully the useful properties of operational amplifiers.

The Feedback Technique

The precision and flexibility of the operational amplifier is a direct result of the use of negative feedback. Generally speaking, amplifiers employing feedback will have superior operating characteristics at a sacrifice of gain.

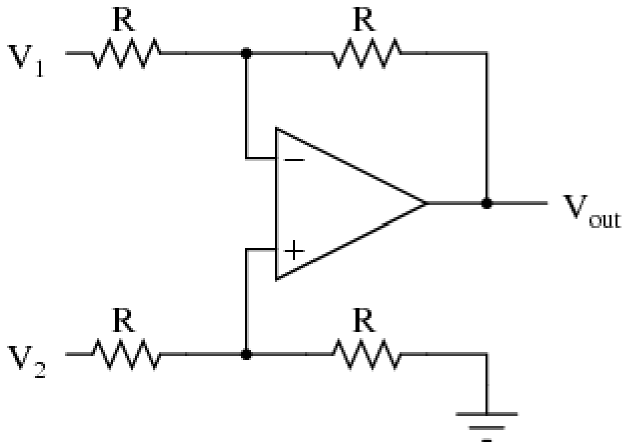
In order to introduce operational amplifier circuitry, we will use an ideal model of the operational amplifier to simplify the mathematics involved in deriving gain expressions, etc., for the circuits. With this understanding as a basis, it will be convenient to describe the properties of the real devices themselves in later sections, and finally to investigate circuits utilizing practical operational amplifiers.
To begin the presentation of operational amplifier circuitry, then, it is necessary first of all to define the properties of a mythical "perfect" operational amplifier.

- Input Impedance: Input impedance is assumed to be infinite. This is so the driving source won't be affected by power being drawn by the ideal operational amplifier.
- Output Impedance: The output impedance of the ideal operational amplifier is assumed to be zero. It then can supply as much current as necessary to the load being driven.
- Response Time: The output must occur at the same time as the inverting input so the response time is assumed to be zero. Phase shift will be 180°. Frequency response will be flat and bandwidth infinite because AC will be simply a rapidly varying DC level to the ideal amplifier.
- Gain: The primary function of an amplifier is to amplify, so the more gain the better. It can always be reduced with external circuitry, so we assume gain to be infinite.
- Offset: The amplifier output will be zero when a zero signal appears between the
- inverting and non-inverting inputs.



The Differential Amplifier

An op-amp with no feedback is already a differential amplifier, amplifying the voltage difference between the two inputs. However, its gain cannot be controlled, and it is generally too high to be of any practical use. So far, our application of negative feedback to op-amps has resulting in the practical loss of one of the inputs, the resulting amplifier only good for amplifying a single voltage signal input. With a little ingenuity, however, we can construct an op-amp circuit maintaining both voltage inputs, yet with a controlled gain set by external resistors.

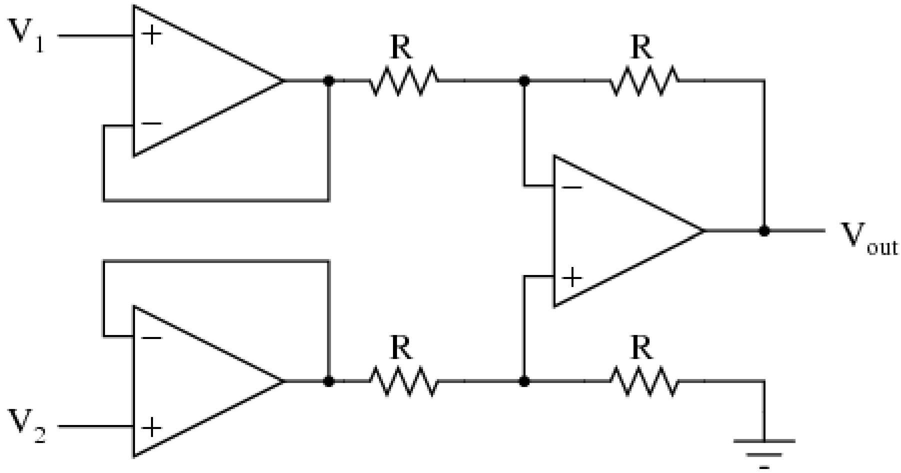


If all the resistor values are equal, this amplifier will have a differential voltage gain of 1. The analysis of this circuit is essentially the same as that of an inverting amplifier, except that the noninverting input (+) of the op-amp is at a voltage equal to a fraction of V_2, rather than being connected directly to ground. As would stand to reason, V_2 functions as the noninverting input and V_1 functions as the inverting input of the final amplifier circuit. Therefore:

$$V_{out} = V_2 - V_1$$

If we wanted to provide a differential gain of anything other than 1, we would have to adjust the resistances in *both* upper and lower voltage dividers, necessitating multiple resistor changes and balancing between the two dividers for symmetrical operation. This is not always practical, for obvious reasons.

Another limitation of this amplifier design is the fact that its input impedances are rather low compared to that of some other op-amp configurations, most notably the noninverting (single-ended input) amplifier. Each input voltage source has to drive current through a resistance, which constitutes far less impedance than the bare input of an op-amp alone. The solution to this problem, fortunately, is quite simple. All we need to do is "buffer" each input voltage signal through a voltage follower like this:



Now the V_1 and V_2 input lines are connected straight to the inputs of two voltage-follower op-amps, giving very high impedance. The two op-amps on the left now handle the driving of current through the resistors instead of letting the input voltage sources (whatever they may be) do it. The increased complexity to our circuit is minimal for a substantial benefit.

Filters: Noise Reduction

From: http://www.electronics-tutorials.ws/

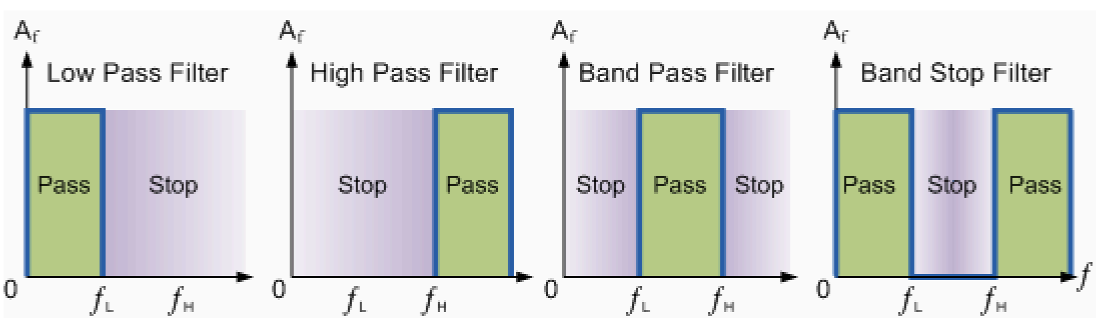
Low Pass Filter Introduction

Basically, an electrical filter is a circuit that can be designed to modify, reshape or reject all unwanted frequencies of an electrical signal and accept or pass only those signals wanted by the circuits designer. In other words they "filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency. In low frequency applications (up to 100kHz), passive filters are usually made from simple RC (Resistor-Capacitor) networks while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components. Passive filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

Filters are named according to the frequency of signals they allow to pass through them. There are **Low-pass filters** that allow only low frequency signals to pass, **High-pass filters** that allow only high frequency signals to pass through, and **Band-pass filters** that allow signals falling within a certain frequency range to pass through. Simple First-order passive filters (1st order) can be made by connecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output of the filter, (Vout) taken from the junction of these two components. Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a **Low Pass Filter** or a **High Pass Filter**.

As the function of any filter is to allow signals of a given band of frequencies to pass unaltered while attenuating or weakening all others that are not wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

Ideal Filter Response Curves



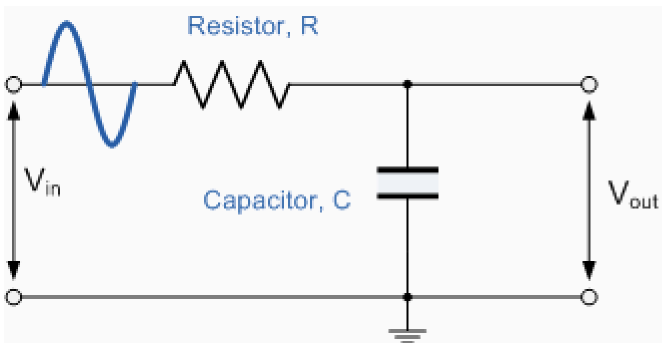
Filters can be divided into two distinct types: active filters and passive filters. Active filters contain amplifying devices to increase signal strength while passive do not contain amplifying devices to strengthen the signal. As there are two passive components within a passive filter design the output signal has a smaller amplitude than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the "cutoff" frequency. The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors.

The Low Pass Filter

A simple passive **Low Pass Filter** or **LPF**, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below. In this type of filter arrangement the input signal (Vin) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (Vout) is taken across the capacitor only. This type of filter is known generally as a "first-order filter" or "one-pole filter", why first-order or single-pole?, because it has only "one" reactive component in the circuit, the capacitor.

Low Pass Filter Circuit



The reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At low frequencies the capacitive reactance, (Xc) of the capacitor will be very large compared to the resistive value of the resistor, R and as a result the voltage across the capacitor, Vc will also be large while the voltage drop across the resistor, Vr will be much lower. At high frequencies the reverse is true with Vc being small and Vr being large.

While the circuit above is that of an RC Low Pass Filter circuit, it can also be classed as a frequency variable potential divider circuit. The following equation is used to calculate the output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

We also know that the capacitive reactance of a capacitor in an AC circuit is given as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

Opposition to current flow in an AC circuit is called **impedance**, symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

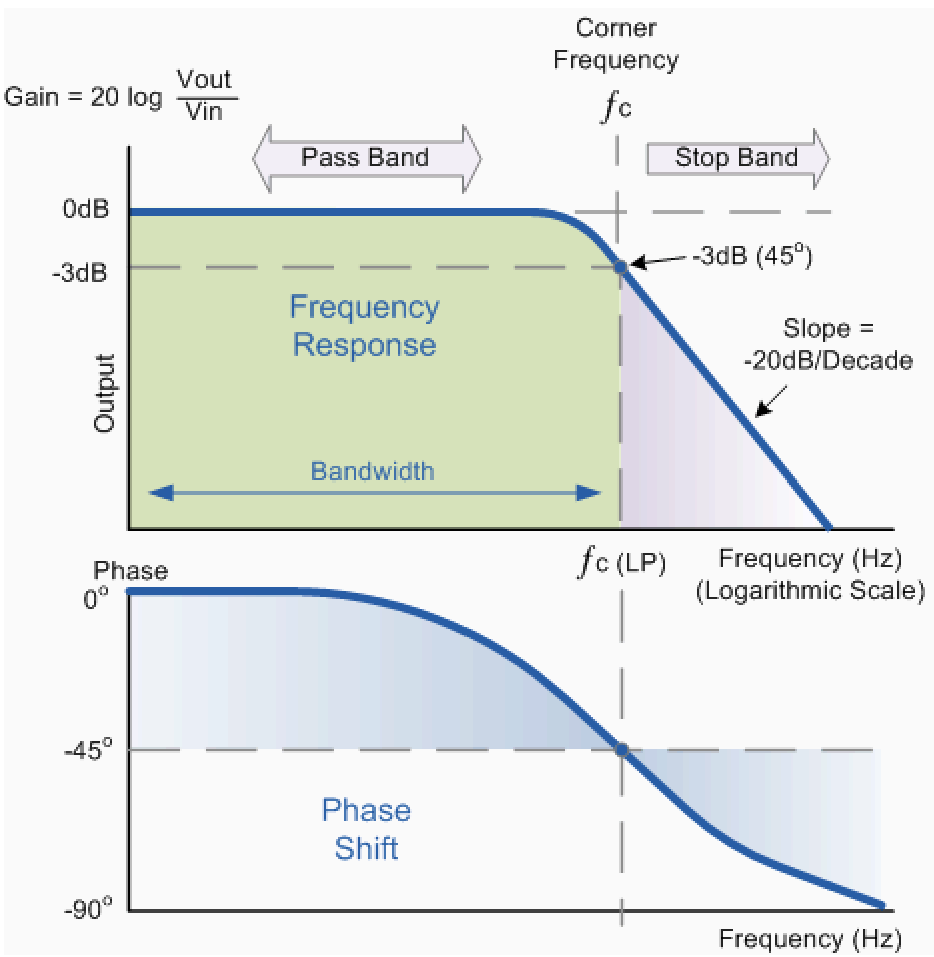
Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Frequency Response

We can see above, that as the frequency increases from 100Hz to 10kHz, the output voltage (Vout) decreases from 9.9v to 0.718v. By plotting the output voltage against the input frequency, the **Frequency Response Curve** or **Bode Plot** function of the low pass filter can be found, as shown below.Frequency Response of a 1st-order Low Pass Filter



The Bode Plot shows the **Frequency Response** of the filter to be nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its **Cut-off Frequency** point (ƒc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor. After this cut-off frequency point the response of the circuit decreases giving a slope of -20dB/ Decade or (-6dB/Octave) "roll-off" as signals above this frequency become greatly attenuated, until at very high frequencies the reactance of the capacitor becomes so low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

For this type of **Low Pass Filter** circuit, all the frequencies below this cut-off, fc point that are unaltered with little or no attenuation and are said to be in the filters **Pass band** zone. This pass band zone also represents the **Bandwidth** of the filter. Any signal frequencies above this point cut-off point are generally said to be in the filters **Stop band** zone and they will be greatly attenuated.

This "Cut-off", "Corner" or "Breakpoint" frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or **-3dB** (20 log (Vout/Vin)) of the input. Although R = Xc, the output is **not** half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input. As the filter contains a capacitor, the Phase Angle (Φ) of the output signal **LAGS** behind that of the input and at the -3dB cut-off frequency (fc) and is -45^{o} out of phase. This is due to the time taken to charge the plates of the capacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) "lagging" behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more "out of phase".

The cut-off frequency point and phase shift angle can be found by using the following equation:

Cut-off Frequency and Phase Shift

$$f c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

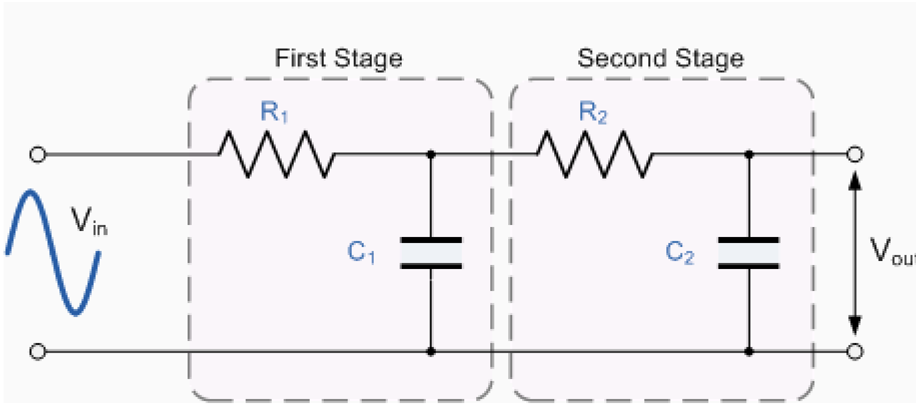
$$\text{Phase Shift } \varphi = -\arctan\,(2\pi f RC)$$

Then for our simple example of a "**Low Pass Filter**" circuit above, the cut-off frequency (fc) is given as 720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45^{o}.

Second-order Low Pass Filter

Thus far we have seen that simple first-order RC low pass filters can be made by connecting a single resistor in series with a single capacitor. This arrangement then gives us a -20dB/decade attenuation of frequencies above the cut-off point at f_{3dB} . However, sometimes this -20dB/decade (-6dB/octave) angle of the slope is not enough to remove an unwanted signal then two stages of filtering can be used as shown.

Second-order Low Pass Filter



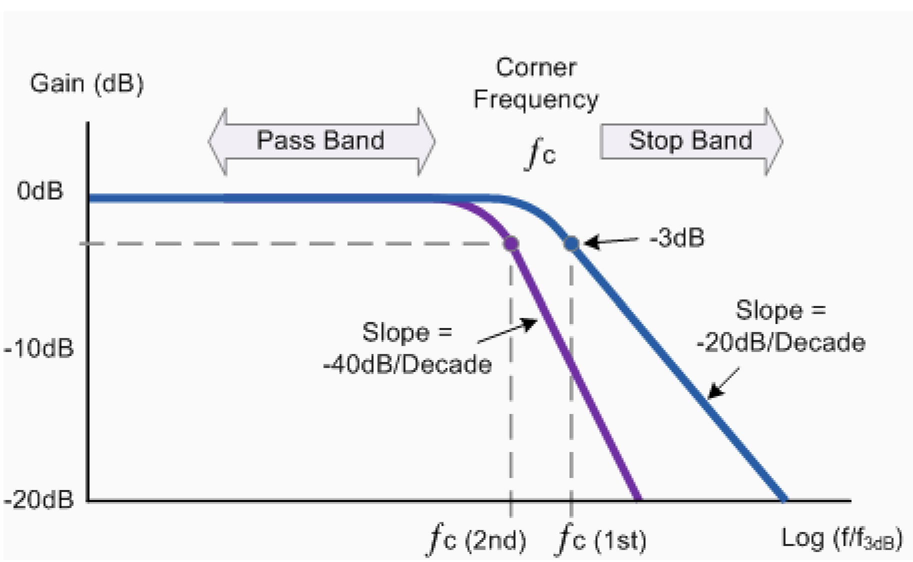
The above circuit uses two first-order low pass filters connected or cascaded together to form a second-order or two-pole filter network. Then a first-order low pass filter can be converted into a second-order type by simply using an additional RC network. If a number (n) of such filters are cascaded together, the resulting filter circuit would be known as an "n^{th}-order" filter with a slope of n x -20dB/decade. For example, a second-order filter would have a slope of -40dB/decade (-12dB/octave), a fourth-order filter would have a slope of -80dB/decade (-24dB/octave) and so on.

Second-order filters are important because higher-order filters can be designed using them. The cut-off frequency, fc is determined by both the resistors and capacitors as follows.

$$f_C = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

Then the frequency response for a second-order low pass filter assuming the same -3dB cut-off point would be:

Frequency Response of a 2nd-order Low Pass Filter



In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so R2 = 10 x R1 and C2 = 1/10th C1. Second-order and above filter networks are generally used in the feedback circuits of op-amps, making what are commonly known as *Active Filters* or as a phase-shift network in *RC Oscillator* circuits.

Low Pass Filter Summary

So to summarize, the **Low Pass Filter** has a constant output voltage from D.C. (0Hz), up to a specified Cut-off frequency, (fc) point. This cut-off frequency point is 0.707 or **-3dB** (dB = -20log Vout/Vin) of the voltage gain allowed to pass. The frequency range "below" this cut-off point fc is generally known as the **Pass Band** as the input signal is allowed to pass through the filter. The frequency range "above" this cut-off point is generally known as the **Stop Band** as the input signal is blocked or stopped from passing through. A simple 1st order low pass filter can be made using a single resistor in series with a single non-polarized capacitor (or any single reactive component) across an input signal Vin, whilst the output signal Vout is taken from across the capacitor. The cut-off frequency or -3dB point, can be found using the formula, fc = 1/(2πRC). The phase angle of the output signal at fc and is -45^{o} for a Low Pass Filter.

The gain of the filter or any filter for that matter, is generally expressed in **Decibels** and is a function of the output value divided by its corresponding input value and is given as:

$$\text{Gain in dB} = 20 \log \frac{Vout}{Vin}$$

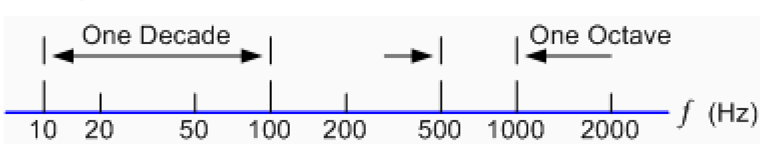
Applications of passive Low Pass Filters are in audio amplifiers and speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or "hiss" type distortion. When used like this in audio applications the low pass filter is sometimes called a "high-cut", or "treble cut" filter.

If we were to reverse the positions of the resistor and capacitor in the circuit so that the output voltage is now taken from across the resistor, we would have a circuit that produces an output frequency response curve similar to that of a *High Pass Filter*,

Decades and Octaves

One final comment about decades and octaves, a **Decade** is a tenfold (factor of 10) increase or tenfold decrease on the frequency scale for example, 2 to 20Hz is 1 decade or 50 to 5000Hz is 2 decades (50 to 500 and then 500 to 5000Hz). An **Octave** is a doubling (factor of 2) or halving (divide by 2) of the frequency scale for example, 10 to 20Hz is 1 octave and 2 to 16Hz is 3 octaves (2 to 4, 4 to 8 and finally 8 to 16Hz). Either way *Logarithmic* scales are used in the frequency domain to denote a frequency value.

Logarithmic Frequency Scale



Since the frequency determining resistors are all equal, and as are the frequency determining capacitors, the cut-off or corner frequency (fc) for either a first, second, third or even a fourth-order filter must also be equal and is found by using our now old familiar equation:

$$fc = \frac{1}{2\pi RC}\ Hz$$

As with the first and second-order filters, the third and fourth-order high pass filters are formed by simply interchanging the positions of the frequency determining components (resistors and capacitors) in the equivalent low pass filter. The overall gain of high-order filters is **fixed** because all the frequency determining components are equal.

High-order filters

Filter Approximations

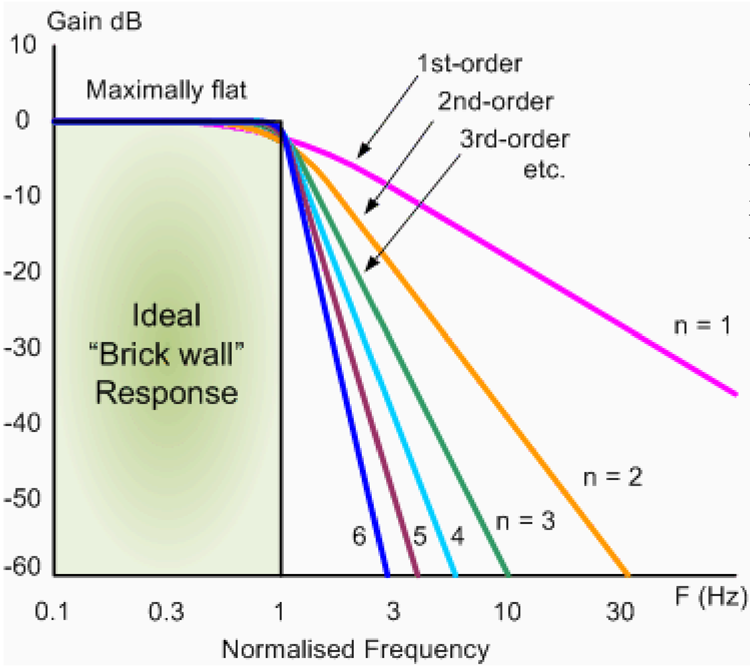
So far we have looked at a low and high pass first-order filter circuits, their resultant frequency and phase responses. An ideal filter would give us specifications of maximum pass band gain and flatness, minimum stop band attenuation and also a very steep pass band to stop band roll-off (the transition band) and it is therefore apparent that a large number of network responses would satisfy these requirements.

Not surprisingly then that there are a number of "approximation functions" in linear analogue filter design that use a mathematical approach to best approximate the transfer function we require for the filters design. Such designs are known as **Elliptical**, **Butterworth**, **Chebyshev**, **Bessel**, **Cauer** as well as many others. Of these five "classic" linear analogue filter approximation functions only **Butterworth Filters** and especially the Butterworth Low Pass filter design will be considered here as its the most commonly used function.

The Butterworth Low Pass Filter

The frequency response of the **Butterworth Filter** approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 0.707. However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter, and the standard Butterworth approximations, for different filter orders are given below.

Ideal Frequency Response for a Butterworth Filter



Note that the higher the order and number of cascaded stages the closer the filter is to the ideal "brick wall" response. However, in practice this "ideal" response is unattainable.

Where the general equation for a Butterworth filters frequency response is given as:

$$H(j\omega) = \frac{1}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2n}}}$$

Where: n represents the filter order, Omega ω is equal to 2πf and Epsilon ε is the maximum pass band gain, (A_{max}). If A_{max} is defined at a frequency equal to the cut-off -3dB corner point (fc), ε will then be equal to one and therefore ε^2 will also be one.

Filter Design - Butterworth Low Pass

Find the order of an active low pass Butterworth filter whose specifications are A_{max} = 0.5dB at a pass band frequency (ωp) of 200 radian/sec, A_{min} = 20dB at a stop band frequency (ωs) of 800 radian/sec. Also design a suitable Butterworth filter circuit to match these requirements.

Firstly, the maximum pass band gain A_{max} = 0.5dB which is equal to a gain of **1.0593** (0.5dB = 20log A) at a frequency (ωp) of 200 rads/s, so the value of epsilon ε is found by:

$$1.0593 = \sqrt{1 + \varepsilon^2}$$

$$\therefore \ \varepsilon = 0.3495 \quad and \quad \varepsilon^2 = 0.1221$$

Secondly, the minimum stop band gain A_{min} = 20dB which is equal to a gain of **10** (20dB = 20log A) at a stop band frequency (ωs) of 800 rads/s.

Substituting the values into the general equation for a Butterworth filters frequency response gives us the following:

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_S}{\omega_p}\right)^{2n}}}$$

$$\frac{1}{10} = \frac{1}{\sqrt{1 + 0.1221\left(\frac{800}{200}\right)^{2n}}}$$

$$\therefore\ 100 = 1 + 0.1221\ x\ 4^{2n}$$

$$4^{2n} = \frac{99}{0.1221} = 810.811$$

$$4^{n} = \sqrt{810.811} = 28.475$$

$$\therefore\ n = \frac{\log 28.475}{\log 4} = 2.42$$

Since n must be an integer (whole number) then the next highest value is n = 3 ie, **"a third-order filter is required"**. Then to produce a third-order filter a second-order stage cascaded with a first-order stage is required.

From the normalised low pass Butterworth Polynomials table above, the coefficient for a third-order filter is given as $(1+s)(1+s+s^2)$ and this gives us a gain of 3-A = 1, or A = 2. As A = 1 + (Rf/R1), choosing a value for both the feedback resistor Rf and resistor R1 gives us values of 1kΩ and 1kΩ respectively, (1kΩ/1kΩ + 1 = 2).

We know that the cut-off corner frequency, the -3dB point (ω_o) can be found using the formula 1/CR, but we need to find ω_o from the pass band frequency ω_p then,

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$3dB \;=\; 1.414 \; at \;\; \omega = \omega_O$$

$$\frac{1}{1.414} \;=\; \frac{1}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$2 \;=\; 1 + \; \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}$$

$$\therefore\; 1 \;=\; \varepsilon \left(\frac{\omega_O}{\omega_P}\right)^{n}$$

$$\omega_O{}^{n} \;=\; \frac{\omega_P{}^{n}}{\varepsilon}$$

$$\omega_O{}^{3} \;=\; \frac{200^3}{0.3495}$$

$$\omega_O{}^{3} \;=\; 22.889 \; x \; 10^6$$

$$\therefore\; \omega_O \;=\; 283.93 \;\simeq\; 284\, rads\,/\,s$$

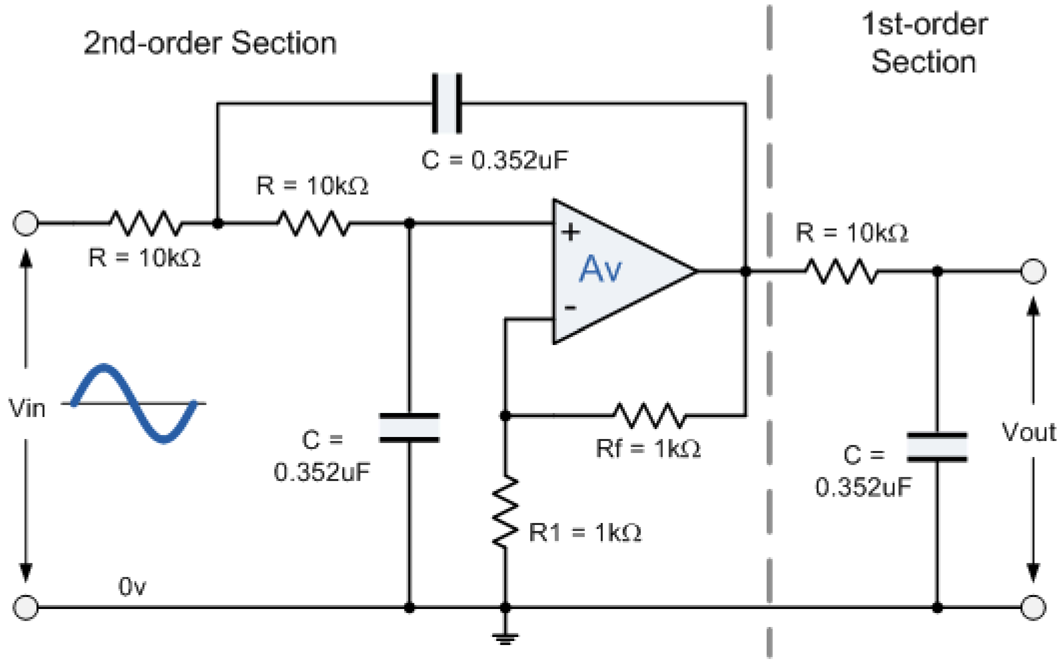
So, the cut-off corner frequency is given as 284 rads/s or 45.2Hz, (284/2π) and using the familiar formula 1/CR we can find the values of the resistors and capacitors for our third-order circuit.

$$284 rads\,/\,s \;=\; \frac{1}{CR} \quad use\; a\; value\; of\; R = 10k\Omega$$

$$\therefore\; Capacitor\;\; C \;=\; \frac{1}{284\, x\, 10{,}000} \;=\; 0.352uF$$

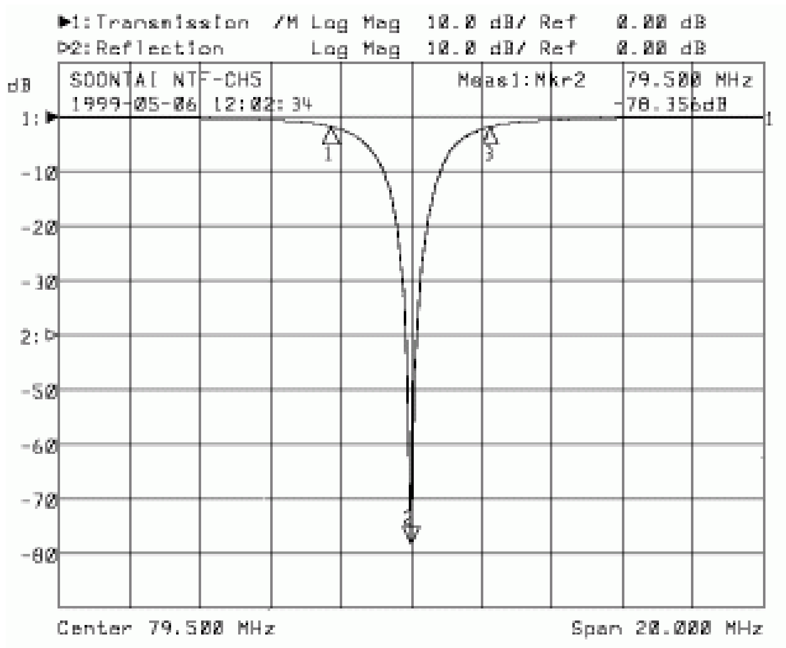
Third-order Butterworth Low Pass Filter

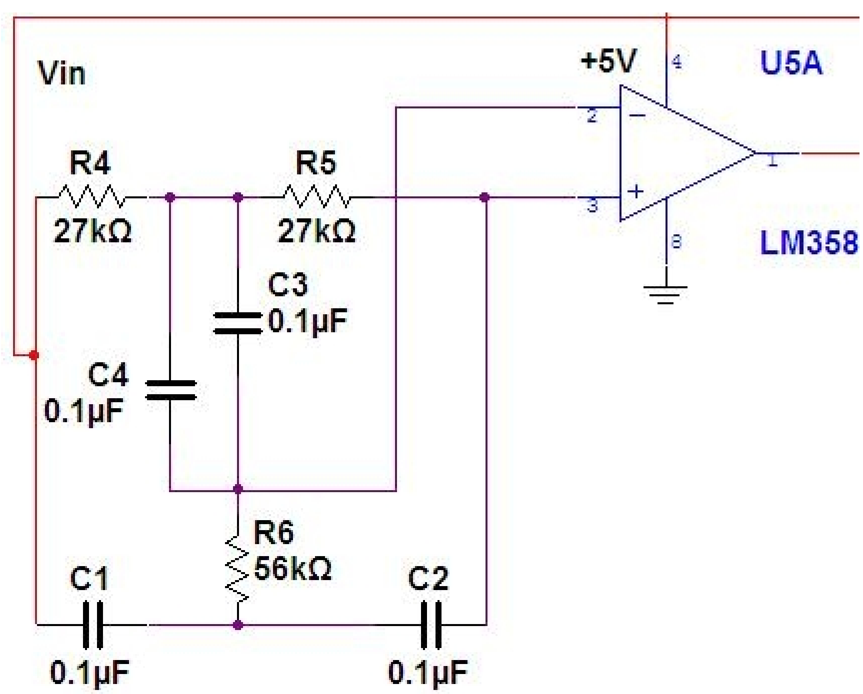
and finally our circuit of the third-order low pass **Butterworth Filter** with a cut-off corner frequency of 284 rads/s or 45.2Hz, a maximum pass band gain of 0.5dB and a minimum stop band gain of 20dB is constructed as follows.



Notch Filter

A notch filter is a filter that attenuates all of the frequencies that are not in the frequency range. A notch filter was used on this project to attenuate the 60Hz signal received on the electrodes due to the electrical appliances.





The Microcontroller

A microcontroller(MCU) is a small computer on a single integrated circuit containing a processor core, memory, and programmable input/output peripherals. Program memory in the form of NOR flash or OTP ROM is also often included on chip, as well as a typically small amount of RAM. Microcontrollers are designed for embedded applications, in contrast to the microprocessors used in personal computers or other general purpose applications.

Microcontrollers are used in automatically controlled products and devices, such as automobile engine control systems, implantable medical devices, remote controls, office machines, appliances, power tools, and toys. By reducing the size and cost compared to a design that uses a separate microprocessor, memory, and input/output devices, microcontrollers make it economical to digitally control even more devices and processes. Mixed signal microcontrollers are common, integrating analog components needed to control non-digital electronic systems.

Some microcontrollers may use four-bit words and operate at clock rate frequencies as low as 4 kHz, for low power consumption (milliwatts or microwatts). They will generally have the ability to retain functionality while waiting for an event such as a button press or other interrupt; power consumption while sleeping (CPU clock and most peripherals off) may be just nanowatts, making many of them well suited for long lasting battery applications. Other microcontrollers may serve performance-critical roles, where they may need to act more like a digital signal processor (DSP), with higher clock speeds and power consumption.

A few microcontrollers are used on this project in order to send compositive video to a NTSC television, and read the EOG using Analog-to-Digital converters. Other concepts such as interrupts and timers were very useful tools as well.

Conclusion

Ever since I began to venture into the programming field when I was 14 years old, my interest in how video games worked began to be one of my greatest further knowledge goals until then. That fact took me to the level of developing my first video game system from scratch when I was 16 years, completely done from my own ideas, and been able to control the game using a remote control I developed sensitive to motion. By the end of fall of 2010 I started to make some researches about the possibility to control that video game using body movements by using sensors sensitive to small voltages and some other electronics. After hearing about a fellow student struggling to communicate just by using his eyes, I started with my EOG design, having a software translate the eye movements into digital data and making it stable enough at my 17 years old.
At the end, what started just as the development of projects oriented to the gaming field, changed these projects to the development of biomedical systems at a very low cost, which makes me glad that I've managed to contribute a little bit to the biomedical field.

Despite having some economic limitations for my projects, I've managed to use that fact to strengthen my system by making this project stable enough in the sense of having properly handled the very small voltages generated by the eye movements, and at the same time attenuating external noise without resorting to expensive equipment both for filters and amplifiers; therefore, have a fully funcional project built with a very low budget.

Given the fact that this is the first stable version of my project due to my lack of time to keep modifying it, I think that the beta version is complete enough to keep doing further tests in order to accomplish an even more reliable system more user friendly for more people according to further statistical data.
Digital and analog electronics, software programming, biomedical engineering and mathematics were very useful fields to successfully finish the prototype of this project, yet this is just the start of one of my greatest endeavors to succeed as a developer.