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A gravity-based three-dimensional compass in the mouse brain

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Summary

Head direction cells in the mammalian limbic system are thought to function as an allocentric neuronal compass. Although traditional views hold that the compass of ground-dwelling species is planar, we show that head-direction cells in the rodent thalamus, retrosplenial cortex and cingulum fiber bundle are tuned to conjunctive combinations of azimuth, pitch or roll, similarly to presubicular cells in flying bats. Pitch and roll orientation tuning is ubiquitous, anchored to gravity, and independent of visual landmarks. When head tilts, azimuth tuning is affixed to the head-horizontal plane, but also uses gravity to remain anchored to the terrestrial allocentric world. These findings suggest that gravity defines all three degrees of freedom of the allocentric orientation compass, and only the azimuth component can flexibly remap to local cues in different environments. Collectively, these results demonstrate that a three-dimensional, gravity-based, neural compass is likely a ubiquitous property of mammalian species, including ground-dwelling animals.

Introduction

51 Head direction (HD) cells, which encode allocentric head orientation analogous to a
52 “neural compass” are a primordial component of the mammalian (Taube 2007; Finkelstein et al.
53 2015) and fly (Seelig and Jayaraman, 2015; Green et al., 2017; Turner-Evans et al., 2017; Kim et
54 al., 2017) spatial navigation system. Originally identified in the rat dorsal presubiculum (Taube
55 et al. 1990a,b; Preston-Ferrer et al. 2016; Simonnet et al. 2018), HD cells have been
56 subsequently found in multiple brain regions, including the anterior thalamus (Taube et al.
57 1995; Peyrache et al. 2015, 2017; Page et al. 2017; Shinder and Taube 2011, 2014, 2018),
58 parasubiculum (Kornienko et al. 2018), retrosplenial cortex (Chen et al. 1994a,b; Cho et al.
59 2001; Jacob et al. 2017; Lozano et al. 2017), and entorhinal cortex (Sargolini et al. 2006;
60 Kornienko et al. 2018; Park et al. 2018).

61 HD cells have been traditionally known for encoding head orientation in the horizontal
62 plane (azimuth) by firing when the animal faces a particular direction. Yet, HD cells in the bat
63 presubiculum were recently shown to respond in vertical planes, i.e. when the head tilts in
64 pitch or roll (Finkelstein et al. 2015). Gravity is a ubiquitous cue for allocentric vertical and may
65 thus provide an Earth constant reference for the brain compass. However, Finkelstein et al.
66 (2015) did not investigate whether tilt tuning is anchored to gravity. In parallel, Laurens et al.
67 (2016) identified gravity-anchored tilt tuning in the anterior thalamus of rhesus macaques, but
68 did not test whether gravity-tuned cells are traditional azimuth-tuned HD cells. Thus, the two
69 observations (gravity-anchored tilt tuning in macaques and 3D HD tuning in bats) have
70 remained segregated. Furthermore, tuning to tilt has never been shown in rodents, and some

71 researchers proposed that it may be restricted to aerial and tree-dwelling species but absent in
72 ground-dwelling species like rodents (Calton and Taube 2005; Shinder and Taube 2018).

73 Here we test the hypothesis that gravity-anchored tilt signals and visually-anchored
74 azimuth signals converge onto rodent HD cells to yield a sense of 3D head orientation (Laurens
75 and Angelaki, 2018; **Fig. 1a**). As in bat presubiculum, we first show that HD cells in the mouse
76 anterior thalamus, retrosplenial cortex and cingulum are tuned to combinations of azimuth and
77 tilt. We then show that, not only does gravity anchor tilt tuning, but it also participates in
78 updating azimuth to preserve allocentric invariance during 3D motion (Laurens and Angelaki,
79 2018). Finally, we also show that gravity-anchored tilt (vertical) tuning and gravity-defined
80 azimuth tuning interact multiplicatively to endow a neural correlate of 3D spatial orientation.
81 Thus, a 3D orientation compass is not a specialized property limited to areal species, but may
82 instead represent a ubiquitous property throughout many chapters of animal evolution.

83 **A 3D compass in the mouse brain**

84 We used tetrodes to record extracellularly from the antero-dorsal nucleus of the
85 thalamus (ADN; n = 4 mice; **Suppl. Table 1**), retrosplenial cortex (RSC; n=2 mice; **Suppl. Table 1**),
86 and cingulum fiber bundle (CIN; n = 4 mice; **Suppl. Table 1**). The CIN carries projection fibers
87 from the ADN and RSC (Domesick 1970; van Groen and Vyss 1990; 1995; Bubb et al. 2018). Cells
88 were exclusively selected based on spike isolation (**Suppl. Fig. 1**) and recording locations were
89 verified post-mortem (**Suppl. Fig. 2**). Based on their head orientation during free foraging in a
90 horizontal arena (**Fig. 1b**; summarized in **Suppl. Fig. 3**), cells were characterized as azimuth (Az)-
91 tuned (i.e. 'traditional' HD) cells in light (**Fig. 1c**, red) and darkness (see example in **Fig. 1c**,
92 black) or azimuth-untuned (see example in **Fig. 1d**).

93 Neurons were then characterized as animals walked on a 3D orientable platform (**Fig.**
94 **1e**) that could be re-positioned so that animals foraged through multiple tilt angles ranging
95 between 0° (upright) and ~60° (**Suppl. Fig. 4**). We computed tilt tuning curves in spherical
96 coordinates, with 2 degrees of freedom: tilt angle from upright: α (range: 0-180°), and tilt
97 orientation: γ (range: 0-360°; see **Suppl. Fig. 5** for definitions of right/left ear-down [RED/LED]
98 and nose-up [NU]/down [ND] orientations). For simplicity of illustration, 2D tilt tuning curves
99 are shown in a planar representation using an equal-area Mollweide projection, and the 1D
100 azimuth axis is unfolded. Next, the neurons' tuning was plotted as a color map in a volume
101 formed by this tilt plane and the azimuth axis (**Fig. 1f,g**; **Suppl. Movie 1**). Both example neurons
102 in **Fig. 1** exhibited tilt tuning in the restricted 3D orientation space, characterized by an
103 increased firing rate at a preferred tilt (**Fig. 1f,g**, NU for both cells; see **Suppl. Movie 2**), with
104 peak-to-trough amplitudes of 32 spk/s and 23 spk/s, respectively when averaged across all
105 azimuth angles. In addition, with the platform in the earth-horizontal orientation, the cell
106 classification as Az-tuned or Az-untuned persisted (**Fig. 1c,d**, dashed pink curves).

107 Next, animals were transferred to a multi-axis rotator (**Fig. 1j**) that sampled 3D
108 orientation uniformly to measure the neurons' full 3D tuning curves (**Suppl. Fig. 6; Suppl.**
109 **Movie 3**). The example azimuth-tuned cell in **Fig. 1c, f** maintained both azimuth (**Fig. 1c**, gray)
110 and tilt (**Fig. 1k**; see also **Suppl. Movie 4**) tuning in the rotator, although its peak firing rate had
111 decreased from ~60 spikes/s to ~10 spikes/s. While the example cell's preferred direction (PD)
112 in azimuth differed across environments (**Fig. 1c**, compare solid red, dashed pink and gray lines,
113 with PDs at -46° , -163° , and 114° , respectively), its tilt PD was relatively constant at NU (**Fig. 1k**:
114 $[\alpha=83^\circ, \gamma=180^\circ]$, as compared to **Fig. 1f**: $[\alpha=56^\circ, \gamma=130^\circ]$; note that tuning was not sampled at
115 higher tilt angles]). Similarly, the example Az-untuned cell exhibited a tilt PD at $[\alpha=97^\circ, \gamma=168^\circ]$
116 (**Fig. 1l; Suppl. Movie 5**), as compared to $[\alpha=48^\circ, \gamma=150^\circ]$ (**Fig. 1g**). Thus, tilt tuning recorded
117 when animals were passively re-oriented in the rotator had similar spatial properties to that
118 when moving freely in 3D.

119 We used identical criteria to classify neurons as azimuth-tuned or tilt-tuned (Methods;
120 **Suppl. Fig. 7**). When tested on the platform, we found that tilt tuning was widespread in Az-
121 tuned (traditional HD) cells classified based on their responsiveness in the horizontal arena (of
122 **Fig. 1b**), as summarized in **Fig. 2**. Specifically, out of 29 ADN neurons recorded on the platform,
123 25 (86%) were classified as azimuth-tuned (**Fig. 2a**, Venn diagram). Amongst these, 24 (96%)
124 were also tuned to tilt and are subsequently called *conjunctive (azimuth and tilt)* HD cells (solid
125 red symbols in **Fig. 2**). A sizeable population of Az-tuned cells were also recorded in RSC and CIN
126 (49% and 38% of recorded cells, respectively). Of these, 58% (RSC) and 75% (CIN) were
127 conjunctive cells.

128 Tilt tuning on the platform was also seen in Az-untuned cells (solid black disks and
129 symbols in **Fig. 2a**). Thus, tilt tuning was common, observed in all areas, regardless of Az-tuning,
130 with 92/139 (66%) tilt-tuned cells. A total of 74/139 (53%) cells were Az-tuned, and tilt and
131 azimuth tuning showed some overlap across neurons. Tilt-tuned cells were slightly (7%) more
132 likely to be Az-tuned and reciprocally Az-tuned cells were slightly (8%) more likely to be tilt-
133 tuned (Chi-square test, $p=0.03$). The peak-to-trough modulation of tilt and azimuth tuning was
134 comparable (**Fig. 2a**; see also **Suppl. Fig. 7d-f**), although tilt tuning amplitude was lower in
135 conjunctive ADN cells (Wilcoxon paired rank test; $p=10^{-4}$ in ADN, $p=0.4$ in RSC, $p=0.6$ in CIN).
136 These results indicate that tilt signals are an inherent component of the mouse HD system
137 during natural behavior; thus, the term "HD cell" should refer to both tilt-tuned cells as well as
138 azimuth-tuned cells. However, 3D characterization in freely moving mice was limited by the
139 explorable space.

140 **Spatial properties of tilt tuning in 3D**

141 To characterize tuning uniformly in 3D space, 514 (60 ADN, 167 RSC, 287 CIN) neurons
142 were tested in the rotator. Seventy-one percent (363) of these cells were significantly tuned to

143 tilt (88% ADN, 65% RSC and 70% CIN). Similar to responses obtained to foraging on the
144 platform, tilt-tuned cells were slightly (4%) more likely to be Az-tuned and reciprocally Az-tuned
145 cells were slightly (6%) more likely to be tilt-tuned (Chi-square test, $p=0.003$).

146 In line with previous studies (Shinder and Taube, 2011; 2014) and the example cell in
147 **Fig. 1c,k**, tuning modulation amplitude was attenuated when animals were restrained in the
148 rotator (scatter plots in **Fig. 2b** vs. **Fig. 2a**). Azimuth tuning amplitude was attenuated by a
149 factor of 4.9 (**Suppl. Fig. 8a**). As a result, only a minority of neurons tuned to azimuth when
150 moving freely were significantly tuned to azimuth when restrained in the rotator (63/267; **Fig.**
151 **2c**, left panel). Nevertheless, the PDs of multiple azimuth-tuned HD cells had consistent angles
152 relative to each other when moving freely and in the rotator (**Suppl. Fig. 8d**), suggesting that an
153 attractor network structure of the population of azimuth-tuned cells was maintained in the
154 rotator. Thus, other than the smaller response magnitude, azimuth (and, as will be shown
155 below, tilt) responses measured in the rotator are representative of the neurons' natural
156 responses.

157 Tilt tuning magnitude was also attenuated in the rotator, although to a lesser extent (by
158 a factor of 2.4; **Suppl. Fig. 8b**). A minority of cells tested both on the orientable platform and
159 the rotator were tilt tuned only during stimulation in the rotator because of the larger sampling
160 of 3D space as compared to foraging on the platform that was limited to 60° tilt. In contrast,
161 some cells were significantly tuned only when moving freely because the response magnitudes
162 were larger (**Suppl. Fig. 8b**). Nevertheless, a large proportion of cells (62%) were tuned to tilt
163 under both conditions (**Fig. 2c**, right panel), indicating that tilt tuning is conserved across free
164 locomotion and restrained, passive motion conditions.

165 We then compared tuning curves in freely moving and restrained animals by computing
166 their pixel-by-pixel correlations. This revealed an important difference between azimuth and tilt
167 tuning. Because azimuth curves shifted randomly between environments (**Suppl. Fig. 8c**), their
168 pixel-by-pixel correlations were uniformly distributed (median=0.07; Kolmogorov-Smirnov test
169 $p=0.15$; **Fig. 2d**, left). In contrast, tilt tuning was preserved (median=0.58; $p<10^{-5}$; **Fig. 2d**, right),
170 as expected if the tilt compass was anchored to a common reference: gravity (**Fig. 1a**; see direct
171 testing of this hypothesis below).
172

173 The majority of 3D tilt tuning curves were unimodal (as in the example responses of **Fig.**
174 **1k,l**, see also **Suppl. Fig. 9a**). However, bimodal (**Suppl. Fig. 9b**) or complex-shaped (**Suppl. Fig.**
175 **9c**) tuning curves were also encountered. Regardless of their shape, tilt tuning curves were well
176 fitted with Gaussian functions (**Suppl. Fig. 9,10**), which allowed quantification of tilt preferred
177 directions in 3D. As illustrated in **Fig. 3a**, PDs were scattered across the full range of angles,
178 suggesting that the population of tilt-tuned neurons could provide a code for tilt orientation.

179 Yet, the distribution was not uniform, with an overrepresentation of ND pitch PDs and an
180 underrepresentation of PDs in the roll plane (LED/RED, grey sectors) ADN: $p < 10^{-6}$; RSC: $p < 2 \cdot 10^{-3}$;
181 CIN: $p < 10^{-10}$; Chi Square). In contrast, there was no significant difference in PDs between the
182 upper vs. lower hemispheres of the 3D space (i.e. tilt angles lower or higher than 90° ($p > 0.1$ in
183 all areas, Chi Square). These results, showing a dominance of pitch-tuned over roll-tuned cells,
184 are consistent with those previously described for bats (Finkestein et al., 2015) and monkeys
185 (Laurens et al., 2016).

186 A fundamental property of traditional HD cells is that their tuning persists in darkness
187 (Taube, 2007; Peyrache et al., 2015). To test whether this property also characterized tilt
188 tuning, we recorded the rotation responses of 186 (23 ADN; 30 RSC; 133 CIN) tilt-tuned cells in
189 complete darkness. Angular differences between PDs recorded in light and darkness were close
190 to zero (PD difference $< 45^\circ$ in 122/186 cells, Kolmogorov-Smirnov test versus the expected
191 distribution if PD were uniformly distributed: $p = 10^{-7}$ in all areas, **Fig 3b**). In addition, the tilt
192 modulation amplitude was highly correlated between light and dark conditions (**Fig. 3c**,
193 Spearman correlation: ADN: $r = 0.8$, slope = 0.9; $p < 10^{-5}$; RSC: $r = 0.81$, slope = 0.96; $p < 10^{-7}$;
194 CIN: $r = 0.71$, slope = 0.9; $p < 10^{-10}$), although slightly lower in darkness in CIN (paired Wilcoxon
195 test: $p < 10^{-4}$; $p > 0.1$ in other areas). These findings suggest that tilt tuning curves have similar
196 properties as azimuth-tuning, further supporting the hypothesis that they represent different
197 components of the same compass. Similar findings were also reported in gravity-tuned cells in
198 the monkey anterior thalamus (Laurens et al. 2016), suggesting that these tilt-tuned neurons
199 may be found along a broad range of animal evolution.

200 In further agreement with Laurens et al. (2016), a small fraction of cells also responded
201 to tilt or azimuth velocity (**Suppl. Fig. 11**). In addition, tilt tuning in the rotator could be
202 reproduced using traditional single-axis rotations like pitch and roll (**Suppl. Fig. 12**). Thus, tilt
203 tuning is anchored to allocentric space, independent of the exact motion trajectory. Finally, we
204 verified that 3D tilt tuning curves were highly reproducible across repetitions of the rotation
205 protocol across different days (**Suppl. Fig. 13**).

206 **Tilt tuning is anchored to gravity**

207 The invariance of tilt tuning in light and dark conditions (**Fig. 3b,c**), and across setups
208 (**Fig. 2d**), supports the hypothesis that gravity could represent the allocentric reference upon
209 which tilt tuning is based. To more explicitly test this hypothesis, we recorded 128 (22 ADN; 27
210 RSC; 79 CIN) tilt-tuned cells with the 3D passive re-orientation protocol after tilting the rotator
211 and visual surround together 60° (**Fig. 1j**, magenta rotation axis). This manipulation dissociated
212 the reference frame defined by the visual cues inside the sphere from the allocentric
213 orientation defined by gravity (**Suppl. Fig. 14a-g**). We compared the cells' tilt tuning curves
214 recorded with the rotator upright (**Fig. 4a**) and when tilted (**Fig. 4b,c**). For the latter, tuning

215 curves were computed in (i) a visual reference frame (**Fig. 4b**) and (ii) a gravity reference frame
216 (**Fig. 4c**). If a neuron's tuning curves measured with the system upright and tilted were identical
217 when expressed in a gravity reference frame, then gravity served as the response anchor
218 constant. If the tuning curves were identical when expressed in a visual reference frame, then
219 vision was the response constant (see **Suppl. Fig. 14h**).

220 As illustrated with the example cell in **Fig. 4a-c**, and with summary data (**Fig. 4d**), tilt
221 tuning curves were better conserved when expressed relative to gravity cues, rather than visual
222 cues. In fact, 76/128 cells (14/22 ADN; 15/27 RSC; 47/79 CIN) were significantly ($p < 0.01$) more
223 correlated to the gravity reference frame than the visual frame, and the remaining 52 cells
224 were not significantly more correlated in either frame. Thus, although the local visual
225 environment was identical between the upright and 60° tilt conditions, the tilt tuning followed
226 (i.e., was invariant to) the orientation of the head relative to gravity. These findings are
227 identical to tilt-tuned cells in the macaque anterior thalamus (Laurens et al., 2016).

228 **Azimuth tuning in 3D**

229 To investigate how tilt (gravity) and azimuth components work together to encode 3D
230 head orientation, we first questioned how to define azimuth when the head tilts away upright.
231 The brain may simply project head direction onto the earth-horizontal (EH) plane and encode
232 azimuth in that plane (**Fig. 5a**). Alternatively, it may represent azimuth in a compass affixed to
233 the head-horizontal plane by tracking head rotations in this plane (yaw; **Fig. 5b**, cyan), ignoring
234 other movements (Yaw-only model, YO, Shinder and Taube 2018). However, this possibility
235 should be excluded on theoretical grounds, as it would not maintain allocentric invariance
236 when the mouse locomotes in 3D (Page et al., 2017; Laurens and Angelaki, 2018). For instance,
237 when completing the trajectory in **Fig. 5b** (red), the compass would register only 3 right-angle
238 turns, i.e. 270°, when the head returns to its initial position. To maintain allocentric invariance
239 when the head faces a horizontal direction (i.e. anywhere on surface 1, when facing -90/90°
240 surface 2, or -180/180° on surface 3 in **Fig. 5b**), a head-horizontal compass must use a “dual
241 updating rule” which includes both head-horizontal and earth (gravity)-horizontal rotations (**Fig.**
242 **5b**, green). The resulting compass was named a “tilted azimuth (TA)” frame (**Fig. 5c**; **Suppl. Fig.**
243 **15a**; see also Page et al. 2017; Laurens and Angelaki, 2018). Thus, in summary, we emphasize
244 that the YO compass loses allocentric invariance during 3D motion, even when returning to
245 upright (**Fig. 5b**). In contrast, EH and TA frames remain invariant and are identical when the
246 head is upright (as the head-horizontal and EH planes are aligned) but differ when the head tilts
247 (Laurens and Angelaki, 2018, **Suppl. Movie. 6**).

248 The data collected during the 3D motion protocol in the rotator allows quantitative
249 testing of the YO, EH and TA azimuth hypotheses. First, we expressed azimuth in all 3 frames
250 and tested whether cells were significantly tuned when the 3D trajectory brought the head

251 close to upright ($<45^\circ$ tilt). As predicted, almost none of the cells observed exhibited significant
252 tuning in a YO frame (6/267 tuned cells, consistent with false positive at $p = 0.01$). In contrast,
253 63/267 cells were tuned as predicted when azimuth was expressed in either the EH or TA
254 frames (ADN: $n=17$; RSC: $n=7$; CIN: $n=39$).

255 Second, when azimuth is expressed in the appropriate coordinate frame, the cell's
256 azimuth PD should be invariant at all head tilts (**Suppl. Fig. 15b,c,d**). To test this, we compared
257 the cells' azimuth tuning near upright ($<45^\circ$ tilt) or when tilted ($>60^\circ$ tilt) (**Fig. 5d**). We observed
258 that the neurons' tuning curves were highly correlated when expressed in a TA frame (median:
259 0.73; [0.63-0.86]CI), but significantly less so when expressed in a EH frame (0.19; [0.04-0.37] CI).
260 In addition, correlations were near zero when expressed in a YO frame (0.08, [-0.07-0.18]CI).
261 We verified (3-way ANOVA, difference between reference frames: $p < 10^{-10}$) that correlations
262 were similar across recorded areas ($p=0.6$) and tilt-tuned or non-tilt tuned cells ($p=0.2$).

263 It was also noted that, regardless of coordinate frame, the PD response amplitude of
264 azimuth tuning often decreased when the animal was tilted beyond 90° from upright (**Suppl.**
265 **Movie 6-9**). As illustrated with an example Az-only cell in **Fig. 5e** (animated curve in **Suppl.**
266 **Movie 7**), azimuth tuning was strong, with a clear PD at $\gamma = -85^\circ$ for small tilt angles (lowest
267 portion of the example cell's tuning curve). However, response at large tilt angles (i.e, upper
268 portion of the 3D tuning curve, **Fig. 5e**) did not exhibit any azimuth tuning. This finding was
269 consistent for all Az-only cells: the average HD tuning curve (aligned to peak at $PD=0^\circ$) had a
270 higher modulation when computed for head tilts close to upright (**Fig. 5f**, red) and almost no
271 modulation close to upside-down (**Fig. 5f**, grey). Thus, for Az-only tuned cells, the response
272 amplitude of azimuth tuning was dependent on the tilt angle, even though the cells were not
273 tuned to tilt.

274 That the average azimuth normalized tuning amplitude decreased monotonically with
275 tilt angle was not only limited to Az-only cells (**Fig. 5g**, grey). Notably, the azimuth tuning
276 amplitude of conjunctive cells decreased in a similar manner, irrespective of whether the cell's
277 tilt tuning favored upright orientation (**Fig. 5g**, red), intermediate tilt (i.e. 90° ; **Fig. 5g**, blue) or
278 upside-down (**Fig. 5g**, green). Normalized tuning amplitudes for all azimuth tuned cells were
279 affected by tilt angle (two-way ANOVA, $p < 10^{-10}$) and varied between groups of cells ($p < 10^{-4}$);
280 however there was no significant interaction effect ($p=0.8$), indicating that the azimuth tuning
281 of all cells was equally affected by tilt. We conclude that HD cells encode azimuth in a TA
282 reference frame, and that their azimuth response decreases when the head tilts away from
283 upright, irrespective of tilt tuning.

284 **Tilt and azimuth tuning components of 3D compass follow multiplicative interaction**

285 In line with experimental results and to gain further insight into the structure of 3D
286 tuning, we created a 3D HD model that incorporates the following properties (**Fig. 6a**): (1) tilt
287 tuning curves are generated by integrating gravity signals into Gaussian tuning functions (**Suppl.**
288 **Fig. 9**), (2) azimuth-tuned cells encode TA with a tilt-dependent gain, independently from tilt
289 tuning, (3) tilt and azimuth tuning interact multiplicatively (an additive model performed worse;
290 **Methods**).

291 We found that these properties composed a model that could fit the tuning curves of
292 conjunctive cells well ($n=16$ ADN, 4 RSC, 33 CIN; median $p=0.88$, [0.86-0.91] CI), as illustrated
293 with two examples (**Fig. 6b,c**; **Suppl. Movie 8,9**). The peak tilt response of the first example cell
294 occurred at a tilt angle $\alpha = 42^\circ$, at which azimuth tuning ($PD=-27^\circ$) had not yet attenuated.
295 Consequently, the cell exhibited tilt and azimuth tuning simultaneously at this tilt angle,
296 resulting in a preferred 3D orientation, visible as a region of maximum firing on both the
297 measured (**Fig. 6b**, top) and fitted (**Fig. 6b**, bottom) curves. This type of tuning was
298 characteristic of conjunctive cells with PDs close to upright.

299 The second example cell exhibited a PD at a large tilt angle ($\alpha = 105^\circ$), where azimuth
300 tuning had already substantially decreased. As a consequence, the cell appeared azimuth-tuned
301 at small tilt angles, where tilt tuning was minimal (**Fig. 6c**, lower horizontal plane) and tilt-tuned
302 at large tilt angles (**Fig. 6c**, upper horizontal plane). This type of tuning curve was characteristic
303 of conjunctive cells with a large preferred tilt angle.

304 Further, the same model properties also quantitatively described 3D tuning curves for
305 azimuth-only (median $p=0.75$, [0.7-0.8] CI) and tilt-only cells (median $p=0.87$, [0.85-0.88] CI).
306 Here again, model fits were significantly lower when 3D curves were computed with azimuth in
307 an EH frame (**Suppl. Fig. 16**), confirming that the TA frame captures the cell's response better
308 than either the EH or YO frames could.

309 Discussion

310 In summary, these findings demonstrate that HD cells in two areas of the mouse navigation
311 system, as well as their output fiber bundle (CIN), are tuned in 3D. HD cells were observed to
312 encode 2D tilt either in isolation or conjunctively with 1D azimuth. Azimuth and tilt tuning
313 appear to represent two independent streams of information converging onto HD cells and the
314 probabilities of individual cells being tuned to tilt and azimuth are nearly independent (**Fig. 2**).
315 In addition, the spatial properties of azimuth tuning are independent of tilt tuning (**Fig. 5g**), and
316 the two are separable; i.e. a cell's entire 3D head orientation tuning curve can be computed
317 given its tilt and azimuth tuning. Finally, azimuth tuning is anchored to visual cues (Taube 2007)
318 and tilt tuning to gravity. At the same time, azimuth tuning is encoded in a tilted reference
319 frame, as suggested in previous studies (Page et al. 2017; Laurens and Angelaki, 2018) which is

320 defined by integrating 3D head rotations and gravity (Page et al. 2017, Laurens and Angelaki
321 2018) and not only rotations in the head horizontal plane (Calton and Taube 2005; Shinder and
322 Taube 2018). Furthermore, the amplitude of azimuth tuning is modulated by gravity..

323 A recent study by Shinder and Taube (2018) concluded that HD cells encode only azimuth
324 computed in 1D by integrating rotations in the head horizontal (yaw) plane. However, in our
325 assessment (Laurens and Angelaki 2019), this study is in fact supportive of the tilted azimuth
326 model, although it wasn't directly tested in that work. Furthermore, Shinder and Taube (2018)
327 reported no evidence of tilt tuning. However, this study is inconclusive because their
328 interpretation of the data confounds the effects of possible tilt tuning with the fact that
329 azimuth tuning strength is reduced as a function of tilt angle (**Suppl. Fig. 17**, see also Laurens
330 and Angelaki 2019), a problem we circumvented by testing responses in the full 3D space. Note
331 that Shinder and Taube observed that azimuth tuning vanishes in inverted rats, similar to
332 (Calton and Taube 2005) and to our results. Altogether, Shinder and Taube (2018) concluded
333 that HD cells are essentially 1D because their experimental manipulations qualitative analyses
334 were insufficient to reveal the 3D tuning properties identified here.

335 The 3D model shown here to quantitatively account for the 3D orientation tuning of both
336 azimuth-tuned and conjunctive HD cells is compatible with the toroid topology proposed for HD
337 cells in bats (Finkelstein et al. 2015) when azimuth is expressed in a TA frame and tilt is
338 restricted to the pitch plane (**Suppl. Fig. 18**). Tilt preferred directions are not uniformly
339 representing 3D space, as the pitch plane is over-represented compared to the roll plane,
340 consistent with previous findings in both macaques and bats (Finkelstein et al. 2015; Laurens et
341 al. 2016). In fact, as summarized above (see also quantitative analysis in **Suppl. Fig. 17**), tilt
342 tuning may also exist in rat HD cells (see also Laurens and Angelaki 2019).

343 We conclude that 3D tuning may be a ubiquitous feature of the mammalian HD system. We
344 suggest that the denomination 'head direction cell' should also apply to tilt-tuned and
345 conjunctive cells as well as previously described azimuth-tuned cells. Thus, the proportion of
346 HD cells is much higher than previously reported, based upon response conditions that only
347 considered the horizontal plane.

348 Our study is the first to record HD cells directly in the cingulum fiber bundle that conveys ADN
349 and RSC projections to parahippocampal regions (ADN and RSC projections), RSC (ADN
350 projections) and cingulate cortex (RSC projections) (Domesick 1970; van Groen and Vyss
351 1990,1995; Bubb et al. 2018). Although recordings of axonal spikes with tetrodes are
352 uncommon, they have been performed previously (Robbins et al. 2013). Furthermore, histology
353 clearly demonstrates that recordings occurred in white matter (**Suppl. Fig. 3c**) and units
354 recorded in the CIN exhibited short spike duration consistent with axonal spikes (**Suppl. Fig.**
355 **19b**; Barry 2015). Furthermore, as the CIN contains projections from the ADN and RSC, the

356 properties of tilt and azimuth responses recorded there were consistent with those in ADN and
357 RSC. The existence of tilt signals in the CIN supports the notion that these signals are
358 communicated between various regions of the limbic system and therefore form an inherent
359 component of the spatial navigation system.

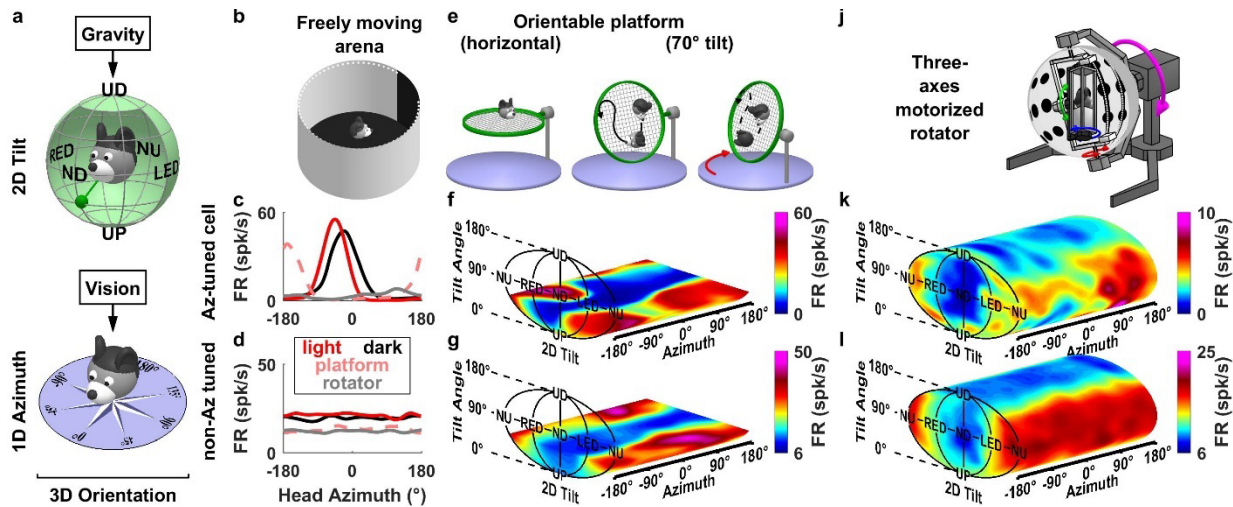
360 A recent imaging study (Kim and Maguire 2018) indicates that the human RSC encodes pitch
361 orientation in a virtual navigation task, although the ADN was found to encode mainly azimuth.
362 It is possible that visually-driven tilt signals arise in the RSC in a virtual environment where
363 visual, but not inertial gravity cues, are present. RSC is involved in visual processing, possibly in
364 transforming visual landmark form an egocentric to an allocentric reference frame (see Clark et
365 al. 2018, Mitchell et al. 2018 for reviews). This raises the possibility that the RSC may use
366 gravity-referenced tilt signals to transform visual signals in 3D.

367 We have demonstrated that tilt signals are anchored to gravity. Indeed, gravity is a veridical
368 vertical allocentric cue, that largely dominates vision in human verticality perception
369 (Vingerhoets et al. 2009). Further, gravity sensing represents one of the most ubiquitous
370 sensory modalities of terrestrial living organisms (Sack 1991, Bender and Frye 2009). The origins
371 of the gravity signal are likely multisensory, involving proprioceptive and vestibular inputs
372 (Clemens et al, 2011; Foisy and Kapoula 2018), as well as computations in central vestibular
373 areas (Laurens and Angelaki 2011; Laurens et al. 2013a,b).

374 Prominent views of azimuth-tuned HD cells posit that they form a neuronal attractor that can
375 memorize azimuth in the absence of sensory inputs and is updated by integrating rotation
376 signals (Clark and Taube 2012, Peyrache et al. 2015, Laurens and Angelaki 2018), although some
377 HD cells in the RSC (Jacob et al. 2017), and parahippocampal regions (Kornienko et al. 2018)
378 may not contribute to the attractor network. Establishing whether tilt-tuned cells also form an
379 attractor will be challenging, especially since gravitational input that anchors tilt tuning is not
380 easily altered, unlike the visual input that anchors azimuth tuning. Alternatively, there may not
381 be a gravity attractor, but rather gravity signals may be computed directly by central vestibular
382 pathways, where the mathematical challenge of integrating 3D inertial cues (Green et al. 2005)
383 is already solved (Laurens et al. 2013b), thus providing a computationally simpler solution for a
384 3D orientation compass. Exploring neuronal computations that underlie 3D orientation coding
385 in the limbic system and their relationship with pre-limbic computations in the brainstem,
386 cerebellum, thalamus and hypothalamus must be targeted in future studies.

387 Tilt and 3D orientation tuning had previously only been identified in aerial (bats) and tree-
388 dwelling (macaques) species, raising the question of whether a 3D compass would be
389 ethologically relevant to rodents. Although laboratory mice (*Mus musculus*) and rats (*Rattus*
390 *norvegicus*) are primarily land-dwelling, they exhibit a rich 3D behavioral repertoire (Makowska
391 and Weary 2016; Mimica et al. 2018) in the wild living in extensive underground burrows

392 (Schmidt and Fischer 2011) and easily learn 3D spatial orientation tasks (Wilson et al. 2015).
393 Furthermore, laboratory mice and rats are physiologically related to tree-dwelling rodents
394 (Arregoitia et al. 2017), including other muroids (e.g. harvest mice, *Micromys minutus*) and non-
395 muroids (e.g. squirrels). It is therefore not surprising that rodents, like bats (Finkelstein et al.
396 2015) and likely macaques (Laurens et al. 2016) and humans (Kim and Maguire 2018), possess a
397 three-dimensional compass, whose properties may be phyla-independent.

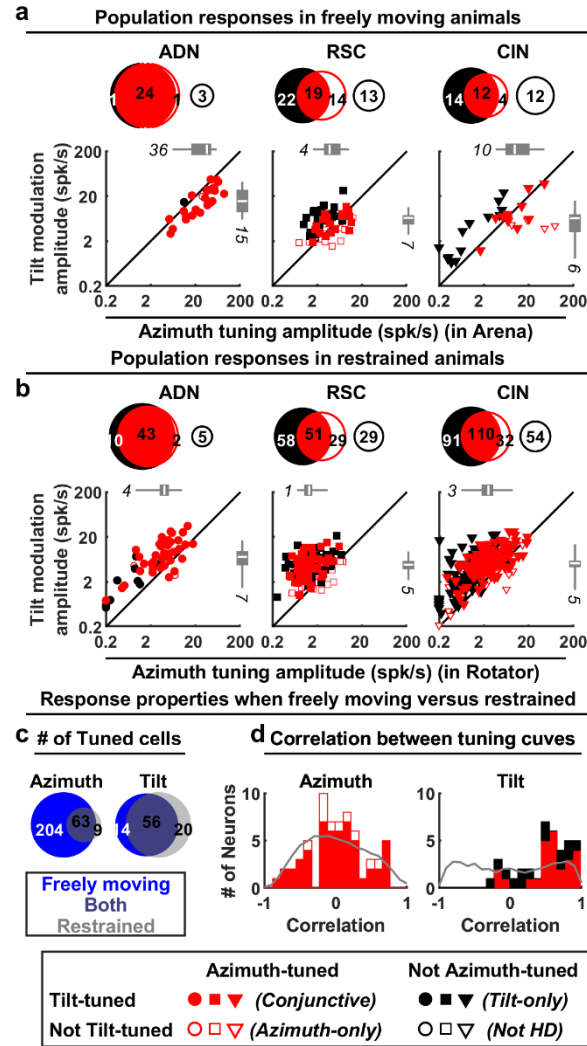


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Figure 1. Example cells tuned to tilt.

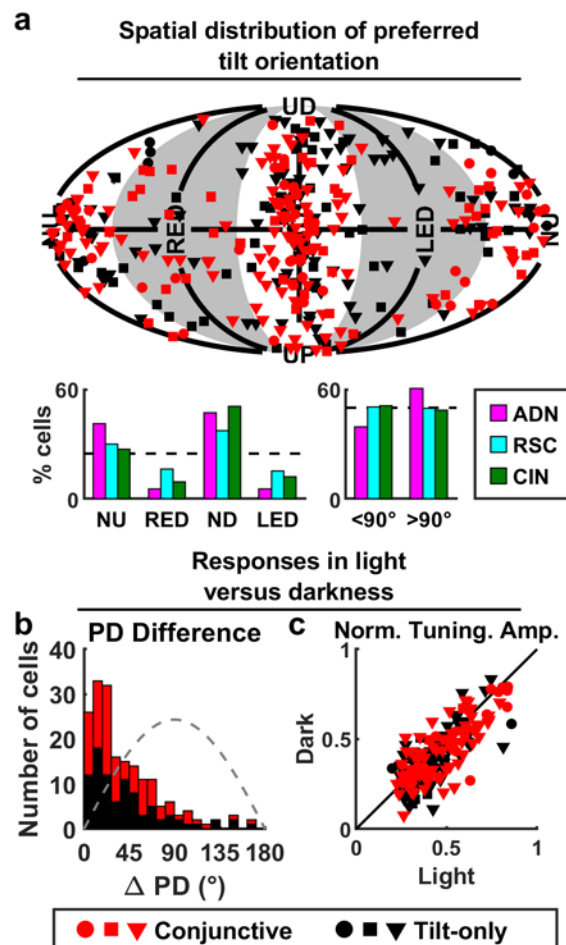
401 (a) Schematic representation of the topologies of tilt (2D, spherical - top) and azimuth (1D, circular - bottom). (b) Schematic of the arena used to identify azimuth-tuned cells during free foraging in the horizontal plane. (c-d) Example azimuth tuning of a 'traditional HD cell', i.e. tuned to azimuth ('Az-tuned') in the ADN (c) and another cell not tuned to azimuth ('non-Az-tuned' cell) in the CIN (d), as the mouse walks freely in light (red) and darkness (black) in a horizontal arena (shown in b), on a platform oriented horizontally (shown in e, left) (broken pink lines) and in the rotator (shown in j) (gray lines). The Az-tuned cell exhibited a significant tuning with different preferred directions (PD) in all setups, although response amplitude was strongly attenuated in the rotator (compare gray with red/pink lines). (e) Schematic of a 3D orientable platform used to measure 3D tuning. Two-degrees-of-freedom allow the platform to be tilted and re-oriented. (f,g) Tuning curves for the two cells in c and d, obtained from responses as the mouse foraged on the orientable platform (shown in e). Firing rate is shown as a heat map in 3D space (see **Suppl. Movies 1,2**). Note that tuning curves are restricted within a narrow sector of $\sim 60^\circ$ around upright (see Methods). (j) Schematic of a 'rotator' used to measure complete 3D tuning curves. (k,l) Tuning curves for the two cells in (c,f) and (d,g) as the mouse was passively re-oriented uniformly throughout the full 3D space using the rotator (see **Suppl. Movie 3,4,5**). Note that, unlike Az-tuning, tilt PDs are maintained across different environments.

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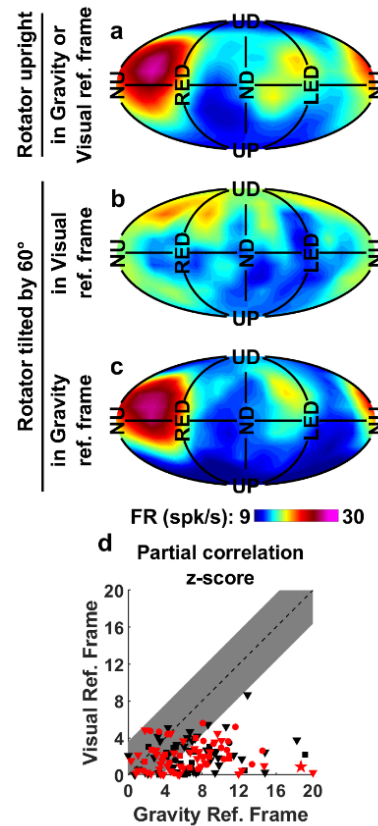


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 420 **Figure 2: Population azimuth and tilt tuning in freely moving vs. restrained animals,**
 421 **segregated by area. (a)** Summary of tuning prevalence during unrestrained motion. Azimuth
 422 tuning was derived from data in the freely moving arena (**Fig. 1b**). Tilt tuning was derived from
 423 data on the 3D platform (**Fig. 1e**). For each panel, Venn diagrams (top) indicate the number of
 424 tilt-tuned (filled black discs) and azimuth-tuned (red disks) cells. Conjunctive cells appear at the
 425 intersection of these disks. Open disks illustrate cells responsive to neither tilt nor azimuth. The
 426 scatterplots (bottom) indicate the peak-to-trough modulation amplitude of responsive cells,
 427 computed from Gaussian (tilt tuning) or von Mises (azimuth tuning) fits; see Methods. The
 428 boxes and whiskers represent the median (white line), 95% confidence interval (boxes) and
 429 upper/lower quartiles (whiskers) of the azimuth modulation of azimuth-tuned cells (top) and tilt
 430 modulation of tilt-tuned cells (right). Different symbols (based on recorded area) are color-
 431 coded based on cell type (Conjunctive: filled red; Azimuth-only: open red; Tilt-only: filled black).
 432 **(b)** Summary of prevalence of tilt tuning in restrained animals rotated passively in the rotator
 433 (**Fig. 1j**) and azimuth tuning (when moving freely). Format as in panel a. **(c)** Comparison of
 434 responsiveness for cells tested in both restrained and freely foraging animals. Venn diagrams
 435 with the number of cells tuned when moving freely (blue) in the arena (azimuth tuning) or 3D
 436 platform (tilt tuning) and restrained in the rotator (grey). Cells tuned under both conditions

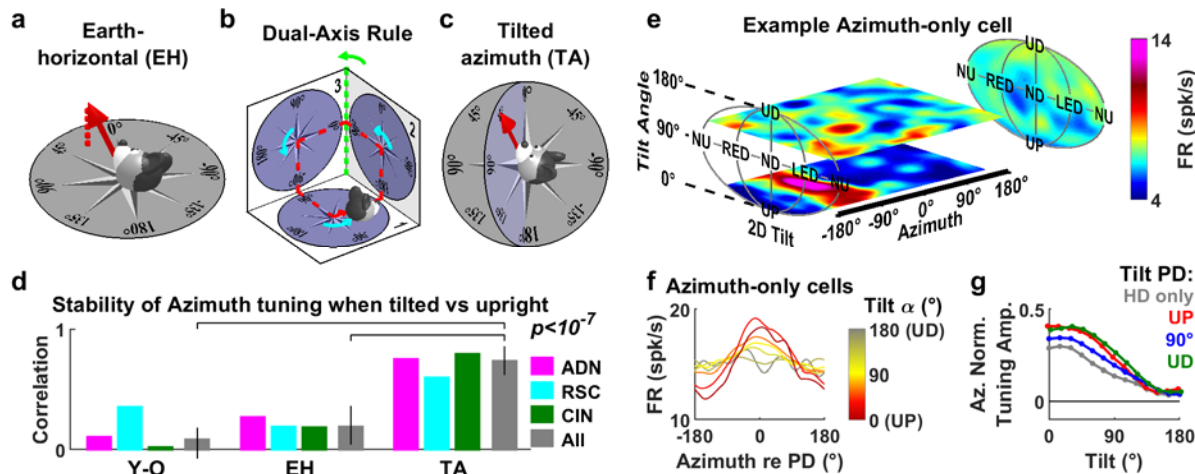
437 appear at the intersection of both discs. **(d)** Pixel-by-pixel correlation of the fitted azimuth (left)
438 and tilt (right) tuning curves (only cells tuned under both freely moving and restrained
439 conditions are included). For tilt tuning, the rotator data were re-analyzed by restricting tilt
440 angles up to 60° (to match the conditions in the platform). Grey: expected distribution if tuning
441 curves shift randomly (H_0), computed by randomly shuffling the cells.
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443
 444 **Fig 3: Summary of 3D tilt tuning (based on full tuning curves measured in the rotator).** (a) Top:
 445 Distribution of tilt PDs. Red: Conjunctive (azimuth and tilt) cells; Black: tilt-only cells. Circles,
 446 squares, triangles: ADN, RSC, CIN, respectively. Bottom: Number of cells with PD in the roll
 447 (RED/LED) or pitch (ND/NU) plane; and in upper (<90° tilt) or lower (>90° tilt) hemisphere,
 448 color-coded separately for each area (AND, CIN, RSC). (b) Distributions of absolute difference in
 449 tilt PD for light vs. darkness for conjunctive (red) and tilt-only (black) cells. Gray dashed line:
 450 expected distribution if PDs are independently distributed on a sphere. (c) Comparison of tilt
 451 peak-to-trough normalized tuning amplitude computed from Gaussian fits (Methods) in
 452 darkness vs. light. Red: Conjunctive (azimuth and tilt) cells; Black: tilt-only cells.

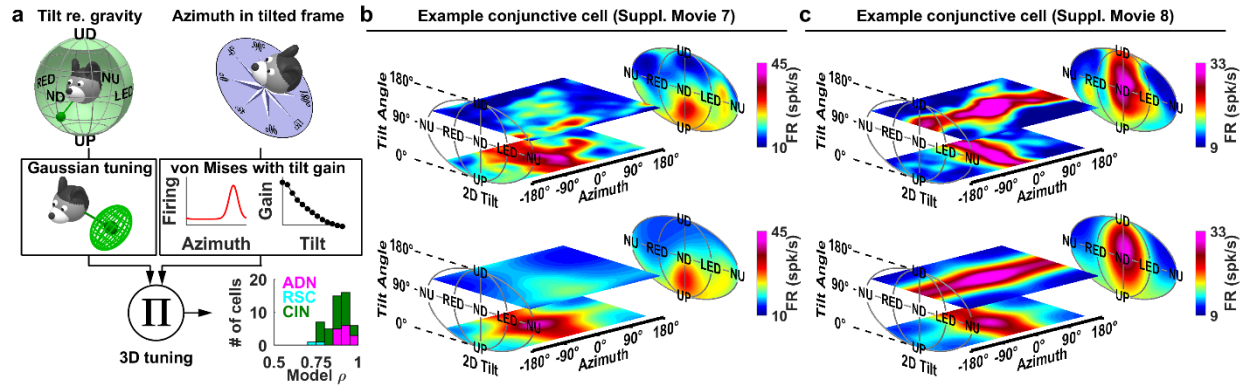


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 454 **Figure 4: Tilt tuning is anchored to gravity.** (a-c) Tilt tuning curves of one tilt-only example cell
 455 obtained from responses in the rotator with the spherical enclosure either (a) upright or (b), (c)
 456 tilted 60°. For the latter, tuning curves are computed in a visual (b) or gravity (c) reference
 457 frame. The visual reference frame tuning curve (b) is markedly distorted and attenuated
 458 relative to the one measured with the rotator upright (a), unlike the tuning curves computed in
 459 a gravity reference frame (c). Correlation coefficients: 0.02 (visual frame) and 0.89 (gravity
 460 frame). (d) Comparison between the z-scored partial correlation coefficients for Visual and
 461 Gravity reference frames across cells. Red: Conjunctive (azimuth and tilt) cells; Black: tilt-only
 462 cells. Circles, squares, triangles: ADN, RSC, CIN, respectively. Star: Example cell in (a-c). Shaded
 463 area: $p > 0.01$.



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465 **Figure 5: Encoding of azimuth is also influenced by gravity.** (a) Illustration of the earth-
 466 horizontal (EH) reference frame, where azimuth direction is projected onto the earth-horizontal
 467 plane (dashed red line). (b) Dual-axis rule for updating an azimuth compass in a head-horizontal
 468 frame, illustrated by an example trajectory (red) where the head travels in 3D across three
 469 orthogonal surfaces (numbered 1 to 3). Head azimuth is updated when the head rotates within
 470 one surface (yaw, cyan arrows) or when it rotates in the earth-horizontal plane (green arrow
 471 when transitioning from surface 2 to 3). If azimuth is updated by yaw rotations only (yaw-only
 472 frame, YO; Shinder and Taube, 2018), the compass will not maintain allocentric consistency
 473 since it will register only 270° when the head returns to its initial position. (c) Tilted azimuth
 474 (TA) reference frame (Laurens and Angelaki, 2018), where head direction is measured in a
 475 compass co-planar with the head-horizontal plane but aligned with the EH compass along the
 476 line intersecting the two planes (here, the 0-180° line). The head has the same 3D orientation in
 477 (a) and (c) but its azimuth is different in the two frames (24° in EH frame and 45° in TA frame).
 478 Updating a TA compass must follow the dual-axis rule in (b). (d) Stability of azimuth PD in YO,
 479 EH and TA frames. For each cell, we computed the correlation between the azimuth tuning
 480 curves when the head was upright (<math><45^\circ</math> tilt) or tilted (>60°). Correlations are shown separately
 481 for each frame and for each recorded area (colored bars) or all cells pooled together (grey
 482 bars). (e) 3D tuning curve of an example azimuth-only cell. Horizontal slices of the tuning curves
 483 (at tilt angles of 30° and 110°) are shown (see animations in **Suppl. Movie 7** for full tuning
 484 curve). The average tilt tuning curve (across all azimuths) is shown on the right panel. (f)
 485 Average azimuth tuning curve of all azimuth-only cells that maintained their azimuth tuning in
 486 the rotator ($n=10$), averaged across all tilt orientations (γ) and computed at tilt angles (α)
 487 ranging from 0 to 180°. (g) Average normalized tuning amplitude of the azimuth tuning curve as
 488 a function of tilt angle, computed for azimuth only cells ($n=10$, grey) and conjunctive cells with
 489 preferred tilt directions near upright (<math><75^\circ</math> tilt, red, $n=16$), near 90° tilt ($\pm 15^\circ$, blue, $n=22$) and
 490 near upside-down (>105°, green, $n=15$).



491

492 **Figure 6: Modelling 3D responses.** (a) Separable model overview. 3D tuning curves are the
493 product of a 2D tilt tuning curve and a 1D azimuth tuning curve. Tilt tuning curves are produced
494 by integrating gravity signals through Gaussian tuning functions (Suppl. Fig. 9,10). Azimuth
495 tuning is expressed in a TA frame, and modelled as von Mises distributions combined with a tilt-
496 dependent gain factor (Fig. 5g). Inset: Distribution of the model's coefficient of correlation (ρ)
497 across areas. (b,c) Experimentally measured 3D tuning curves (top) from two conjunctive cells
498 that maintain their azimuth tuning in the rotator and fitted tuning curves (bottom), represented
499 as color maps in 3D space (animated in Suppl. Movies 8,9).

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- 620

621 **On-line Methods**

622 ***Animals***

623 A total of 10 male adult mice (C57BL/6J), 3-6 months old, were used in this study (**Suppl. Table**
624 **1**). Animals were prepared for chronic recordings by implanting a head-restraint bar and a
625 microdrive/tetrode assembly under general anesthesia (Isoflurane) and stereotaxic guidance.
626 Two skull screws were implanted in the vicinity of the target region, and a circular craniotomy
627 (~1.5mm diameter) was performed above the target region. Animals were single-housed on a
628 reversed [12/12] light/dark cycle. Experimental procedures were conducted in accordance with
629 US National Institutes of Health guidelines and approved by the Animal Studies and Use
630 Committee at Baylor College of Medicine.

631 ***Neuronal recordings***

632 Neurons were recorded using 6 (mice AA1/AA2) or 4 (all other mice) tetrode bundles
633 constructed with platinum-iridium wires (17 micrometers diameter, polyimide-insulated,
634 California Fine Wire Co, USA) and platinum-plated for a target impedance of 200k Ω using a
635 Nano-Z (Neuralynx, Inc) electrode plater. Tetrodes were cemented to a guide tube (26-gauge
636 stainless steel) and connected to a linear EIB (Neuralynx EIB/36/PTB). The tetrode and guide
637 tube were attached to the shuttle of a screw microdrive (Axona Ltd, St Albans, UK) allowing a
638 travel length of ~5mm into the brain.

639 The stereotaxic coordinates for each tetrode implant was based upon Bregma as a reference
640 point. The coordinates used to target both the ADN and the CIN were 0.2 mm posterior and 0.7
641 mm lateral to Bregma. The granular/dysgranular RSC were targeted by implanting 2.0 mm
642 posterior and 0.07/0.7 mm lateral to Bregma respectively.

643 *Postmortem verification of electrode sites:* At the end of the study, brains were removed for
644 histological verification of electrode location. The animals underwent transcardial perfusion
645 with 4% paraformaldehyde (PFA). The brains were postfixed in 4% PFA and then transferred to
646 30% sucrose overnight. Brain sections (40 μ m) were stained (Nissl or neutral red staining), and
647 examined using bright-field microscopy to localize tetrode tracks (**Suppl. Figure 2**). Photographs
648 of histological slides were corrected for brightness, contrast, gamma and color balance.

649 *Firing properties:* We confirmed that the distribution of average firing rates and CV2 were
650 similar for all cells in the CIN, ADN, and RSC (**Suppl Fig. 19a**). Furthermore, we observed that
651 most spikes recorded in CIN had small (<0.3ms) trough to peak durations, indicative of axonal
652 spikes (**Suppl Fig. 19b**).

653 ***Experimental apparatus:***

654 *Freely moving recordings:* In order to identify traditional HD cells, we first recorded as mice
655 explored freely in a circular arena (50 cm diameter, 30 cm height; **Fig. 1b**). The walls of the
656 arena were white with a 45° black card to provide a visual orientation cue. To record tilt tuning
657 in freely moving mice, the arena was replaced by a movable platform that was constructed by
658 mounting an oblong nylon mesh (20x30cm, 1.5cm mesh) onto a manually operated three-axis

659 gimbals system; **Suppl. Fig. 4a**. The system was placed at the center of a large cylinder (130 cm
660 diameter, 2 m height), its door was left open during recording to provide a visual landmark and
661 to allow the experimenter to monitor each mouse. In both systems, neuronal data were
662 acquired at 22 kHz using a MAP system (Plexon Inc.). The microdrive's EIB was plugged to a
663 tethered head stage that included two LEDs (one red and one infra-red, 4 cm apart) for optical
664 tracking (Cineplex, Plexon Inc.). In addition, mice's head were equipped with a digital 6-degree-
665 of-freedom inertial measurement unit (IMU; SparkFun SEN-10121) for measuring head tilt
666 relative to gravity. Perspective effects that could affect optical tracking when the head tilted
667 away from horizontal were corrected based on the IMU data.

668 Motorized rotator: To measure the 3D orientation tuning using a uniform representation of tilt
669 angles, we tested animals using a motorized rotator. It also allowed us to separate visual from
670 gravity representations. We gently restrained each mouse's body and fixed its head rigidly, and
671 placed it in the center of a rotation simulator (**Suppl. Fig. 6a**) composed of a motorized three-
672 axis motion system (Axes I-III in **Suppl. Fig. 6a**) inside a visual surround sphere (1.8 m diameter)
673 (Acutronics Inc., Switzerland). The inside of the sphere was painted in white, with three
674 horizontal lines of dots (10° diameter, 30° spacing) to provide horizon and optokinetic cues.
675 Three vertical LED stripes were placed 22.5° apart to provide a horizontal orientation cue. A
676 fourth rotation axis (Axis IV) allowed tilting the rotator and the sphere together (sphere door
677 closed). Neuronal data were acquired at 30 kHz using a neural data acquisition system
678 (SpikeGadget, San Francisco, California). The position of the rotator's axes (and therefore the
679 3D orientation of the head) was measured with potentiometers installed in each rotation axis
680 and digitized at 833 Hz.

681 All recorded data was organized in a custom-made database using Datajoint (Yatskeno et al.
682 2015)

683 **Experimental protocols (see Suppl. Table 2):**

684 Experiment 1: Characterization of HD tuning in the arena: We recorded neuronal responses
685 during five 8-minute sessions. A first recording session was performed in light (*Experiment 1-*
686 *L0*). We then performed the other protocols described below on a moving platform and rotator,
687 before returning the mouse to the same arena and performing three separate 8-min sessions,
688 first in light (*Experiment 1-L1*), then in darkness (*Experiment 1-D*), then we repeated a session in
689 light (*Experiment 1-L2*).

690 Experiment 2: Tilt tuning in freely moving animals: We recorded neural responses when mice
691 walked freely on a platform. Recordings were performed in 5-min blocks during which the
692 setup's axis II and III were fixed. Within a single block with the platform tilted, freely
693 movements on the platform's surface changed the mice's head azimuth (Az) and tilt orientation
694 (angle γ) together, and these variables are therefore correlated (**Suppl. Fig. 4b**, yellow,
695 magenta). Rotating the base (rotation along the blue arrow) between blocks added an offset to
696 azimuth, while leaving the range of head tilt unchanged (e.g. **Suppl. Fig. 4b**, yellow vs.
697 magenta). This manipulation allowed coverage of all possible head azimuth and tilt orientations

698 (plane in **Suppl. Fig. 4b**), thereby allowing coverage of a large portion of 3D space relatively
699 uniformly (up to $\alpha=60^\circ$; **Suppl. Fig. 4c**).

700 We perform one additional manipulation of the space covered by the animal: Half-way through
701 each block, the platform was rotated using axis I (**Suppl. Fig. 4a**). This manipulation served as
702 control for the following potential confounding factor: As long as only Axis III is operated, then
703 local azimuth on the platform is anchored to gravity, e.g. the same side of the platform is
704 always placed downward. Therefore, if a cell's firing was anchored to the azimuth on the
705 platform itself, and not to the tilt, its response could be misinterpreted as a tilt response.
706 Changing Axis I multiple times within each block randomizes local azimuth relative to tilt, which
707 prevents this potential confound.

708 We performed 17 blocks (~68 min) with the following organization: (1) one block where the
709 platform was horizontal (duration: 8 min), (2) eight blocks where the platform was tilted 45°
710 and the base was rotated in steps of 45° (duration: 2.5 min) and (3) eight blocks where the
711 platform was tilted 65° and the base was rotated in steps of 45° (duration: 5 min). Note that
712 mice tended to upright their head, therefore tilting the mesh 45° and 70° resulted in average
713 head tilts of $\sim 35^\circ$ and 60° , respectively (e.g., **Suppl. Fig. 4c**). Together, these 13 blocks allow a
714 relatively uniform sampling of 3D head orientation (at tilt angles up to $\sim 60^\circ$) while mice were
715 unrestrained and locomoting freely.

716 To ensure that tilt space was adequately sampled, we computed the occupancy distribution d
717 (i.e. the time spent) across 73 tilt positions (uniformly distributed in tilt space for up to 60°).
718 Next, we computed the entropy of d $E(d)=-\sum p(d) \cdot \log_2(p(d))$, ranging from $\log_2(73)=6.19$ (uniform
719 distribution) to 0 (if the mouse occupies a single point). We excluded cells where $E(d)<5.6$,
720 which corresponds to mice sampling less than 2/3 of the tilt space. 45% of recorded cells (not
721 counted in **Suppl. Table 1**) were excluded based on this criterion.

722 *Experiment 3: Three-dimensional tuning in the rotator:* The rotator was programmed to scan 3D
723 rotation space uniformly using preprogrammed trajectories that sample 200 head tilt
724 orientations uniformly (**Suppl. Fig. 6b**, red; **Suppl. Movie 3**); the distance between adjacent
725 points being $\sim 15^\circ$. We computed four distinct trajectories (no overlap, **Suppl. Fig. 6c**, different
726 colors), each of which visited all points once, and in different order. Trajectories travelled
727 through each point in a straight line at a constant velocity ($30^\circ/s$) and changed direction
728 between points (**Suppl. Fig. 6b,c**). All trajectories were replayed forward and backward. This
729 technique ensures that the 2D space of head tilt is covered uniformly. While the desired head
730 tilt is achieved by controlling the two innermost axes (I and II), azimuth is varied by rotating axis
731 III (outer) of the rotator at a constant velocity ($\pm 15^\circ/s$; **Suppl. Fig. 6d**, red; the velocity is
732 reversed every 4 rotations). During the trajectory, mice always faced at least 90° away from the
733 second axis (black in **Suppl. Fig. 6a**) to ensure that the visual field in front of the mouse is not
734 obstructed. We performed the following variants of the protocol: (i) with the LED stripes
735 (placed inside the visual enclosure) on (*Experiment 3-L*), (ii) off (*Experiment 3-D*), and (iii) LED
736 on, after the rotator and the visual enclosure were tilted en bloc 60° relative to vertical by
737 operating Axis IV (**Suppl. Fig. 14**) (*Experiment 3-T*).

738 Experiment 4: Yaw/pitch/roll rotations: The rotator was programmed to rotate each mouse
739 back and forth in yaw, pitch or roll at a constant velocity of 30°/s. Starting from a velocity of 0°,
740 each movement included an acceleration phase of 1s to 30°/s, then 380° of rotation at constant
741 velocity and finally a deceleration period of 1s. To exclude any potential response to
742 accelerations or decelerations, only data recorded during the central 360° of constant-velocity
743 rotation period was used in the analysis.

744 **Data analysis**

745 Neuron selection and classification: All well isolated neurons recorded during at least one
746 foraging session in light in the arena (Experiment 1- L0, L1 or L2), and during Experiment 2 or
747 Experiment 3-L have been included for analyses. We first classified neurons as azimuth-tuned or
748 non-azimuth-tuned based on their responses in the freely moving arena. Neurons could also be
749 classified as azimuth-tuned or azimuth-untuned based on their responses in the platform and
750 rotator. However, because azimuth responses have lower amplitude in the rotator, they often
751 didn't reach significance level. Therefore, throughout the study, "azimuth-tuned" refers by
752 default to the classification based on freely moving data in the arena (Experiment 1).

753 Similarly, neurons were classified twice as tilt-tuned or not, based on recordings on the
754 orientable platform and in the rotator independently. As with azimuth tuning, neurons that
755 exhibited significant tilt tuning when moving freely may not be significantly tuned in the
756 rotator, because responses in the rotator had lower amplitude. On the contrary, some neurons
757 that exhibited significant tilt tuning in the rotator weren't significantly tuned when moving
758 freely because this protocol sampled a limited range (~1/3) of head tilt. Nevertheless,
759 differences were small, and the majority of neurons significantly tilt-tuned in one setup were
760 also tuned to the other (**Fig. 2**).

761 Importantly, we confirmed that azimuth and tilt tuning in the rotator and freely moving were
762 correlated in terms of amplitude and consistent in terms of spatial characteristics for neurons
763 that were significantly tuned to azimuth or tilt in both experiments (**Fig. 2c,d; Suppl. Fig. 8**).

764 Tuning curves: For each recorded neuron, we computed the following tuning curves:

- 765 (1) To evaluate azimuth tuning, we computed 1D azimuth tuning curves in all conditions of
766 *Experiment 1*, in *Experiment 2* when the platform is horizontal, in *Experiment 4-Yaw*;
767 and in data points where head tilt was less than 45° during *Experiment 3-L*.
- 768 (2) To evaluate tilt tuning for *Experiment 2* and *Experiment 3-L,D,T*, data were averaged
769 across azimuth. We also computed pitch/roll tuning curves based on *Experiment 4*.

770 Note that tilt tuning is different from a recent finding that RSC HD cells encode azimuth in a tilt-
771 dependent manner (Page et al. 2017), which has been explained by a framework called 'the
772 dual-axis rule' (Page et al. 2017) or 'tilted azimuth' (Laurens and Angelaki 2018); see **Suppl. Fig.**
773 **15**. Although tilt influences the azimuth reference frame, tilted azimuth signals do not carry any
774 information about head tilt. Reciprocally, the tilt tuning identified here doesn't carry any
775 information about azimuth. Thus, 2D tilt tuning and 1D tilted azimuth encode different
776 dimensions of 3D head orientation (see also **Suppl. Movie 1**).

777 Coordinates used to encode 3D head orientation: Head tilt was expressed in spherical
778 coordinates (α, γ ; **Suppl. Fig. 5**), where α is the tilt angle: $\alpha=0^\circ$ in upright orientation (UP) and
779 $\alpha=180^\circ$ in upside-down orientation (UD); and γ encodes tilt orientation: $\gamma = 0^\circ$ and $\gamma = 180^\circ$
780 correspond to nose-down (ND) and nose-up (NU) tilt (pitch); $\gamma = 90^\circ$ and $\gamma = -90^\circ$ correspond to
781 left-ear-down (LED) and right-ear-down (RED) tilt (roll). For fitting purposes (e.g. **Suppl. Fig. 9**),
782 head tilt was also expressed in Cartesian coordinates (G_x, G_y, G_z) that represent the orientation
783 of the gravity vector (normalized to a length of 1) in head coordinates. Spherical coordinates
784 are transformed into Cartesian coordinates and vice-versa by $G_x = \sin(\alpha) \cdot \cos(\gamma)$; $G_y = \sin(\alpha) \cdot \sin(\gamma)$;
785 $G_z = -\cos(\alpha)$; and $\alpha = \arccos(-G_z)$; $\gamma = \arctan2(G_y, G_x)$.

786 Neuronal responses were evaluated by computing tuning curves, which were smoothed using
787 Gaussian kernels with standard deviation of 15° on both azimuth and tilt. We computed 3D
788 azimuth tuning curves in 3 different ways, by expressing azimuth in a yaw-only (Y-O), an earth-
789 horizontal (EH) or a tilted (TA) frame, and found that the latter accounted for neuronal
790 responses better (**Fig. 5; Suppl. Fig. 15**). Earth-horizontal azimuth is computed by defining a
791 “forward” pointing vector N , aligned with the head’s naso-occipital axis, and encoding its
792 orientation in an earth-fixed reference frame (i, j, k), i.e. $N = (N_i, N_j, N_k)$. Earth-horizontal azimuth
793 is defined as the orientation of N on the earth-horizontal (i, j) plane, i.e. $EHAz = \arctan2(N_j, N_i)$. EH
794 azimuth can be transformed into tilted azimuth by the following equation: $TA = EHAz - \gamma -$
795 $\arctan2(-\sin(\gamma), \cos(\alpha) \cdot \cos(\gamma))$.

796 Tuning curve fitting: To quantify tuning curves, von Mises and/or Gaussian functions were fitted
797 and standard shuffling analysis was used to evaluate the statistical significance of azimuth and
798 tilt tuning.

799 2D tilt tuning curves were fitted with Gaussian distributions (see **Suppl. Fig. 9,10**), where tilt
800 was expressed in Cartesian coordinates and $FR_{Ti}(\alpha, \gamma) = FR_0 + A \cdot N_{M,C}(G_x, G_y, G_z)$ where $N_{M,C}(G_x,$
801 $G_y, G_z)$ is a 3D Gaussian distribution centered on M and with covariance matrix C .

802 Azimuth tuning curves were fitted with circular normal (von Mises) distributions. Preliminary
803 analysis revealed that the PD of azimuth tuning is maintained when the head tilts (when
804 azimuth is expressed in a tilted frame) but that its gain changes. To account for this, we defined
805 a tilt-dependent gain $G(\alpha)$ and expressed azimuth tuning as:

$$806 \quad FR_{Az}(Az, \alpha) = G(\alpha) \cdot \exp(\kappa \cdot \cos(Az - PD)) / (1 + (1 - G(\alpha))),$$

807 where κ is the parameter of the von Mises distribution. For convenience, we normalized
808 $FR_{Az}(Az, \alpha)$ such that its average value across all azimuths is 1 (by setting l to the average value
809 of $\exp(\kappa \cdot \cos(Az - PD))$).

810 Finally, we evaluated the interaction between azimuth and tilt tuning by fitted 3D tuning curves
811 as the product of the azimuth and tilt tuning curves defined above, i.e.

$$812 \quad FR(Az, \alpha, \gamma) = FR_{Az}(Az, \alpha) \cdot FR_{Ti}(\alpha, \gamma)$$

813 Tilt tuning curves had 11 free parameters: FR_0 , A , M (3-dimensional) and C (a covariance matrix,
814 i.e. 6-dimensional). The normalized azimuth tuning curves had 2 free parameters (κ and PD).

815 The tilt-dependent gain $G(\alpha)$ was fitted independently at 13 tilt angles α ranging from 0 to 180°
816 by increments of 15°, resulting in 13 additional free parameters. The 3D tuning curves were
817 computed from experimental data at 184 uniformly distributed tilt orientations and 24 azimuth
818 orientations, i.e. 4416 points, and fitted to the 3D curve model by gradient ascent (Matlab
819 function `lsqnonlin`). Note that since the average value (across all azimuths) of $FR_{Az}(Az, \alpha)$ was 1,
820 the average tilt tuning curve (across all azimuth) was $FR_{Ti}(\alpha, \gamma)$.

821 We tested an additional model that assumes that azimuth and tilt tuning interact additively, i.e.
822 $FR(Az, \alpha, \gamma) = FR_{Az}(Az, \alpha) + FR_{Ti}(\alpha, \gamma)$. We found that this model didn't fit 3D tuning curves as well as
823 the multiplicative model: its correlation coefficient was significantly lower in 33/53 conjunctive
824 cells, and better only in 1/53 conjunctive cell (Fisher r to z transform, at $p < 0.01$), and
825 significantly lower at the population level (median $\rho = 0.85$, [0.80-0.87] CI versus 0.88, [0.86-
826 0.91] CI, $p < 10^{-8}$, paired Wilcoxon test). Therefore, we used the multiplicative model to model
827 3D responses in this study.

828 Quantification of tuning strength: The length of the mean vector (i.e. the normalized Rayleigh
829 vector), ranging from 0 to 1, is used commonly to assess how strongly a cell is tuned ($|R| = 0$, un-
830 tuned cell; $|R| = 1$, maximally tuned cell) independently from its average firing rate. It allows
831 comparing cells with a large range of peak firing rates. Thus, azimuth tuning was quantified
832 consistently with previous studies by computing the mean vector $R = c \cdot \sum FR(Az) \cdot \exp(-i \cdot Az) / \sum$
833 $FR(Az)$, where $FR(Az)$ was sampled at 100 positions separated by 3.6° and $c =$
834 $3.6 \cdot \pi / 180 / 2 / \sin(1.8)$ (Zar, 1998).

835 Mean vectors can be generalized to a 2D distribution by expressing tilt in Cartesian coordinates
836 $G = (G_x, G_y, G_z)$ and computing $R = \sum FR(G) \cdot G / \sum FR(G)$. The resulting 2D vector has a length of 1 if
837 all spikes occur at the same tilt and 0 if spikes are distributed uniformly or symmetrically.

838 However, because mean vectors computed in 1D and 2D can't be compared directly, we
839 developed an alternative measure called normalized tuning amplitude (NTA; **Suppl. Fig. 7**). The
840 normalized tuning amplitude of an 1D azimuth or 2D tilt tuning curve was defined based on the
841 maximum (FR_{max}) and minimum (FR_0) firing rate, as $A_N = (FR_{max} - FR_0) / FR_{max}$. Thus, normalized
842 tuning amplitude ranged from 1 (when a tuning curve ranged from 0 spk/s to a peak value) to 0
843 (when a cell was unmodulated). Note that normalized tuning amplitude measures the cell's
844 modulation amplitude, but not the sharpness of the tuning curve.

845 Statistical procedures to determine significant tuning: We used a shuffling procedure (Langston
846 et al. 2010; Yartsev et al; 2011) to assess the statistical significance of azimuth or tilt tuning.
847 Each sample was generated by (1) shifting the entire spike train circularly by a random value of
848 at least $\pm 10s$, (2) recomputing the tuning curve, (3) performing the Gaussian fit and (4)
849 computing the azimuth and/or tilt normalized tuning amplitude. We computed the mean value
850 m and standard deviation σ of the normalized tuning amplitude across 100 shuffled samples.
851 The statistical p -value of the normalized tuning amplitude NTA measured in the un-shuffled
852 data was computed as $1 - F(\text{NTA}, m, \sigma)$, where F is the cumulative Gaussian distribution with
853 average m and standard deviation σ .

854 We considered azimuth or tilt tuning to be significant if (1) the p-value computed as described
855 above was less than 0.01 and (2) the normalized tuning amplitude NTA was equal to or higher
856 than 0.25. The second criterion was equivalent to selecting cells where the modulation was at
857 least one third of the baseline firing rate, and was used to eliminate cells with very small but
858 significant modulation (how this threshold compares to criteria used in other studies is
859 analyzed and discussed in **Suppl. Fig. 7**).

860 We combined data from multiple repetitions of Experiment 1-L to assess if cells were significantly
861 tuned to azimuth when the animal was moving freely. We used two techniques to combine
862 multiple repetitions: (1) we analyzed each repetition independently and computed the median
863 p-value and amplitude across repetitions, and (2) we computed a p-value and amplitude based
864 on data pooled across all repetitions. We tested if the values obtained with technique (1) or
865 with technique (2) passed the criteria described above, and classified cells as azimuth tuned if
866 they passed any of these tests. This two-technique approach was used because pooling data
867 yields a greater statistical power but fails if the cell's PD shifted between sessions, whereas the
868 second approach isn't affected by shifts in the PD. In total, 37% cells passed both tests; 6.5%
869 passed the first test only, 7.5% passed the second test only, and 51% cells passed one or the
870 other and were classified as azimuth-tuned.

871 We used data from Experiment 3-L to assess if cells are significantly tuned to tilt, by testing if
872 the normalized tuning amplitude of the average tilt tuning curve (across all azimuth) passed the
873 criteria described above. We also tested if cells are significantly tuned to azimuth in the rotator
874 by computing the azimuth tuning curve based on data for up to 45° tilt and testing if its
875 normalized tuning amplitude passes the criteria described above. In some cells, Experiment 2 or
876 Experiment 3-L were repeated multiple times. We found that the preferred direction of tilt and
877 azimuth tuning were stable across repetitions, and pooled data across all repetitions.

878 *Tilt and azimuth velocity analysis:* We performed another Gaussian curve fitting to test whether
879 neurons carry a mixture of tilt and tilt derivative information. We expressed head tilt measured
880 during Experiment 3-L in Cartesian coordinates (G_x , G_y , G_z) and then computed the time
881 derivative of the gravity vector (dG_x/dt , dG_y/dt , dG_z/dt) as a measure of tilt velocity. Next, we
882 fitted neuronal firing rate with $FR = FR_0 + A \cdot N_{M,C}(G_x, G_y, G_z) + A' \cdot N_{M',C'}(dG_x/dt, dG_y/dt, dG_z/dt)$.
883 We computed the normalized tuning amplitude of tilt and tilt velocity and used the same
884 shuffling method and criterion ($p < 0.01$, $A_N > 0.25$) as in other Gaussian fits to assess whether
885 cells were significantly tuned to the gravity derivative. Note that the tilt tuning curves obtained
886 with this method were identical to those obtained in previous Gaussian fit.

887 We investigated whether cells encode azimuth velocity (dAz/dt) by computing azimuth velocity
888 tuning curves for each cell (i.e. average firing rate as a function of dAz/dt) using all data from
889 Experiment 1-L. The tuning curves were evaluated at all velocities ranging from -200 to 200°/s
890 by increment of 20°/s and smoothed using a 8°/s Gaussian kernel. We computed the amplitude
891 and normalized tuning amplitude of these curves, and used the same shuffling method and
892 criterion ($p < 0.01$, $A_N > 0.25$) to assess whether cells were significantly tuned.

893 Experiment 3-T: To analyze the results of Experiment 3-T, we computed neuronal tuning curves
894 in both gravity and visual reference frames. Tuning curves in a gravity frame (curve G) were
895 computed based on the actual tilt of the head relative to gravity. Tuning curves in a visual frame
896 (curve V) were computed as if Axis IV of the system had not been tilted. Next, we performed a
897 linear regression of the tuning curve recorded with the rotator upright (Experiment 3-L) and the
898 curves G and V, computed the partial coefficients of correlation of curves G and V, and z-
899 transformed these coefficients (with 43 d.o.f. since curves were sampled at 46 points).
900 Statistical significance was assessed at $p < 0.01$ if the absolute difference between the z-scored
901 partial correlations was $> \sqrt{2} * 2.56$.

902 Experiment 4P/R: Using the full 3D tuning curve data from Experiment 3-L, we “predicted” the
903 responses to pitch and roll rotations by sampling the 3D tuning curve at the head orientations
904 visited during pitch and roll rotations. The pitch and roll tuning curves measured during
905 Experiment 4 and predicted based on Experiment 3 were then fitted with 1D Gaussians, and the
906 resulting modulation amplitudes and preferred direction were compared. Data during
907 Experiment 3 and 4 in light and darkness were averaged.

908

909


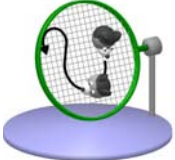
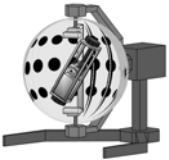

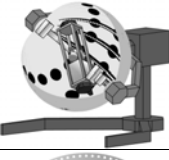

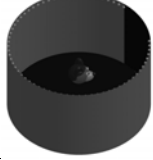

910

Animal name	H71M	H72M	I10M3	I29M	H65M	H68M	H69M	H74M	AA1	AA2	Total
Region	ADN	ADN	ADN	ADN	CIN	CIN	CIN	CIN	RSC	RSC	
Total	35	13	6	15	72	137	61	27	53	124	543
<i>Azimuth tuned</i>	33	9	6	4	34	68	28	17	26	58	283
<i>not Azimuth tuned</i>	2	4	0	11	38	69	33	10	27	66	260
Experiment 2	19	0	5	5	0	19	14	9	28	40	139
Platform											
<i>Conjunctive (Az&Tilt)</i>	16	0	5	3	0	5	4	3	5	14	55
<i>Tilt-only</i>	0	0	0	1	0	6	4	4	6	16	37
<i>Azimuth-only</i>	1	0	0	0	0	1	2	1	9	5	19
<i>Not HD</i>	2	0	0	1	0	7	4	1	8	5	28
Experiment 3L,D,T	30	13	3	14	72	137	59	19	53	114	514
<i>Rotator - in Light</i>											
<i>Conjunctive (Az&Tilt)</i>	30	8	2	3	28	47	24	11	14	37	204
<i>Tilt-only</i>	0	3	0	7	23	45	20	3	15	43	159
<i>Azimuth-only</i>	0	1	1	0	6	21	3	2	12	17	63
<i>Not HD</i>	0	1	0	4	15	24	12	3	12	17	88
<i>Rotator - in Darkness</i>	12 (12)	5 (4)	2 (1)	8 (6)	47 (33)	98 (68)	43 (32)	0 (0)	0 (0)	44 (30)	259 (186)
<i>Rotator - Tilted</i>	12 (12)	5 (3)	1 (1)	8 (6)	41 (26)	55 (38)	22 (16)	0 (0)	0 (0)	38 (27)	182 (129)
Experiment 4: yaw, pitch, roll	0 (0)	0 (0)	0 (0)	0 (0)	40 (28)	18 (14)	11 (8)	0 (0)	0 (0)	0 (0)	69 (50)

911

912 **Supplemental Table 1: Number of recorded cells and categories.** For each mouse (and each
 913 area), the table indicates the number of cells recorded during Experiment 1 (numbers in top
 914 row), 2, 3 and 4), along with their categorization. The last column shows the total number of
 915 neurons tested for each experimental protocol.

916

Experiment name	Illustration	Description	Goal
Experiment 1-L0		Free motion in arena.	Measure azimuth tuning using traditional method. Control for tuning stability (by comparing with Experiment 1-L1,2).
Experiment 2		Free motion on orientable platform.	Measure 3D tuning in freely moving animals at up to 60° tilt.
Experiment 3-L		3D tuning curve scanning in light.	Measure tuning uniformly in entire 3D space.
Experiment 3-D		3D tuning curve scanning in darkness.	Test that tilt tuning depends of gravity and not visual cues.
Experiment 3-T		3D tuning curve scanning in a tilted visual surround.	Test that tilt tuning depends of gravity and not visual cues.
Experiment 1-L1		Free motion in arena.	Measure azimuth tuning using traditional method. Control for tuning stability (by comparing with Experiment 1-L0,2).
Experiment 1-D		Free motion in arena, in darkness.	Test that azimuth tuning is maintained in darkness.
Experiment 1-L2		Free motion in arena.	Measure azimuth tuning using traditional method. Control for tuning stability (by comparing with Experiment 1-L0,1).

917 **Supplemental Table 2: Description and order of experimental protocols.** For each mouse (and
 918 each area), the table indicates

919

920 **Supplemental Movie 1: Volumetric representation of 3D head orientation.** Head tilt relative to
921 vertical is represented on a plane (labelled “2D Tilt”; see **Suppl. Fig. 5**) and head azimuth is
922 represented on a third (unfolded) axis to form a 3D volume. The animation represents a mouse
923 head placed at 5 different tilt orientations (upright, 60° LED, 90° ND and NU, 150° LED) and
924 moved through all possible azimuths. The center of the head is placed at the corresponding 3D
925 position in the volumetric plot. The movements that change azimuth without changing head
926 tilt, correspond to rotations around an earth-vertical axis, as represented by red arrows.

927 **Supplemental Movie 2: 3D tuning curve of the example neuron in Fig. 1g.** The tuning curve of
928 Fig. 1g is shown as animation sweeping through tilt angles from 0 to 60°. The lower right panel
929 shows the average azimuth tuning curve at each tilt angle, computed by averaging across all tilt
930 orientations. The cell is tuned to tilt, with a preferred orientation at 48° ND, but not to azimuth.

931 **Supplemental Movie 3: Three-dimensional passive re-orientation protocol in rotator (Fig. 1j;**
932 **see also Suppl. Fig. 6) and response of a tilt-tuned RSC cell.** Upper left panel: an animated
933 model of the 3D rotator (the rotator is ~2.4 m height; the size of the mouse head is
934 exaggerated) displays its position recorded in real-time using potentiometers during
935 Experiment 3-L. Lower panel: 3D head orientation is shown in volumetric space. Instantaneous
936 head orientation is represented by a black circle; the travelled trajectory by a grey line, and
937 recorded spikes by red dots. The trajectory is also projected on a tilt plane (right panel). A clock
938 shows the time elapsed since the beginning of the recording. The first 10s of recording are
939 shown in real time and then the movie accelerates to span the entire experiment. Spikes clearly
940 cluster in a space that corresponds to ND tilt between 120 and 150°, independently of azimuth.
941 The end of the movie (starting at t=50s) shows the complete tuning curve as a color map. The
942 animation sweeps through all tilt angles from 0 to 180° to allow visualizing the entire 3D
943 volume. The cell is tuned to tilt, with a preferred orientation at 135° ND, but not to azimuth.

944 **Supplemental Movie 4: 3D tuning curve of the example neuron in Fig. 1k (conjunctive cell).**
945 The movie pauses at a tilt angle of 10° to show azimuth tuning, which is strongest in the vicinity
946 of upright. The cell is also tuned to tilt, with a peak response in NU orientation. Same format as
947 in **Suppl. Movie 2**.

948 **Supplemental Movie 5: 3D tuning curve of the example neuron in Fig. 1l (tilt-only cell).** Same
949 format as in **Suppl. Movie 2**.

950 **Supplemental Movie 6: Azimuth tuning of an example neuron in YO, EH and TA frames.** Upper
951 left: animated model of the rotator during Experiment 3-L. Right panels: head tilt (α) versus
952 azimuth computed in a yaw-only frame (upper panel), earth-horizontal frame (middle) or tilted
953 azimuth frame (bottom). The tuning curves of an example conjunctive ADN cell, obtained by
954 averaging the 3D tuning curve across all tilt orientations (γ), are shown as a color map. As the
955 animation runs, the position of the head and the spikes emitted by the cells are indicated in
956 each panel. The position is identical in the 3 frames during the first few seconds of the
957 experiment, because the head is about upright where all frames are identical. As soon as the
958 head tilts beyond 90°, the 3 frames diverge. When the head comes back to upright orientation,
959 the position is again identical in the EH and TA frame because these frames maintain allocentric

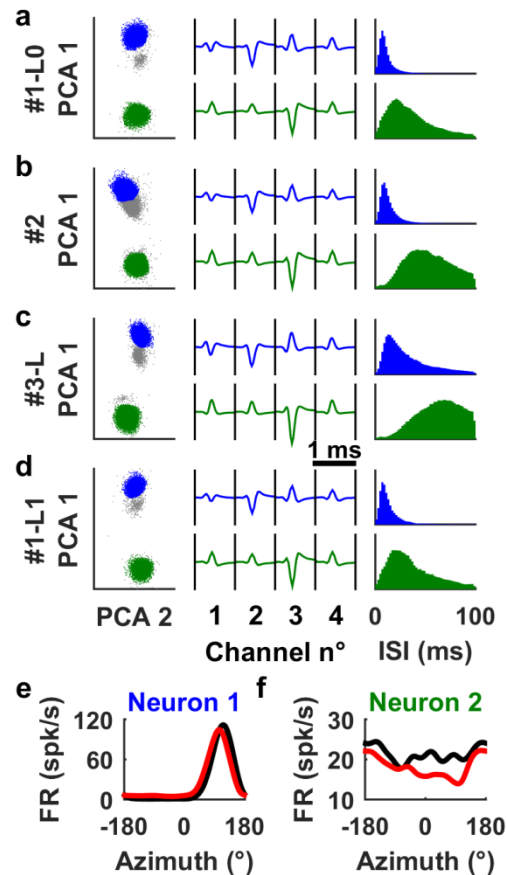
960 invariance. In contrast, azimuth in the yaw-only frame can't track allocentric head position (as
961 shown in **Fig. 5b**). As shown by the spiking activity and the tuning curves, the cell is tuned to
962 azimuth in the EH and TA frame. Beyond 90° tilt, where the EH and TA frame diverge most,
963 azimuth tuning is lost in the EH frame but maintained, although weakened, in the TA frame. The
964 final sequence of the movie shows the cell's full 3D tuning curve. Same format as in **Suppl.**
965 **Movie 2**.

966 **Supplemental Movie 7: 3D tuning curve of the example azimuth-only neuron in Fig. 5e.** The
967 movie pauses at tilt angles of 30° and 110°. As shown by the azimuth tuning curve (lower right
968 panel), azimuth tuning fades away when tilt angle increases. Same format as in **Suppl. Movie 2**.

969 **Supplemental Movie 8: 3D tuning curve of the example conjunctive neuron in Fig. 6b.** The
970 neuron is tuned to tilt with a preferred orientation at 42° ND, and to azimuth with a PD of -27°.
971 Consequently, the cell fires maximally at 3D orientations centered on these coordinates. Same
972 format as in **Suppl. Movie 2**.

973 **Supplemental Movie 9: 3D tuning curve of the example conjunctive neuron in Fig. 6c; Suppl.**
974 **Fig. 13.** The neuron is tuned to tilt with a preferred orientation at 105° ND, and to azimuth with
975 a PD of 5°. Although azimuth tuning is clear in the lowest portion of the tuning curve, it
976 decreases with tilt angle and is therefore minimal at 105°, where the cell appears to be only
977 tuned to tilt. As a consequence, the cell seems to alternate from azimuth tuning when the head
978 is close to upright to tilt tuning at large tilt angles. Same format as in **Suppl. Movie 2**.

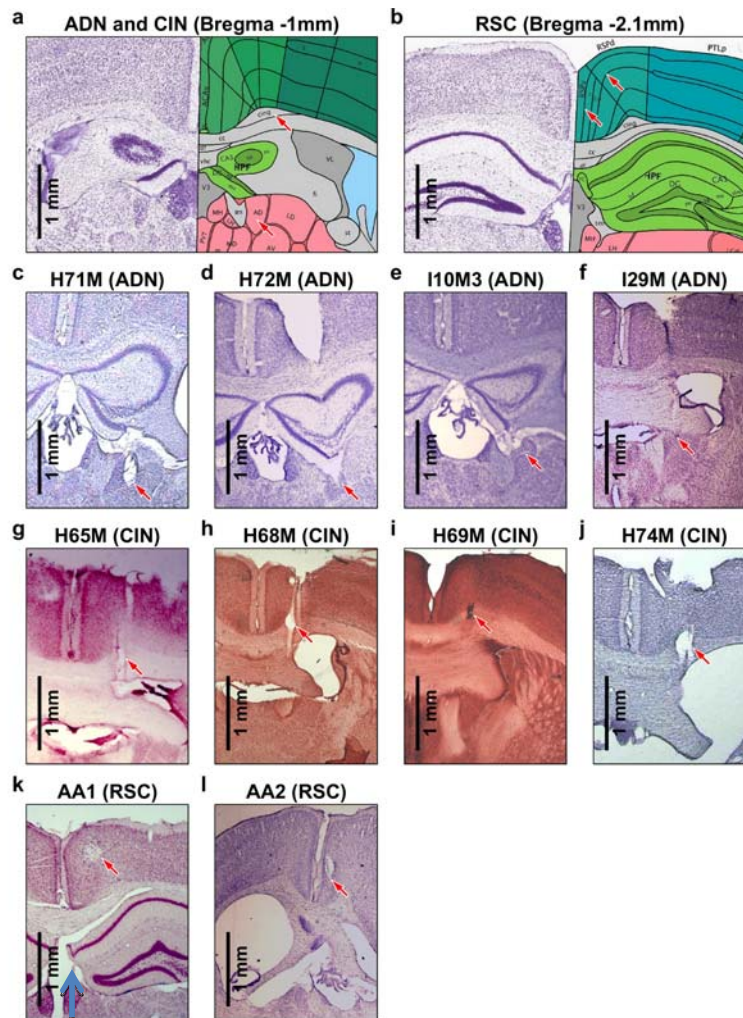
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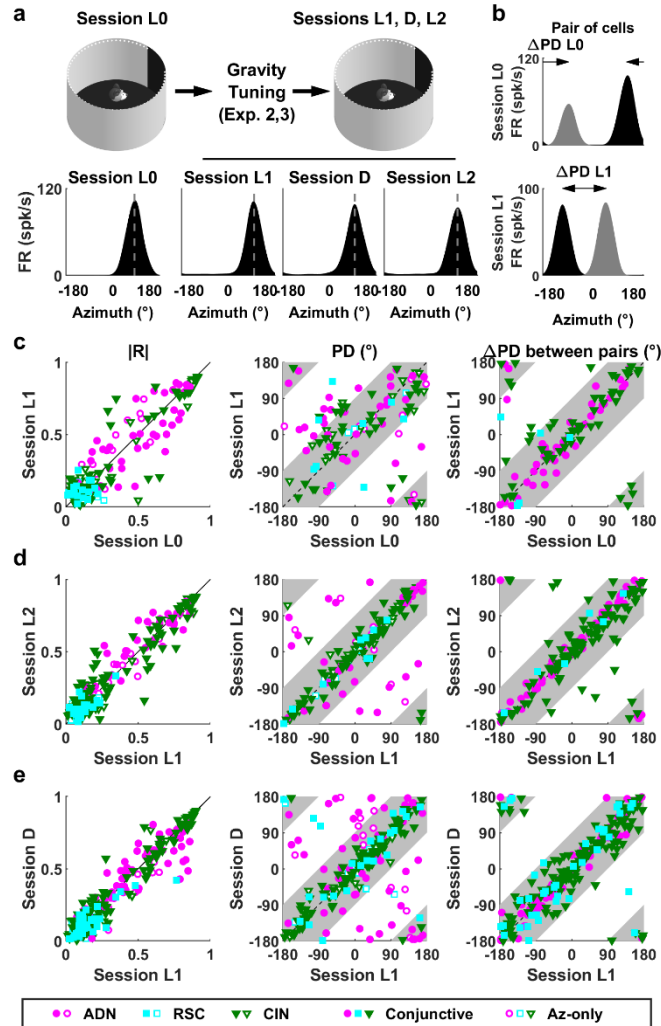
981 **Supplemental Figure 1: Recording stability across sessions.** (a-d) Two neurons (color-coded in
982 blue and green; animal H68M, CIN) were recorded simultaneously during Experiment 1-LO (a),
983 Experiment 2 (b), Experiment 3-L (c) and Experiment 1-L1 (d). Left panels: spikes were extracted
984 by manual clustering based on the maximum (peak) and minimum (valley) voltage of the spikes,
985 combined with factor analysis (the first two components, PCA 1 and PCA 2, are shown here).
986 When represented on a 2D plot, the spikes form two clearly distinct clusters (blue and green).
987 Grey dots represent unsorted events that result from noise and background activity. Middle
988 panels: average spike waveforms across all channels. Right panels: the Inter-spike interval (ISI)
989 histograms are conserved across Experiment 1-LO and 1-L1 (a, d) but shift rightward during
990 Experiment 2 (for neuron 2) and Experiment 3-L for both neurons. This shift reflects the
991 reduction of average firing rate during Experiments 2 and 3 due to the general attenuation of
992 neuronal firing in the rotator (see **Suppl. Fig. 8a,b**). (e-f) Azimuth tuning curves of the neurons,
993 recorded in the initial (black) and second (red) freely moving session, i.e. before and after
994 Experiments 2 and 3-L, are similar. This comparison serves as confirmation of recoding stability
995 across sessions (see also **Suppl. Fig. 3**).

996



997

998 **Supplemental Figure 2: Histological localization of neuronal recordings.** (a,b) Annotated
999 histology slides from the Allen Mouse Brain Atlas (Lein et al., 2007; [http://mouse.brain-](http://mouse.brain-map.org/static/atlas)
1000 [map.org/static/atlas](http://mouse.brain-map.org/static/atlas)). The location of the sections relative to Bregma are indicated in the title.
1001 Red arrows indicate the ADN (labelled 'AD'), CIN (labelled 'cing') in (a), the I-IIth layer of the
1002 granular RSC (labelled RSCv; left arrow) and the Vth layer of the dysgranular RSC (RSCd; right
1003 arrow) in (b). Note that the ADN extends from Bregma -0.4mm to Bregma -1.1mm. Nissl-stained
1004 sections of all animals included in this study are shown; tetrode tracks are indicated by arrows.
1005 (c-f) Recordings in the ADN. In all animals, the ADN appears as a characteristic triangular-
1006 shaped and densely stained nucleus. In H71M, H72M and I10M3, the ADN appears below the
1007 hippocampus, i.e. more caudal than usually indicated in brain atlases (Praxinos and Franklin
1008 2004, Lein et al. 2007). However, we confirmed that the nucleus marked by an arrow is indeed
1009 the ADN in each mouse by examining all microscopic sections and locating the anterior
1010 extremity of the thalamus as well as the anterior and posterior extent of the ADN. (g-j)
1011 Recordings in the cingulum fiber bundle. (k,l) Recordings in the RSC (AA1: dysgranular RSC,
1012 likely layer V; AA2: granular RSC; layer indeterminate).

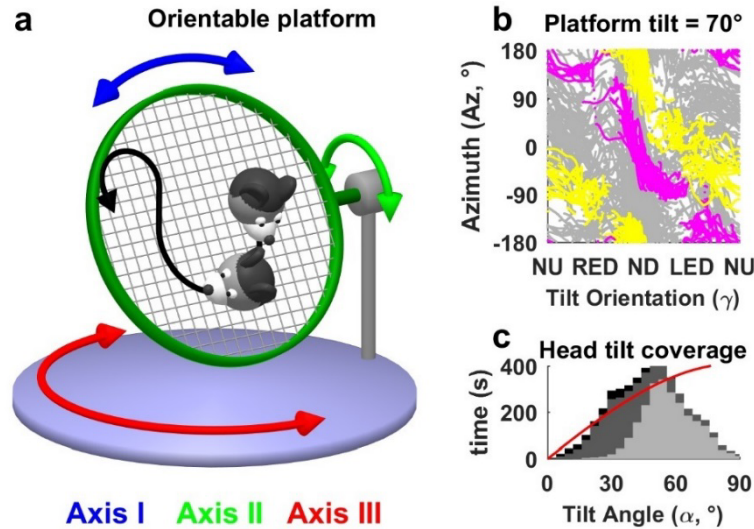


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1014 **Supplemental Figure 3: Response properties of Azimuth-tuned cells during unrestrained**
 1015 **motion in a traditional arena (Experiment 1).** As summarized in this analysis, azimuth-tuned
 1016 cells conform to well-established properties (see e.g. Taube 2007, Peyrache et al. 2015). (a)
 1017 illustration of the sequence of recordings and example cell. At the beginning of an experimental
 1018 day, azimuth tuning is recorded in light as the mouse forages freely in a circular arena (session
 1019 L0). The mouse is then transferred to the platform and rotator to characterize its 3D tuning
 1020 (Exp. 2-3; see **Suppl. Table 2** and **Methods**), upon completion it is returned to the freely moving
 1021 arena and azimuth tuning is measured in light again (session L1), then in darkness (D), then
 1022 again in light (L2). An example azimuth-tuned cell with stable preferred direction (PD) in all
 1023 sessions in the arena (L0, L1, D, L2). (b) Two simultaneously recorded cells (grey and black tuning
 1024 curves) that changed PDs between sessions L0 and L1 (see also other mouse studies: e.g., Yoder
 1025 et al. 2009). Importantly, both cells shift together, such that the difference angle between their
 1026 PD (ΔPD) remains constant. Thus, comparing ΔPD across cell pairs allows testing whether
 1027 azimuth-tuned cells form a coherent neuronal compass even when this compass drifts from one
 1028 session to another. (c) Azimuth response stability between sessions L0 and L1. Left: There is no
 1029 significant difference in tuning strength (Mean vector length $|R|$): signed rank tests, $p > 0.5$ for all

1030 groups; Bonferroni correction applied; data from all cells significantly tuned to azimuth in at
1031 least one session; n = 54 ADN; 33 RSC; 83 CIN). Middle: Comparison between the PD of
1032 individual cells. Only HD cells significantly tuned ($p < 0.01$) in both sessions are included. PDs of a
1033 small subpopulation may drift between session L0 and L1 (PD shift $> 90^\circ$ in 14/46 ADN; 3/12
1034 RSC; 7/56 CIN cells, i.e. 21% cells total). Grey bands represent sectors where the PDs shift by
1035 less than 90° . Right: Δ PD between pairs of simultaneously recorded cells. Only cells significantly
1036 tuned ($p < 0.01$) in both sessions are included. PD differences are stable ($< 90^\circ$ shift) in 93%
1037 (45/46 ADN; 4/5 RSC; 50/55 CIN) of cells pairs. Thus, although PD may shift between L0 and L1,
1038 the PD of all cells tend to shift together, in line with predictions of an attractor network
1039 (Peyrache et al. 2015). **(d)** Azimuth response stability between sessions L1 and L2 (same legend
1040 as in c). There is no significant difference in tuning strength ($p = 0.03$ for ADN, $p > 0.5$ for other
1041 groups; n=55 ADN; 33 RSC; 135 CIN). Only 10% of cells (15/48 ADN; 0/15 RSC; 1/92 CIN) drift
1042 more than 90° . PD differences are stable ($< 90^\circ$ shift) in 97% (61/61 ADN; 6/6 RSC; 102/108 CIN)
1043 of cell pairs. Thus, PD are more stable between L1 and L2 compared to L0 and L1, likely because
1044 of the shorter time interval between L1 and L2 and/or the use of 3D stimuli in-between sessions
1045 L0 and L1. **(e)** Azimuth response stability between sessions L1 and D (same legend as in c,d).
1046 Tuning strength is slightly attenuated in darkness in RSC (linear regression slope=0.71, $p < 10^{-5}$;
1047 n=70) but not in other areas (ADN: n=70, $p = 0.14$; CIN: n=143; $p = 0.3$). Only 14% (25/63 ADN;
1048 4/37 RSC; 0/113 CIN) of PDs drift more than 90° . PD differences are stable ($< 90^\circ$ shift) in 99%
1049 (84/84 ADN; 38/39 RSC; 146/148 CIN) of cell pairs.

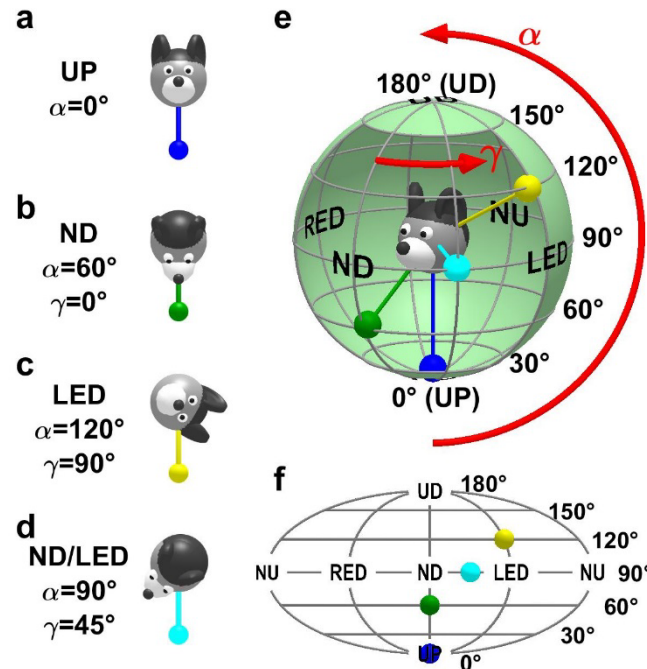
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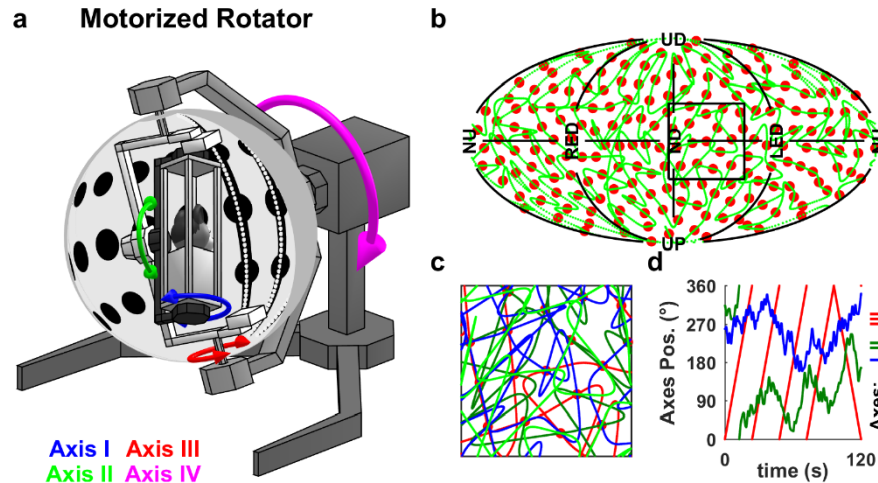
1052 **Supplemental Figure 4: Freely moving setup and protocol for measuring 3D tuning in**
1053 **unrestrained, exploring mice. (a)** Illustration of the 3D orientable platform setup: mice walk
1054 freely (black arrow) on a meshed platform that can rotate around 3 axes (blue, green, red).
1055 Walking on the platform changes tilt orientation (γ ; see **Suppl. Fig. 5**) and azimuth
1056 simultaneously. Rotating the base (Axis III) changes azimuth but not tilt orientation. Axis II is
1057 used to change tilt angle (α ; see **Suppl. Fig. 5**). Recordings are performed in 5-8-minute blocks
1058 where mice walk freely while axis II-III are set to a static position. Axis I is rotated randomly in
1059 the middle of each block to prevent tilt orientation relative to gravity from being coupled to
1060 head orientation relative to local cues on the platform itself. Because of this added complexity,
1061 earth-horizontal azimuth tuning in Fig. 2a was evaluated based on data recorded in the arena.
1062 **(b)** Distribution of azimuth (Az) and tilt orientation (angle γ) in eight 5-min blocks where the
1063 platform was tilted 70° (average head tilt = $60 \pm 15^\circ$, as mice tend to partially compensate with
1064 their head). Yellow, magenta: data recorded during two 5-min blocks corresponding to different
1065 configurations (same tilt angle, but different positions of Axis III). Azimuth and tilt orientation
1066 vary together as animals walk within one block, but azimuth is offset when Axis III is rotated
1067 between blocks, thus allowing to scan the entire tilt orientation/azimuth plane. **(c)** Distribution
1068 of head tilt angle (α) in the same recording session (black/grey/light grey: data collected with
1069 the mesh tilted 0°, 45°, 70°, respectively). Red curve: distribution required for uniform sampling
1070 of tilt orientation, illustrating relatively uniform sampling up to $\sim 60^\circ$.

1071



1072
 1073 **Supplemental Figure 5: Spherical coordinate system for tilt orientation and Mollweide**
 1074 **projection.** The rationale for the coordinate system is the following: each tilt orientation
 1075 relative to gravity is equivalent to, and defined as, the corresponding orientation of the gravity
 1076 vector relative to the head. **(a-d)** Four example tilt orientations, expressed in a spherical
 1077 coordinate system (α, γ) where α is the tilt angle and γ is tilt orientation: $\gamma = 0^\circ$ and $\gamma = 180^\circ$
 1078 correspond to nose-down (ND) and nose-up (NU) tilt; $\gamma = 90^\circ$ and $\gamma = -90^\circ$ correspond to left-
 1079 ear-down (LED) and right-ear-down (RED) tilt. The colored pendulum/ball represents the gravity
 1080 vector. **(e)** Spherical topology of tilt orientation. When head tilt spans all possible orientations,
 1081 the tip of the gravity vector spans a sphere surrounding the head. The tilt variable α
 1082 corresponds to the latitude on the sphere. Upright (UP, $\alpha=0^\circ$) and upside-down (UD, $\alpha=180^\circ$)
 1083 orientations correspond to the lower and upper pole respectively. The orientation variable γ
 1084 corresponds to the longitude. 90° tilt in ND, LED, NU and RED orientations are marked. **(f)**
 1085 Planar representation of the sphere using an equal-area Mollweide projection. The 4 tilt
 1086 orientations in (a-d) are marked with color balls.

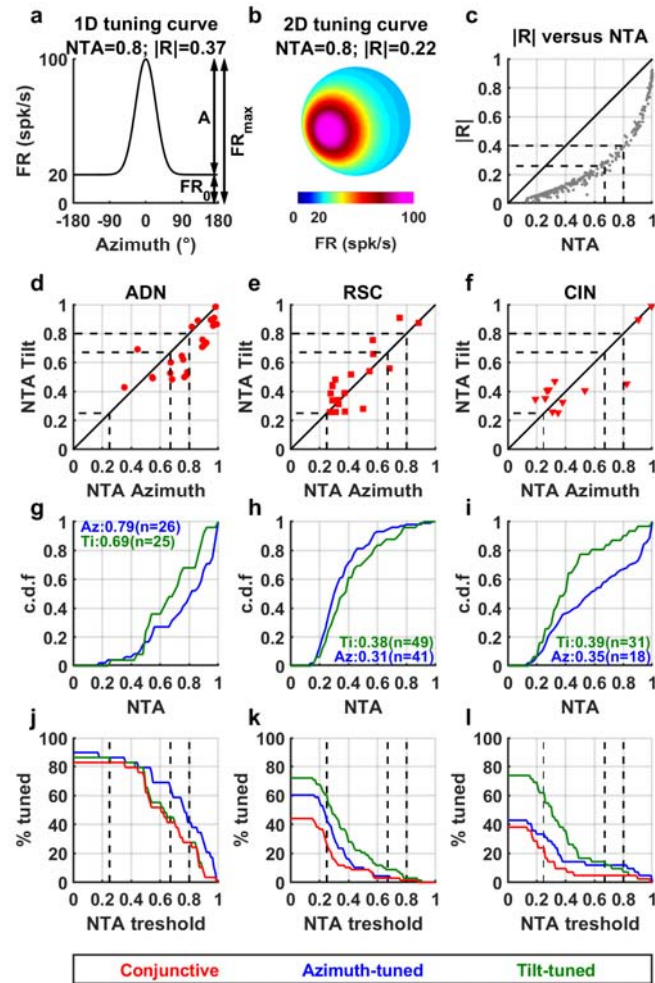
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1089 **Supplemental Figure 6: Motorized rotator setup and protocol for measuring 3D tuning in**
1090 **restrained mice. (a)** Illustration of the motorized rotator; the 4-rotation axes are indicated by
1091 colored arrows. **(b)** Pseudo-random trajectory (green curve) used to measure tilt tuning. The
1092 trajectory visits 200 uniformly distributed tilt positions (red dots). The full protocol scans the
1093 entire tilt space 8 times by running through 4 distinct trajectories, each of which is ran twice in
1094 opposite directions. **(c)** Detail of the highlighted square in (b), with 4 distinct trajectories. **(d)**
1095 Position of the rotator's 3 inner axes during a 2 min segment of the motion. Axes I (inner yaw,
1096 blue) and II (middle, pitch/roll, green) are used to manipulate 2D head tilt relative to vertical,
1097 while axis III (outer yaw, red) is used to continuously vary azimuth. Axis IV is used to tilt the
1098 setup in Experiment 3-T (**Suppl. Table 1**).

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1101 **Supplemental Figure 7: Normalized tuning amplitude vs. mean vector length for quantifying**
 1102 **tuning strength for fair comparison between tilt- and azimuth-tuned cells.**

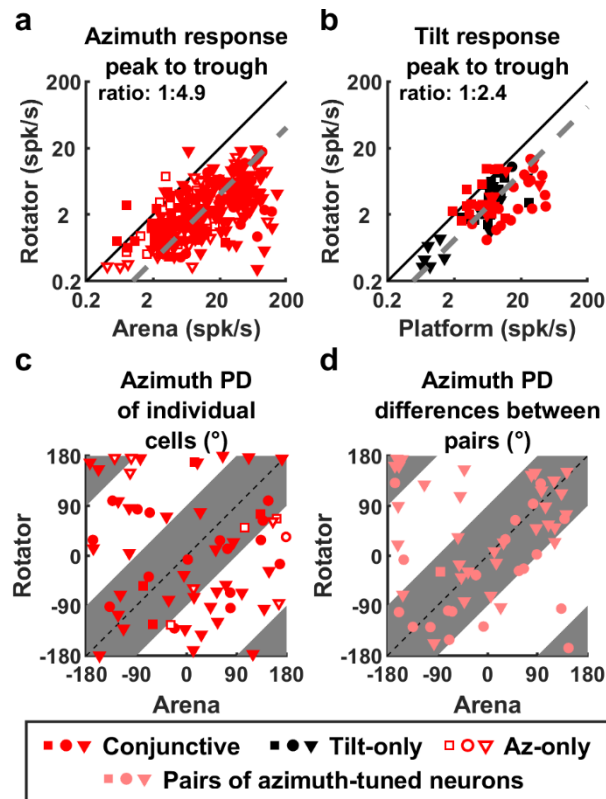
1103 (a) We define a measure of tuning strength called *normalized tuning amplitude (NTA)*,
 1104 illustrated here for a 1D tuning curve. We decompose firing rate into a baseline firing FR_0 and a
 1105 modulation amplitude A . The peak firing rate is $FR_{max}=FR_0+A$. Normalized tuning amplitude is
 1106 defined as $NTA=A/FR_{max}$. The normalized tuning amplitude of this example is $NTA = 0.8$; and the
 1107 mean vector length is $|R|=0.38$. (b) Illustration of a 2D tuning curve on a sphere, used to model
 1108 tilt tuning. This distribution has identical baseline, amplitude and standard deviation as in (a)
 1109 and therefore, the normalized tuning amplitude is also $NTA = 0.8$. However, the mean vector
 1110 length is $|R|=0.22$; i.e. lower than in (a). This is because the area of baseline firing on a sphere
 1111 (cyan in b) amounts to a larger portion of the tuning curve than in a 1D tuning curve, resulting
 1112 in a lower $|R|$ value. Therefore, the mean vector length is inappropriate for comparing azimuth
 1113 (1D) and tilt (2D) tuning. (c) Comparison of the mean vector length ($|R|$) and normalized tuning
 1114 amplitude for azimuth tuning in Experiment 1-L (Suppl. Table 2; data from all significantly-
 1115 tuned cells). Although the scaling between the two measures is non-linear, there is a clear
 1116 relationship between them, indicating that they provide the same information. *Therefore, the*

1117 *normalized tuning amplitude is as suitable as the mean vector for quantifying tuning strength,*
1118 *with the advantage that it allows for a fair comparison between 1D and 2D tuning curves.*

1119 **(d-f)** Comparison of the normalized tuning amplitude of azimuth and tilt tuning for all
1120 conjunctive cells (paired data from Experiment 1-L and Experiment 2; **Suppl. Table 2**; only cells
1121 tested in both experimental protocols and significantly tuned to both Az and tilt have been
1122 included; ADN: n=24; RSC: n=19; CIN: n=12). Azimuth tuning is slightly higher than tilt tuning in
1123 ADN (median $A_N = 0.79$ versus 0.7, $p = 0.002$, signed rank test) but not in RSC (0.37 versus 0.44,
1124 $p = 0.2$) nor in CIN (0.34 versus 0.41, $p = 0.8$). Previous studies have classified neurons as HD
1125 cells when $|R| > 0.26$ (Jacob et al. 2017) or $|R| > 0.4$ (Yoder et al. 2009). We infer from the
1126 population response in (c) that these values correspond to $NTA \approx 0.67$ and $NTA \approx 0.8$, respectively.
1127 In the present study, we classified cells as azimuth or tilt-tuned using statistical criteria: if tuning
1128 passed a shuffling test (at $p < 0.01$), as long as $NTA > 0.25$ (to exclude low-modulating cells). The
1129 three thresholds ($NTA = 0.25, 0.67, 0.8$) are indicated by vertical dashed lines. Although the
1130 statistical criterion we used was more inclusive compared to the fixed threshold of previous
1131 studies, the 3D tuning properties described here are found in those cells that pass the more
1132 restrictive criteria of previous studies, as detailed in subsequent panels.

1133 **(g-i)** Cumulative distribution of NTA for all azimuth-tuned (blue) and tilt-tuned (green) cells that
1134 passed the shuffling test (not only conjunctive cells as in panels d-f). Median values of NTA and
1135 number of cells are indicated in the panels. The median values of tilt and azimuth tuning are
1136 comparable in all regions.

1137 **(j-l)** Percentage of cells that would be classified as azimuth-tuned (blue), tilt-tuned (green) and
1138 conjunctive (red) by passing the shuffling test ($p < 0.01$) and exceeding a variable NTA threshold,
1139 expressed as a function of that threshold, but including all recorded cells in both the arena and
1140 platform setups. Note that the threshold value of $NTA = 0.25$ used in the present study allows
1141 for a large fraction of cells to be classified as azimuth-tuned, tilt-tuned or conjunctive (e.g. 86%
1142 azimuth-tuned and 86% tilt-tuned in ADN). Using a more stringent threshold (e.g. $NTA = 0.8$) to
1143 select cells with very vigorous responses, we find that 41% ADN cells are classified as azimuth-
1144 tuned, 28% as tilt-tuned, and 28% as conjunctive. Thus, even when a stringent criterion is used,
1145 over a fourth of ADN cells exhibit large 3D responses. A sizeable, but lower, fraction of CIN cells
1146 (12% azimuth-tuned, 7% tilt-tuned, 5% conjunctive) exhibit similar strong responses. In
1147 contrast, only a minority (<3%) of RSC cells pass this threshold, indicating that significant HD
1148 responses exist in RSC but are generally weaker than in ADN, which is a known property already
1149 (Chen et al. 1994a). Importantly, the proportion of tilt-tuned cells is typically higher than Az-
1150 tuned cells in all areas, regardless of NTA threshold (green vs. green curves).



1151

1152 **Supplemental Figure 8: Comparison of azimuth and tilt responses when moving freely and**
1153 **restrained.**

1154 **(a)** Comparison of azimuth response amplitude when moving freely in the arena (Experiment 1-
1155 L0,1,2; **Suppl. Table 2**) and when restrained in the rotator (Experiment 3-L, considering data for
1156 tilt < 45°). Data from cells significantly modulated to azimuth in Experiment 1 and recorded
1157 during Experiment 3-L (n=267). Responses were largely attenuated when mice were restrained
1158 (median amplitude ratio= 1:4.9 [1:4.4 - 1:6.3] CI; grey). **(b)** Comparison of tilt response
1159 amplitude when moving freely on the platform (Experiment 2; **Suppl. Table 2**) and when
1160 restrained in the rotator (Experiment 3-L; re-analyzed based on data when head tilt < 60° to
1161 match the range of tilt sampled in Experiment 2). Data from cells significantly modulated to tilt
1162 when moving freely and recorded during Experiment 3-L (n = 70). Responses were attenuated
1163 when mice were restrained (median amplitude ratio= 1:2.4 [1:1.9 1:2.8] CI; grey). Together,
1164 panels (a, b) indicate that restraining mice leads to an attenuation of both azimuth and tilt
1165 response amplitude. Azimuth tuning is attenuated to a larger extent than tilt tuning, which is
1166 the reason tilt tuning curves can be reliably measured in the rotator in most neurons, but
1167 azimuth tuning curves only in a minority of neurons (**Fig. 2c**).

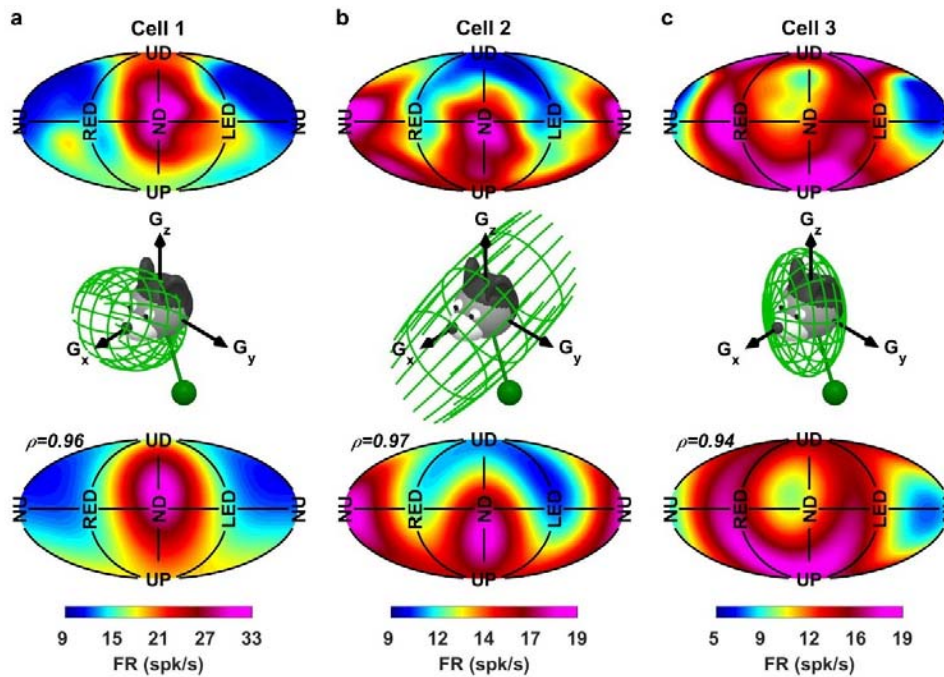
1168 **(c,d)** Inconsistency in azimuth PDs, but consistency of difference in azimuth PD between pairs of
1169 cells across setups. The arena and the rotator are different setups located in separate rooms
1170 and don't share a common azimuth reference. Therefore, the azimuth compass may anchor to
1171 a priori random orientations in each setup, which would cause the PD of individual cells to vary
1172 randomly. Accordingly, PDs shift by more than 90° in 29/63 cells (panel c, p=0.6, Binomial test).

1173 However, if azimuth-tuned cells are part of a neural compass, then the PD of simultaneously
1174 recorded cells should remain anchored one relative to the other. Therefore, the difference in
1175 PD between pair of cells should be identical in the arena and in the rotator. Indeed, PD
1176 differences in the arena vs. rotator were significantly conserved in panel d (within 90° in 44/54
1177 pairs of cells, $p < 10^{-5}$, Binomial test). Note, however, that unlike the azimuth compass that
1178 anchors to visual landmarks, a tilt compass anchored to gravity (**Fig. 1a**), should have identical
1179 PDs on the platform and in the rotator. This is indeed the case, as shown in the pixel-by-pixel
1180 correlation of the fitted tilt tuning curves up to 60° tilt that could be tested in both setups (**Fig.**
1181 **2d**; We used the correlation analysis for tilt tuning, because the PD of most cells can't be
1182 measured on the platform, which is restricted to 60° tilt. Also note that together, panels a-d
1183 and **Fig. 2** indicate that the spatial tuning characteristics of both azimuth and tilt tuning are
1184 conserved across free locomotion and restrained, passive motion, and that, other than the
1185 smaller response magnitude, the 3D responses measured in the rotator are representative of
1186 the neurons' natural responses.

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1191 **Supplemental Figure 9: Example tilt tuning curves fitted with a Gaussian function. (a)**
1192 Example cell with a single hill of activity (representative of the majority of cells). Top panel:
1193 planar representation of the tilt tuning curve (PD close to ND). Middle panel: illustration of the
1194 Gaussian fitting. Bottom panel: planar representation of the fitted firing rate. The correlation
1195 between the measured and fitted firing rate (top and bottom panel) was $\rho=0.96$.

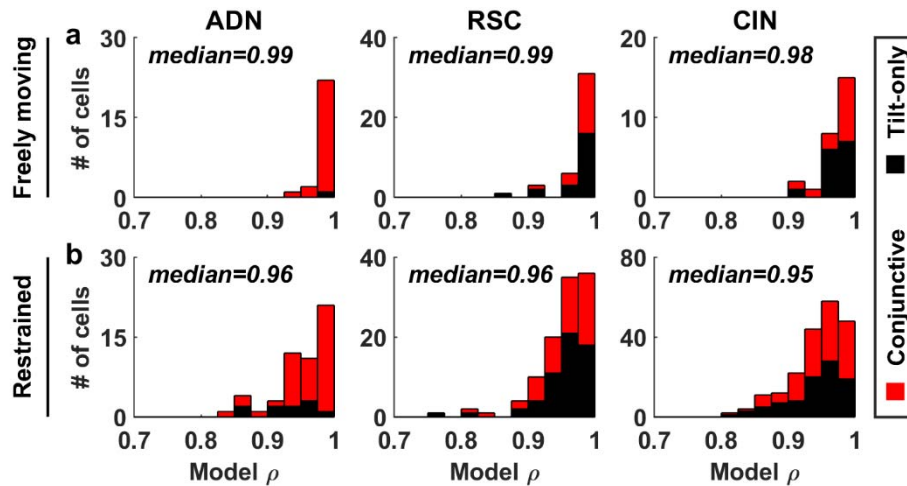
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1197 **(b)** Example cell with a bimodal response. The cell responds preferentially in NU and ND
1198 orientations. Middle panel: When gravity is expressed in Cartesian coordinates, NU and ND
1199 orientations correspond to the gravity vector being aligned with the naso-occipital axis (G_x), and
1200 therefore $G_y \approx 0$ and $G_z \approx 0$. A 3D Gaussian with a large standard deviation along G_x but small
1201 standard deviation along G_y and G_z will have a large value at all positions close to the G_x axis
1202 and therefore resemble the cell's tuning curve. Accordingly, the cell's firing was fitted with an
1203 elongated 3D Gaussian whose long axis was aligned with G_x (shown as a green wireframe
1204 ellipse that corresponds to one std. dev.). The correlation between the measured and fitted
1205 firing rate (top and bottom panel) was $\rho=0.97$.

1206

1207 **(c)** Example cell with a tuning curve exhibiting a ring-shaped hill of activity (top panel),
1208 responding preferentially in UP, UP, RED and LED but not in ND or NU. Middle panel: In
1209 Cartesian coordinates, the cell's response corresponds to gravity being aligned with the G_y , G_z
1210 plane. Accordingly, the cell's response was fitted with a flat, pancake-shaped 3D Gaussian that
1211 has a large value along the G_y , G_z , but small values along the G_x axis ($\rho=0.94$).

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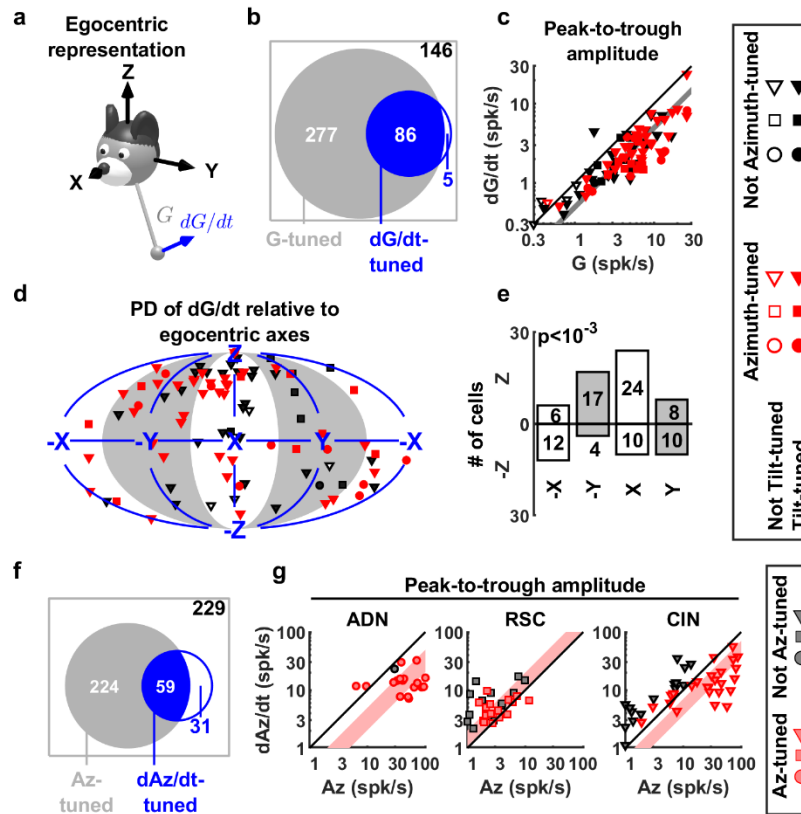


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1215 **Supplemental Figure 10: Summary of goodness of fit (coefficient of correlation, ρ) of Gaussian**
1216 **functions to 3D tuning curves, separated by brain area. Red/black: conjunctive and tilt-only**
1217 **cells. (a) Gaussian fit to the tilt tuning curves measured when moving freely. The correlation**
1218 **doesn't vary between regions or between conjunctive and tilt-only cells (2-way ANOVA: $p=0.33$**
1219 **and $p=0.45$ respectively). (b) Gaussian fits to the tilt tuning curves measured when mice are**
1220 **restrained in the rotator. The correlation doesn't vary between regions or between conjunctive**
1221 **and tilt-only cells (2-way ANOVA: $p=0.13$ and $p=0.26$ respectively). The model's correlation is**
1222 **higher (median: 0.99 versus 0.96, $p<10^{-10}$, Wilcoxon rank sum test), likely because of the small**
1223 **response amplitude when restrained.**

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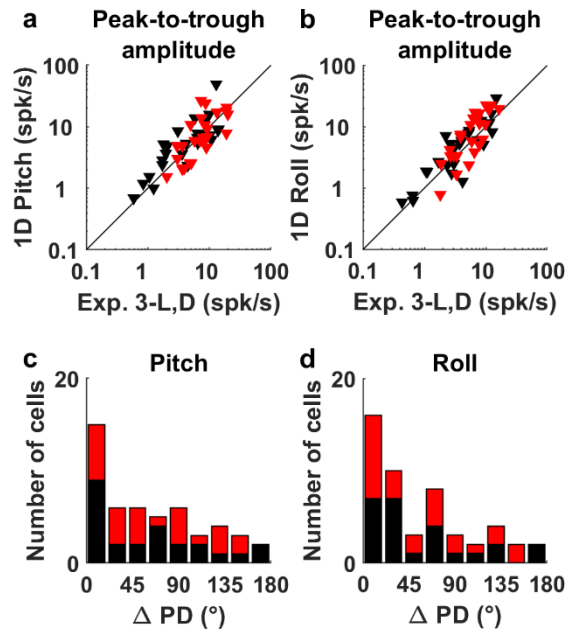
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1228 **Supplemental Figure 11: Responses to tilt and azimuth velocity.** We used identical criteria to
 1229 assess whether cells were significantly tuned to azimuth velocity (dAz/dt) and tilt velocity (i.e.
 1230 the time derivative of gravity, dG/dt) (see Methods). **(a)** Gravity derivative was expressed in an
 1231 egocentric (X,Y,Z) frame, similar to gravity, and the responses to gravity derivative was fitted
 1232 with Gaussian functions (as in **Suppl. Fig. 10**). **(b)** A small percentage of cells (91/514, 18%)
 1233 exhibited significant tuning to tilt velocity (data from Experiment 3-L; **Suppl. Table 2**).
 1234 Furthermore, the majority (86/91; 95%) of these were also tuned to tilt. Tilt-tuned cells were
 1235 more likely to be tuned to dG/dt ($p < 10^{-7}$, Chi square test). **(c)** For most cells, tilt velocity
 1236 responses had a lower amplitude than tilt position responses (geometrical average ratio=0.5,
 1237 [0.44 0.55] CI; data from $n=86$ cells with significant tilt and tilt velocity tuning). There was no
 1238 significant difference between areas (Kruskal-Wallis ANOVA, $p=0.05$).

1239 **(d)** Distribution of PDs for tilt velocity. 'X', 'Y', 'Z' indicate that cells fire preferentially when
 1240 $dG_X/dt > 0$, $dG_Y/dt > 0$, and $dG_Z/dt > 0$, respectively. '-X', '-Y', '-Z' indicate that cells fire
 1241 preferentially when $dG_X/dt < 0$, $dG_Y/dt < 0$, and $dG_Z/dt < 0$, respectively. **(e)** Number of cells in all 8
 1242 quadrants of panel d. Most cells prefer $dG_Z/dt > 0$, and $dG_X/dt > 0$, i.e. when the gravity vector
 1243 moves forward and upward in head coordinates, which corresponds from instance to ND pitch
 1244 movements when starting from an upright condition.

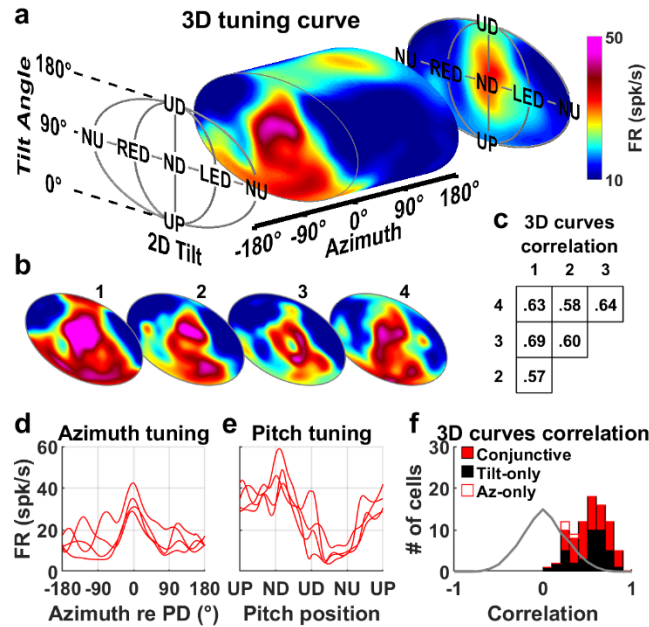
1245 **(f)** A small percentage of cells (90/543, 17%) was also tuned to azimuth velocity (dAz). Azimuth-
 1246 tuned cells were more likely to be tuned to dAz (66% vs. 49%; $p=0.005$, Chi square test). **(g)** For
 1247 cells tuned to both Az and dAz , Az velocity responses had a lower amplitude than Az position

1248 (direction) responses in ADN (median ratio: 1:3.1, [2-4.9] CI; $p < 10^{-3}$, signed rank test), but in
1249 contrast were slightly larger RSC (median ratio 1.5:1, [1.1-2] CI; $p < 10^{-3}$). The ratio in CIN was
1250 intermediate (2:1 in favor of Az responses, [1.4-2.8] CI; $p < 10^{-5}$). The peak-to-trough amplitude
1251 of dAz tuning curves, measured across the range of $\pm 200^\circ/\text{s}$, had a median value of 13 spk/s
1252 ([11-15] CI) in ADN, 5.5 spk/s; [3.7-6.6] CI) in RSC. The distribution of responses in CIN
1253 resembled a mixture of ADN cells (with high Az and lower dAz responses) and RSC cells (with
1254 low Az and dAz responses).



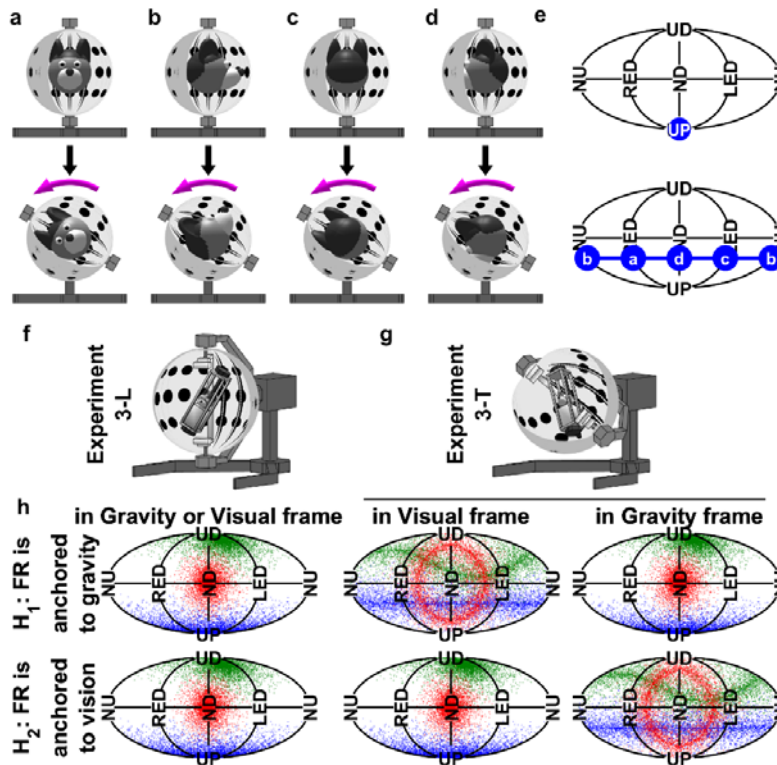
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1257 **Supplemental Figure 12: Comparison between modulation amplitude and preferred direction**
1258 **of tilt-tuned cells during pitch/roll rotation (Experiment 4; Suppl. Table 2) and 3D rotation**
1259 **(Experiment 3-L,D) (a), (b) Peak-to-trough modulation amplitude measured during pitch/roll**
1260 **(ordinate) vs. predicted based on tilt tuning curves measured in Experiment 3-L,D (abscissa).**
1261 **Amplitudes are significantly correlated ($p < 10^{-10}$ for both pitch and roll; $n=50$ tilt-tuned cells,**
1262 **data averaged across recordings in light darkness). The responses are slightly higher during**
1263 **single axis rotation in roll (median=5.9 vs 3.9 spk/s; $p = 10^{-3}$, signed rank test) but not pitch**
1264 **(median=6.4 vs 4.4 spk/s; $p = 0.3$, signed rank test). (c), (d) Distributions of absolute difference**
1265 **in tilt preferred direction (PD) between 1D and 3D stimuli for pitch and roll planes, respectively.**
1266 **Both are significantly aligned with 0 (Kolmogorov-Smirnov tests to test the difference with a**
1267 **uniform; pitch: $p < 10^{-4}$; roll: $p = 2.10^{-3}$). Red symbols/bars: Azimuth-tuned cells; Black**
1268 **symbols/bars: Not-azimuth-tuned cells.**
1269



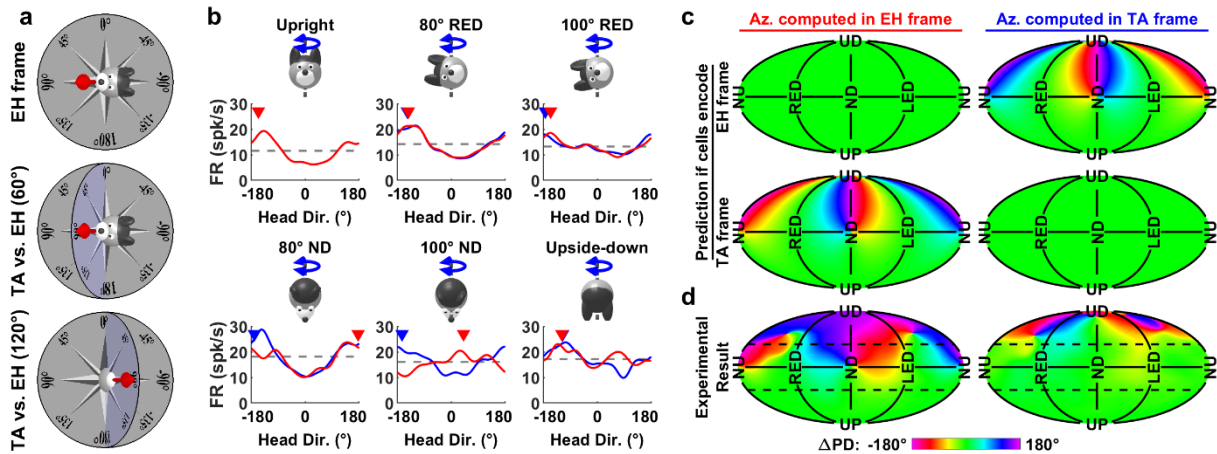
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1271 **Supplemental Figure 13: Reproducibility of 3D tuning across days.** (a) 3D tuning curve of an
 1272 example conjunctive CIN cell (same as in **Suppl. Movie 8**). A vertical section of the 3D tuning
 1273 curve is shown at an azimuth of 0° that corresponds to the cell's PD. Data averaged across all
 1274 repetitions of Experiment 3-L. (b) Reproducibility of the tuning curve. The cell's response was
 1275 recorded 4 times during Experiment 3-L, on 4 distinct days spanning a 2-week period. 3D tuning
 1276 curves were recomputed for each repetition. The same vertical section as in (a) is shown for all
 1277 repetitions (labelled 1 to 4), using the same color scale. Peak firing occurs consistently in the
 1278 vicinity of ND orientation. (c) Pixel-by-pixel correlation, across the entire 3D space, between all
 1279 4 tuning curves. Each curve is correlated with all others ($p < 10^{-10}$). The average correlation is
 1280 0.62. (d) Azimuth tuning curve in upright orientation extracted from the 4 3D tuning curves, and
 1281 centered on each curve's PD. (e) Pitch tuning curve extracted from the 3D tuning curves, at the
 1282 azimuth corresponding to each curve's PD. Analyses in panels b to e indicate that 3D responses
 1283 (or responses along 1D yaw and pitch trajectories) were stable across several days in the
 1284 example cell. (f) Distribution of the average correlation between repetitions of Experiment 3-L
 1285 (as in panel c), for all tuned cells ($n = 88$ cells; 45 conjunctive, 39 tilt only, 3 azimuth-only). The
 1286 median correlation is 0.52 ([0.49-0.58] CI) and is similar for conjunctive and tilt-only cells
 1287 (Wilcoxon rank sum test, $p = 0.77$). Grey: expected distribution if tuning curves shift randomly
 1288 between days (H_0), computed by randomly shuffling the cells (Kolmogorov-Smirnov test of
 1289 observed distribution versus H_0 : $p < 10^{-10}$). This analysis demonstrates that 3D responses are
 1290 stable across days.



1291

1292 **Supplemental Figure 14: Protocol 3-T. (a-e)** Change of head tilt resulting from rotating Axis IV.
 1293 Panels a-d illustrate 4 situations where the head is initially upright and facing various direction
 1294 in the horizontal plane. When Axis IV is tilted 60° (magenta arrow), the head reaches 60° in
 1295 RED, NU, LED or ND, depending of its initial orientation in the horizontal plane, i.e. relative to
 1296 Axis IV. (e) Representation of the initial (UP) and final orientations in the 2D tilt plane (the
 1297 points labelled a, b, c, d correspond to the respective panels). In general, starting from UP, the
 1298 head may reach $\alpha=60^\circ$ with any orientation γ (defined in **Suppl. Fig. 1**). These possible
 1299 orientations are represented by a blue line and correspond to any tilt position that is exactly
 1300 60° from UP. This rule can be generalized as follows: *following a rotation of Axis IV by x° , the*
 1301 *head may reach any tilt position that is exactly x° from its initial tilt, depending of its initial*
 1302 *orientation in the horizontal plane.* (f-g) Illustration of Experiments 3-L,T: the 3D rotation
 1303 protocol (f) is repeated after tilting the entire enclosure (g). (h) Responses of 3 hypothetical
 1304 cells are shown in red, green and blue. *Top: Hypothesis H₁. We expect that neuronal tuning will*
 1305 *appear re-organized (each hill of activity being transformed in a circular tuning curve based on*
 1306 *the rule illustrated in panels a-e) in a Visual reference frame (compare middle vs. left column).*
 1307 *In contrast, tuning will remain relatively unchanged when expressed in a Gravity reference*
 1308 *frame (right vs. left column). Bottom: Hypothesis H₂ with reverse properties.*



1309

1310 **Supplemental Figure 15: Azimuth tuning is spatially invariant when expressed in a Tilted (TA)**
 1311 **frame.**

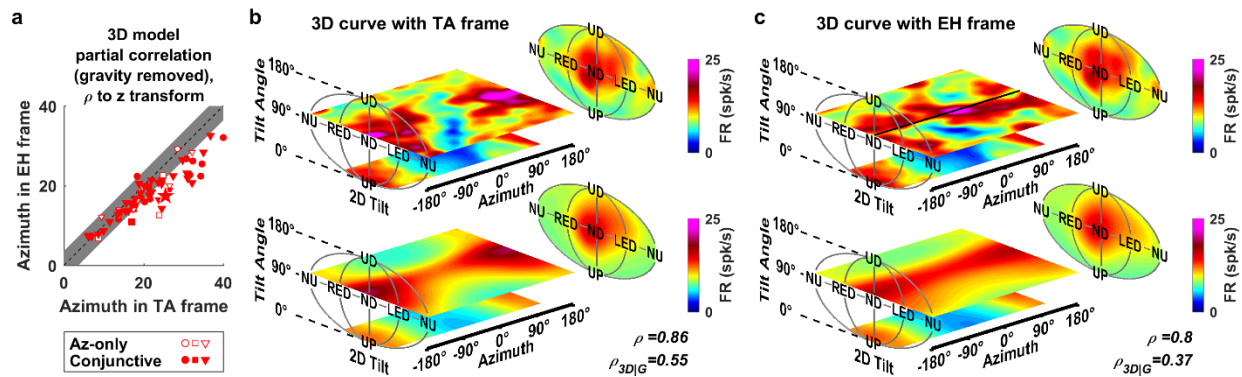
1312 **(a)** Illustration of the EH and TA model (equivalent to the dual-axis rule; see Page et al., 2017;
 1313 Laurens and Angelaki, 2018). In an EH frame (top panel), head direction is projected onto the
 1314 EH plane (grey). In a TA frame, head direction is measured in a compass (blue) that is coplanar
 1315 with the head horizontal plane and oriented such that the azimuth measured in the EH and TA
 1316 frame coincide along the line of intersection of both planes (the 0-180° axis here). In other
 1317 words, the TA frame is anchored to the allocentric reference frame along the earth-horizontal
 1318 direction. It is defined by rotating a horizontal compass to align with head direction, instead of
 1319 projecting head direction onto the horizontal plane (EH frame). In the example orientations
 1320 shown here, the head pitches upward by 60° (middle panel) and 120° (bottom panel). In a TA
 1321 frame, it faces 90° in both panels. When projected on the EH plane, its direction reverses from
 1322 90° to -90° when pitch angle exceeds 90°, as reported by Finkelstein et al. (2015). Note that, if
 1323 the head is facing the 0° (or 180°) direction, it would be rolling instead of pitching, and azimuth
 1324 reversal would not occur since these directions coincide in the EH and TA frames. As a general
 1325 rule, TA is reversed relative to EH azimuth when tilt angle exceeds 90° in the pitch plane, but
 1326 not in the roll plane. In intermediate tilt planes, the difference between EH and TA azimuth
 1327 depends of tilt angle (see panel c).

1328 **(b)** Azimuth tuning curves of an example cell, extracted from the full 3D tuning curve measured
 1329 in Experiment 3-L and computed in EH (red) or TA (blue) reference frames. Each curve
 1330 represents the firing rate for all possible azimuths at a single tilt angle, which correspond to the
 1331 positions attained by tilting the head to a given orientation and rotating around an earth-
 1332 vertical axis (see **Suppl. Movie 1**). Note that the azimuth response of the example cell is modest
 1333 since azimuth tuning is reduced when measured in a rotator (see **Suppl. Fig. 8a,b**). In an EH
 1334 frame, the cell's PD (-157°) was conserved for tilt orientations in the roll plane (-143° and -156°
 1335 at 80° and 100° RED) but reversed abruptly for tilt angles larger than 90° in the pitch plane
 1336 (from 177° to 41° at 80° and 100° ND). However, there is no such abrupt reversal when azimuth
 1337 is computed in the TA frame.

1338 **(c)** Predicted change of the cell's azimuth PD in tilted orientations (Δ PD, expressed relative to its
1339 PD when upright), displayed as a color map. For a given 3D head orientation, azimuth differs
1340 when computed in a EH or TA frame. The difference between both azimuths depends on the
1341 head's orientation relative to gravity (see **Online Methods**; Page et al., 2017; Laurens and
1342 Angelaki, 2018). If cells encode azimuth in an EH frame, we expect their PD to be invariant
1343 across all head tilts (i.e. Δ PD \approx 0) when azimuth is expressed in the EH frame (upper left panel)
1344 but to vary with head tilt, and in particular to reverse when the head is pitched beyond 90° (i.e.
1345 between NU/ND and UD) when azimuth is expressed in TA frame (upper right panel).
1346 Reciprocally, if cells encode azimuth in a TA frame, we expect their PD to be invariant when
1347 expressed in a TA frame (lower right panel) but to vary when expressed in EH frame (lower left).

1348 **(d)** Average Δ PD across all azimuth-tuned cells significantly tuned to azimuth in Experiment 3-L
1349 (17 ADN, 7 RSC, 39 CIN). The azimuth PD varies when expressed in an EH frame but remains
1350 invariant (except close to UD) when expressed in a TA frame, in line with predictions of the TA
1351 model. Note that the PD becomes more variable close to UD, likely because azimuth tuning
1352 amplitude is near zero close to UD (see **Fig. 5g**), making data unreliable at this orientation.

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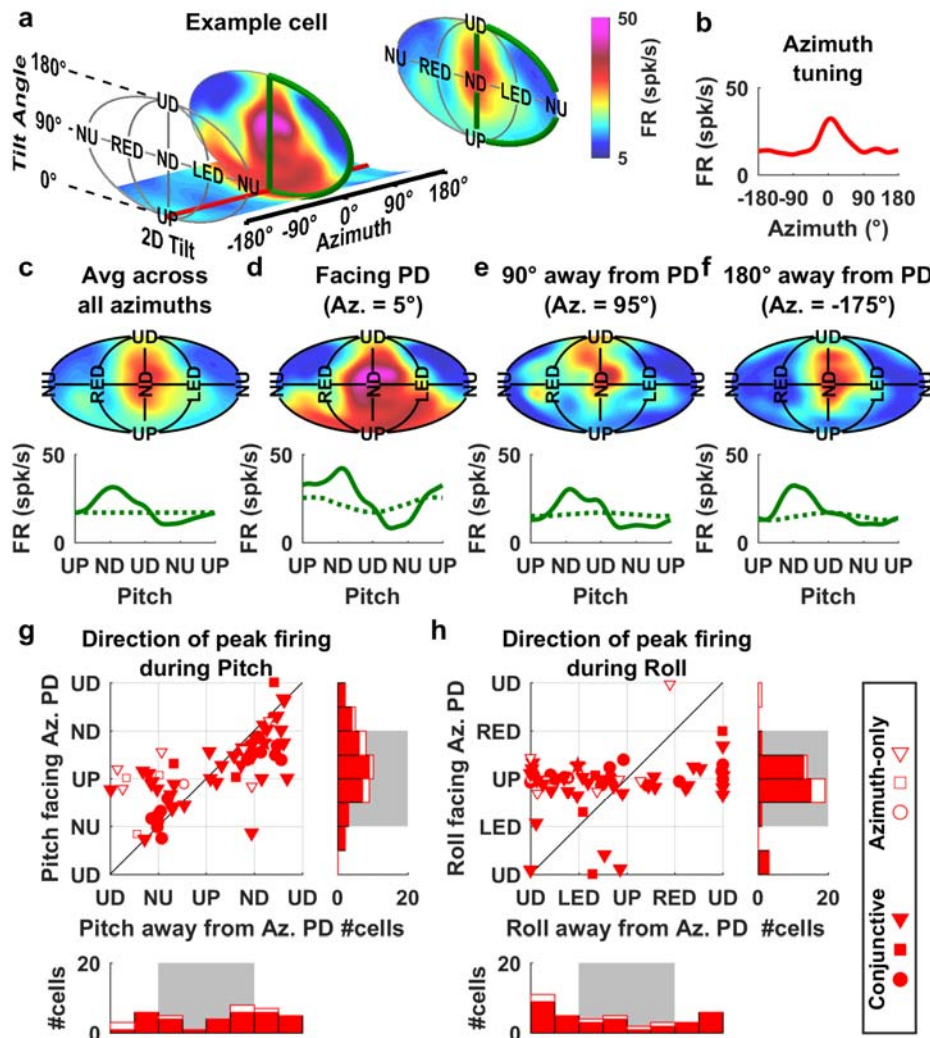


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1356 **Supplemental Figure 16: Comparison of 3D azimuth/tilt separable model fitting performed**
 1357 **with azimuth in TA or EH frame. (a)** Partial correlation of the model fits (shown as z-score),
 1358 with the correlation attributable to gravity removed so that the partial correlation reflects how
 1359 the model fits azimuth tuning in 3D. Data for all Az-tuned neurons that maintained their
 1360 azimuth tuning in the rotator (Experiment 3-L) when the head is close to upright (<45° tilt; 17
 1361 ADN, 7 RSC, 39 CIN). Grey band: zone where partial correlations are not significantly different at
 1362 $p < 0.01$. Partial correlations were significantly higher when azimuth was expressed in a TA frame
 1363 in 24/63 neurons, and significantly higher in a EH frame in only 1 ADN neuron (in this neuron,
 1364 the difference between both frames was weak and vanished if only data for > 90° tilt was
 1365 analyzed, indicating that it is likely a false positive). This analysis confirms that neuronal
 1366 responses are more consistently expressed in a TA frame. The absence of significant difference
 1367 in a large fraction of neurons (38/63) is explained by both the similarity between TA and EH
 1368 frames at small tilt angles and the tendency of azimuth responses to decrease with tilt angle
 1369 (see **Fig. 5f,g**). An alternative explanation, which would be that cells encode a mixture of EH and
 1370 TA azimuth, may be rejected because the two frames are mutually exclusive. **(b,c)** 3D tuning
 1371 curve of an example neuron computed in both frames (upper panels) and corresponding model
 1372 fits (lower panels). This neuron was tuned to tilt, with a PD at $\alpha = 100^\circ$ tilt in ND orientation ($\gamma = -5^\circ$),
 1373 as well as azimuth with a PD at -175° when upright (lower planes in the 3D curve). TA and
 1374 EH frames are identical near upright and, accordingly, tuning appears similar. Next, we examine
 1375 tuning at a tilt angle of 100° (upper planes), where the TA and EH frames diverge sharply (as in
 1376 **Suppl. Fig. 15a,c**). In a TA frame (b), the cell still exhibited a clear azimuth tuning with a similar
 1377 PD (168°) as in upright. The 3D model (lower panel) captured the 3D curve by multiplying a tilt
 1378 tuning centered on 100° ND with an azimuth tuning curve centered on 175° , leading to a total
 1379 correlation of $\rho = 0.86$ and a partial correlation of $\rho_{3D|G} = 0.55$. In contrast, azimuth tuning was
 1380 largely distorted when expressed in a EH frame (upper plane on panel c). In ND orientation
 1381 (marked by a black line), the cell's response reversed and peaked at an azimuth of 18°
 1382 (magenta). In contrast, it shifted back to $\pm 180^\circ$ on either side of the line, i.e. when head
 1383 orientation neared RED and LED. This pattern, where **azimuth tuning reverses in ND but not**
 1384 **RED or LED**, corresponds to the reversal of TA azimuth relative to EH azimuth (**Suppl Fig. 15a,c**)
 1385 and is **expected if azimuth is encoded in a TA frame**. Therefore, the cell's azimuth PD was not
 1386 invariant relative to head tilt when expressed in an EH frame. Since this violates the assumption
 1387 of the 3D model, the correlation decreased to $\rho = 0.8$ and $\rho_{3D|G} = 0.37$ (note that the correlation
 didn't decrease to zero since the model could still fit azimuth tuning at low tilt angles).

1388



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1390 **Supplemental Figure 17: Possible bias when measuring pitch and roll tuning in azimuth-tuned**
 1391 **cells** (see also Laurens and Angelaki 2019). A recent study in rat ADN (Shinder and Taube, 2018)
 1392 failed to identify tilt responses in a sample of 24 azimuth-tuned HD cells. In that study, mice
 1393 were positioned upright, facing the azimuth PD, and rotated in pitch and/or roll. The authors
 1394 observed that most cells fired more in tilt positions near upright and concluded on that account
 1395 that there is “limited evidence that cells contained conjunctive firing with pitch or roll position”
 1396 (sic). Here we demonstrate that the experimental protocol used by Shinder and Taube (2018),
 1397 where mice were tilted while facing the cell’s PD, tends to conceal pitch/roll tuning, because it
 1398 is superimposed on a strong azimuth tuning, whose strength is reduced as a function of head
 1399 tilt (**Fig. 5f,g**).

1400 **(a)** 3D tuning curve of an example conjunctive cell (measured during Experiment 3-L; same cell
 1401 as in **Fig. 6c** and **Suppl. Fig. 13**). When averaged across all azimuths (rightmost plane), the cell is

1402 tuned to tilt with a PD in ND. The cell is also tuned to azimuth, with a PD at 5°. A vertical section
1403 (i.e. firing rate for all tilt position at a given azimuth) of the tuning curve is shown at the
1404 azimuth PD. When exclusively tested during pitch in this plane (green line; as Shinder and
1405 Taube, 2018, did), the cell's tilt modulation is much broader and the cell's firing at ND is barely
1406 above its firing when the animal is upright (red). **(b)** Average azimuth tuning curve when
1407 upright, peaking at 5°.

1408 **(c)** Analysis in this study: Upper panel: Tilt tuning curves averaged across all azimuth angles (as
1409 in panel a). Lower panel: Firing rate measured along a pitch trajectory; Solid green curves:
1410 actual data; Dashed green curves: simulated data (see below). **(d)** Experiment by Shinder and
1411 Taube (2018): Tilt tuning was tested when animals faced the azimuth PD. Note that firing is
1412 elevated in the vicinity of upright. Even though the firing is largest in ND, the preferred pitch
1413 direction, evaluated by fitting a von Mises function, is biased towards upright (67° ND tilt in d
1414 versus 108° ND tilt in c). **(e,f)** Tilt tuning tested 90° (e) or 180° (f) away from the cells' azimuth
1415 PD. Firing measured during pitch rotation away from the PD is similar as the average curve in
1416 (c). We use the 3D model fit to demonstrate that the curve in (d) is affected by azimuth tuning.
1417 We fit the cell's 3D tuning curve, then alter the model's parameter to eliminate tilt tuning (by
1418 setting A to 0 and FR_0 to the cell's average firing in $FR_{Ti}(\alpha, \gamma)$; see **Methods**). Next we simulate
1419 the "pitch tuning curve" (dashed green curves) that is now influenced entirely by its azimuth
1420 tuning. The resulting curve peaks in UP orientation in (d), but is flat in other panels. This
1421 indicates that azimuth tuning affects the cell's response when facing the azimuth PD (d), such
1422 that it biases the firing rate towards upright (by interacting multiplicatively) but has little effect
1423 when facing away from the azimuth PD (e,f) or when data are averaged across all azimuths (c).
1424 We note that ***the green curve in (d) resembles most example pitch or roll tuning curves shown***
1425 ***by Shinder and Taube (2018)***. Based on these simulations, we predict that, had the authors
1426 analyzed individual pitch/roll tuning curves recorded when the mouse faced away from the
1427 cell's PD (e, f), they would have seen tilt tuning with preferred tilt away from upright.

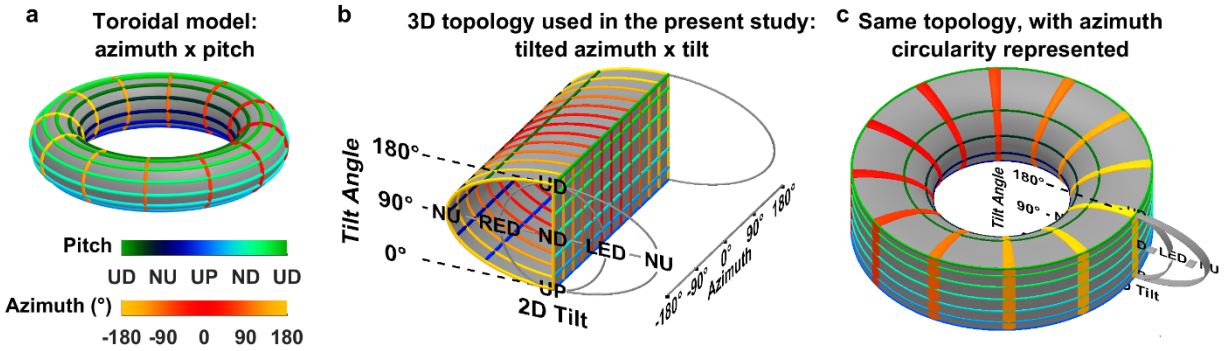
1428 **(g,h)** Same analysis, at the population level. We simulated pitch/roll rotations for all azimuth-
1429 tuned cells that were also azimuth tuned in the rotator ($n=63$; 53 conjunctive and 10 azimuth-
1430 only cells). Top: Scatter plot showing how pitch rotations while facing the cell's azimuth PD can
1431 bias conclusions. Peak responses (by fitting von Mises functions) to pitch and roll rotations
1432 when facing the azimuth PD (ordinate; as in Shinder and Taube, 2018) and when facing away
1433 from the azimuth PD (abscissa). Bottom and right: Marginal distributions are shown as
1434 histograms.

1435 When pitching while facing the azimuth PD (panel g), most conjunctive cells fire preferentially
1436 close to upright right-side histogram, grey zone, red bars (41/53, 78%, $p < 10^{-4}$), similar to the
1437 example cell in a-f. When adding azimuth-only cells (open symbols/bars), the proportion of cells

1438 firing preferentially close to upright is maintained at 49/63 (77%). The bias is even more drastic
1439 in roll (panel h; because tilt tuning is weaker in roll), with the 49/53 conjunctive cells firing
1440 preferentially around upright (lower histogram, grey zone, red) . In contrast, when pitching or
1441 rolling away from the azimuth PD (g,h; abscissae), half of the conjunctive cells (solid red
1442 symbols/bars, 24/53 in g, 19/53 in not significantly different from 50%, $p=0.6/0.05$ respectively)
1443 fire preferentially close to upright (grey band in the marginal distribution) and the other half
1444 fire preferentially closer to UD. Thus, while recording from 'our' neurons, ***if we had done the***
1445 ***experiment (pitch/roll when animal faced azimuth PD in a small sample of cells) and analyses***
1446 ***as in Shinder and Taube (2018), we would likely not have been able to identify tilt tuning.***

1447 We conclude that our dataset and quantitative analyses predict that, even though the PD of tilt
1448 tuning is distributed uniformly between upright and inverted orientation (**Fig. 3a**), conjunctive
1449 cells would appear to respond preferentially in upright orientation when recorded and analyzed
1450 as in (Shinder and Taube, 2018). The results published in our study are therefore entirely
1451 compatible with those described by Shinder and Taube (2018). The conclusions are opposite
1452 because the systematic scanning of 3D orientation (rather than a limited subset) allowed us to
1453 reveal tilt tuning, that was concealed by azimuth tuning in the Shinder and Taube (2018) study.

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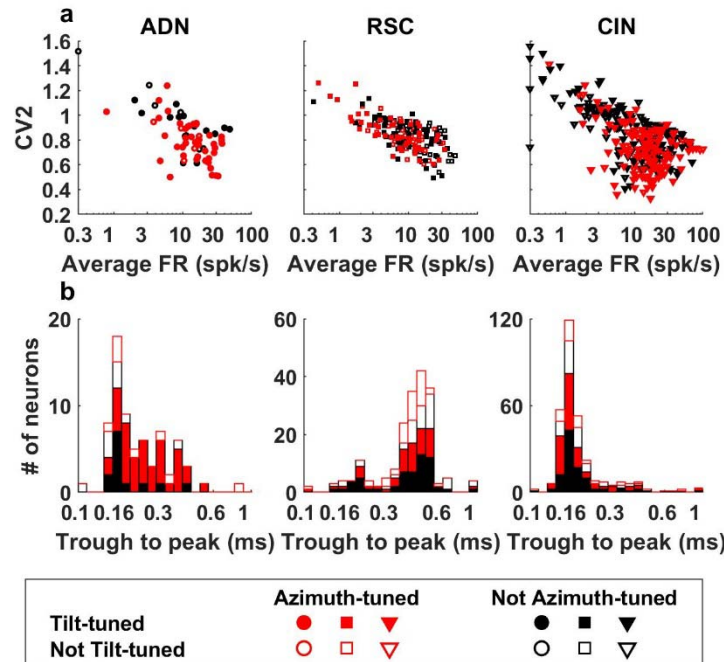


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1456 **Supplemental Figure 18: Comparison between the separable, multiplicative model of Fig. 6**
1457 **and toroid topology (Finkelstein et al. 2015).**

1458 **(a)** The toroid model is restricted to tilt movements in pitch, and assumes that azimuth and
1459 pitch are independent, i.e. pitch movements don't change azimuth. Combination of pitch and
1460 azimuth can be represented on the surface of a torus. 'Iso-pitch' lines (green/blue/black color
1461 code), that correspond to one pitch orientation and all possible azimuths, form horizontal
1462 circles. 'Iso-azimuth' lines (yellow/red color scale), that correspond to one azimuth angle and all
1463 possible pitch tilts form vertical lines. **(b)** Representation of the same iso-pitch and iso-azimuth
1464 lines in the 3D topology used in this study, when azimuth is expressed in a TA frame. Each iso-
1465 azimuth line forms a 'D-shaped' curve that passes through UP, NU, UD and ND orientation, and
1466 each iso-pitch line forms a horizontal line. **(c)** When the diagram in (b) is 'looped' upon itself to
1467 account for the circularity of azimuth (note that this representation was not used outside of this
1468 figure because it distorts volumes), the surface formed by iso-azimuth and iso-pitch lines
1469 adopts a toroidal topology identical to (a). Thus, the 3D model used here is equivalent to the
1470 toroidal topology in (Finkelstein) if (1) tilt is restricted to the pitch plane and (2) azimuth is
1471 expressed in the TA frame.

1472



1473

1474 **Supplemental Figure 19: Spike waveform and firing properties in sampled areas.**

1475 **(a)** Scatter plot of CV2 vs. average firing rate during freely moving in the arena. Median firing rate: 14 spk/s in ADN, 10 spk/s in RSC, 13 spk/s in CIN. Median CV2: 0.83 in ADN, 0.84 in RSC,
 1476 0.79 in CIN. Firing rate varied significantly across areas (Kruskal-Wallis non-parametric ANOVA, $p = 0.006$), but CV2 was similar ($p=0.04$). **(b)** Trough to peak duration of action potentials. The
 1477 median duration in ADN and CIN (0.2 ms and 0.17 ms, respectively) was significantly smaller
 1478 (Kruskal-Wallis non-parametric ANOVA, $p < 10^{-10}$) than in RSC (0.44 ms). The CIN is a fiber bundle
 1479 and neuronal activity recorded therein is therefore expected to consist of axonal spikes, that
 1480 can be recorded by tetrodes (Robbin et al. 2013) and typically exhibit small duration (Barry,
 1481 2015). Note that most units recorded in the ADN also had short-duration spikes. To our
 1482 knowledge, no studies has reported the spike duration of HD cells in the ADN. Investigating the
 1483 physiology of neuronal activity underlying ADN HD cells would be an interesting topic for future
 1484 studies.
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