Uniform Field Electrodes

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Uniform field electrodes are generally employed to effect glow discharge with large cross-sections in the operation of TE gas lasers. In practice, electrodes have finite boundaries. The discharge electric field is obtained by dividing the applied voltage to the electrodes by the inter electrode distance. This is really true , for two infinite parallel plates. Practical electrodes with finite boundaries will have much enhanced field at the edges compared to that at the center. The sharper the edges, the higher is the resulting field, for the same potential. In case of uniform field electrodes , the edge has a designed (calculated) profile, such that the field is maximum at the center, and gradually diminish towards the edges. As the breakdown voltage of the gap is a strong function of E/p (E being the field, and p is the pressure), the discharge is constrained within a region, at whose boundaries, the field is 97% of the central field [(1)]. Beyond this region , to contain the size of the electrodes, one deviates from the calculated profile, and the electrode is rounded off. This rounding is done at a point , where the field has reduced to at least 70% of the field[(2)] at the center. The rounding off will increase the field there, but it would remain less than 97% of the central field, and the discharge is constrained to the central region.

The calculation of the profile of the electrode is carried out in two dimensions (x & y), whereas, the electrode is a three dimensional object. To fabricate the electrode , the cutting edge of a Milling cutter, is made according to the calculated profile. A rectangular bar of Aluminium (Nickel in case of Excimer Laser) of required dimension is taken and the profile is generated along the edges of the Electrode. The ends are generated , by rotating the whole electrode, with respect to the cutter edge[(3)].

In a homogeneous charge free region, the potential distribution between the electrodes is a solution of the Laplace equation,

$$\nabla^2 V = 0 \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

In Cartesian coordinates ,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Electrostatic problems can be solved by the theory of complex variables. Let us take a function z , defined as $z = x + i y$. We can define another function $W(z) = U + i V$. In this, U is the flux function and ,V is the potential function. If the function $W(z)$ has a derivative $W'(z)$, then the function is called an

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$$

$$\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$$

analytic function, if Cauchy –Reimann condition is satisfied, namely,

If the above conditions are true , it is easy to show by simple differentiation, that

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0, \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

and, thus both U and V satisfy Laplace equation. The two families of curves $U(x,y) = C_1$, and $V(x,y) = C_2$ are orthogonal to each other. Three types of profiles are extensively used for profiling electrodes for gas discharges used for CO_2 Lasers and Excimer Lasers. These are Rogowski Profile [4a, 4b], Chang Profile [5], and Ernst Profile [6].

In case of Rogowski Profile, the analytic function used is

$$z = \frac{A}{\pi}(W + e^W) \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

The Rogowski profile calculates the edges for two semi infinite electrodes with a mid plane of infinite extent between them. The distance between an electrode and the mid plane is A/2 . The x and y values of the profiles are given by

$$x = \frac{A}{\pi}(U + e^{U}\text{Cos V})$$

$$y = \frac{A}{\pi}(V + e^{U}\text{Sin V}) \text{ ……………………(4)}$$

The method of designing is to take a value of V[4b]. V is chosen as $\pi/2+\theta$, where θ is an angle in radians. This value substituted in the equation for y , will give a value for U. The value of U can now be used to find x .

Chang proposed the following analytic function

$$z = W + K \text{ Sinh } W \text{ ………………………(5)}$$

In this equation K is an empirical constant, which can be chosen arbitrarily, and each value of K will give a different "xy " profile. In the W plane, the equipotential plane is given by constant values of V and variable value of U. In the z plane , the corresponding equipotential surfaces is

$$x = U + K \text{ Cos V Sinh U}$$

$$y = V + K \text{ Sin V Cosh U …………………(6)}$$

The value of electric field on the electrode surface is given by

$$E^{-2} = (dz / dW)^2$$

$$= (1+ K \text{ Cos V Cosh U})^2 + (K \text{ Sin V Sinh U})^2 \text{…..(7)}$$

Chang showed that in order to get a flat distribution of field near the center of the electrode , V should be chosen as

$$V = \pi/2 + \text{arc Sin } K \text{ …………………(8)}$$

Now various values of K can be chosen (e.g. K=0.2, K=0.06, K=0.01 etc.), and V is defined. So for a choice of K and V, U can be found out from the equation for x and a particular value of x . This value of U can be used to find corresponding y. The lower the value of K , the broader is the electrode.

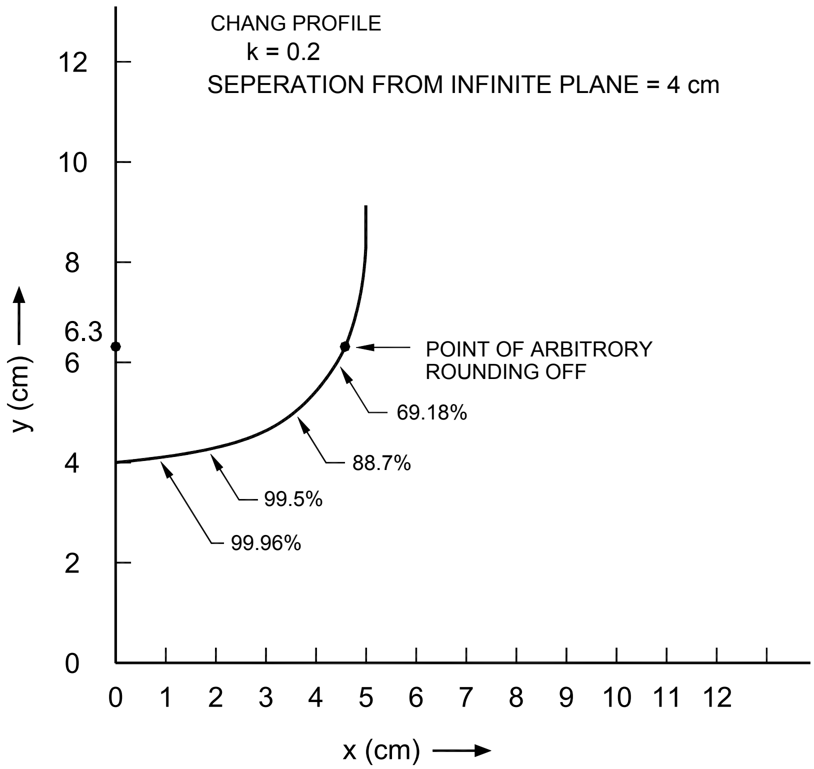

Fig.1. The Chang Profile . This is the profile used in the Milling Cutter. The profile is as per calculation up to the point of arbitrary round off. The calculated fields on the profile at various points are shown. The rounding off is done where the field is Less than 70 %

For our 3 cm X 3cm X 50cm discharges (for CO_2 Laser), we have designed Chang profile, with K=0.2.This Profile is shown in Fig.1 .The field distribution on the profile and the point of rounding off is also shown. These electrodes has been designed for a separation from a infinite plane of 4 cm and a maximum inter electrode separation of 8 cm. When used for 3 cm separation discharges they produce square 3 cm X 3cm CO_2 Laser discharges. For the 10cmX 10cm X 50cm discharges , we have designed Rogowski profiled electrode with $V = \pi/2 + 20°$, and a separation from a infinite plane of 5.3 cm and a maximum inter electrode separation of 10.6 cm .This profile is shown in Fig.2 .For our 1.5cm X 1.5cm X 50 cm discharges we use Ernst profile. This profile is shown in Fig.3 . What is the basis of Ernst profile?

The Ernst Profile is an improvement over the Chang Profile , and results in more compact electrodes. Ernst added more terms to the analytic expression used by Chang (refer to equation 5),

$$z = W + K_0 \text{ Sinh } W + K_1 \text{Sinh } 2W + K_2 \text{Sinh } 3W \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

It is possible to optimize the field distribution on the electrode surface by choosing proper values of the parameters K_0, K_1, K_2 and V. V is usually having a

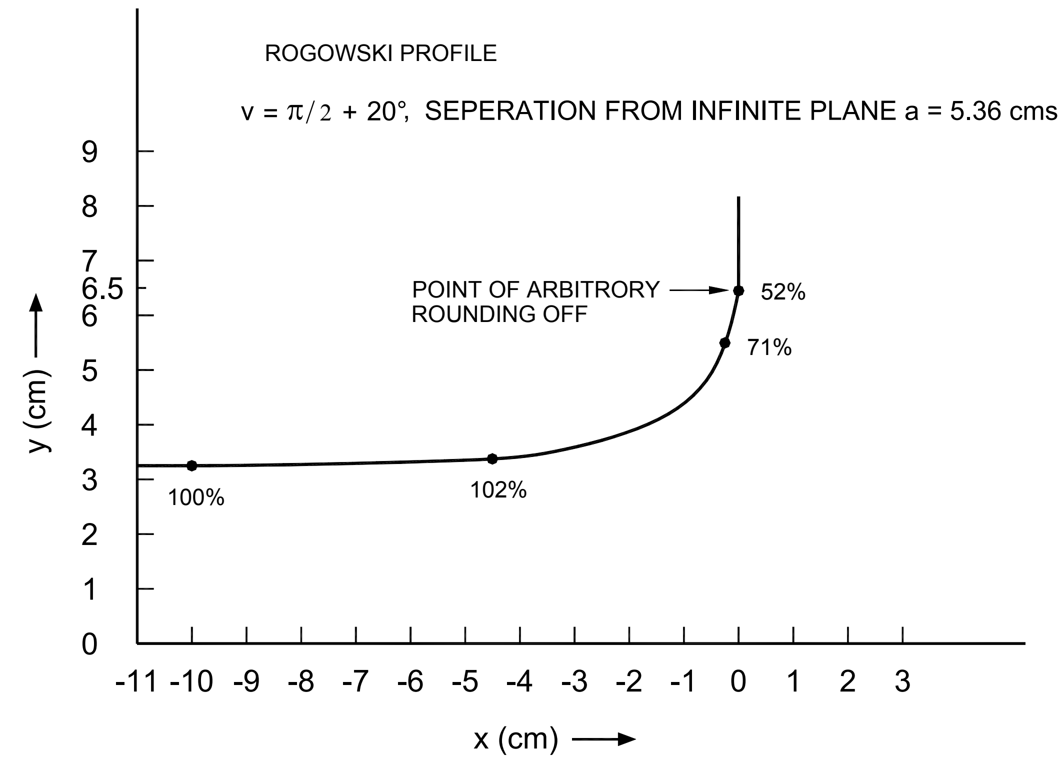

Fig2. The Rogowski Profile. This is the profile used in the Milling Cutter. The profile is as per calculation up to the point of arbitrary round off. The calculated fields on the profile at various points are shown. The rounding off is done where the field is less than 70 %, in this case 52%.

value close to $\pi/2$. Ernst gives iterative relations, using which , for a particular value of K_0, K_1 and K_2 can be found out. The x and y values for the profile are given by

$$x = U + K_0 \text{ Cos } V \text{ Sinh } U + K_1 \text{ Cos } 2V \text{ Sinh } 2U + K_2 \text{ Cos } 3V \text{ Sinh } 3U$$

$$y = V + K_0 \text{ Sin } V \text{ Cosh } U + K_1 \text{ Sin } 2V \text{ Cosh } 2U + K3\text{Sin } 3V \text{ Cosh } 3U \ldots.(10)$$

Ernst in his paper also gives the expression for calculating the Electric Field, as a function of all the parameters used , U, V, K_0, K_1 and K_2 . An unique feature of the Ernst profile is that the profile exhibits a maximum width , after which the profile folds back. This is not apparent in Fig.3. , which shows the Ernst Profile and will occur at higher values of y.

We have developed a Program in FORTRAN to calculate the Ernst Profile. In this Program we input to the Program, the desired discharge height , width and the width of the electrode. U is the running variable. We take different values of K_0
And choose the one which produces the correct value of electrode width. The value of V is also determined from the condition that the field strength at the center point of the electrode surface (starting point of the calculation),should be within 10^{-7} of the required field strength. We have designed an Electrode , to give 15 x 15 mmxmm discharge between two such electrodes. The Electrodes have maximum width of 30 mm. The optimum values of various parameters were : K_0= 0.069, K_1= 5.963 X 10^{-4},

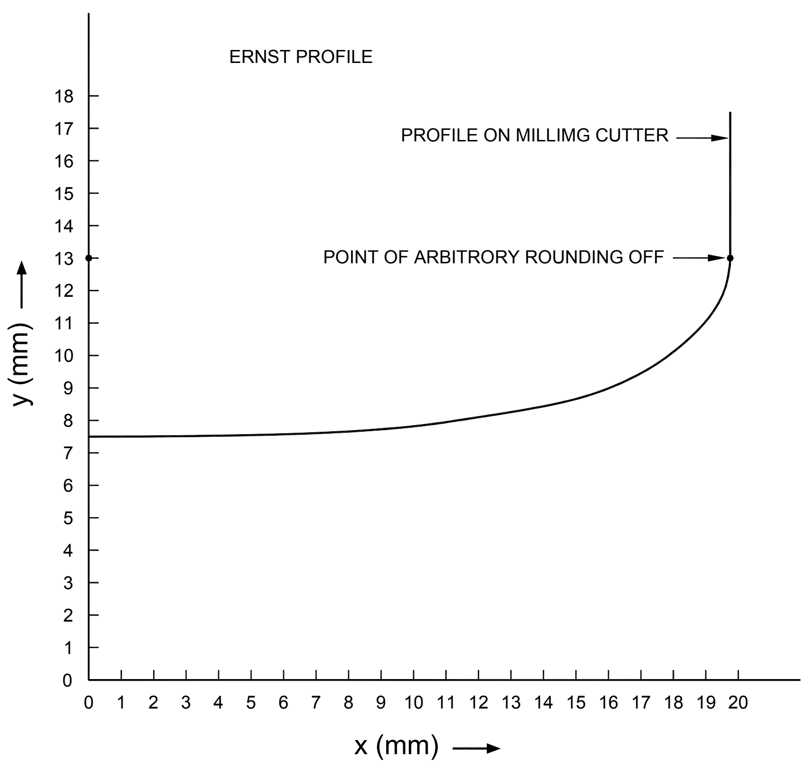

Fig. 3. Ernst Profile. For parameters see text. X axis represents the infinite plane.

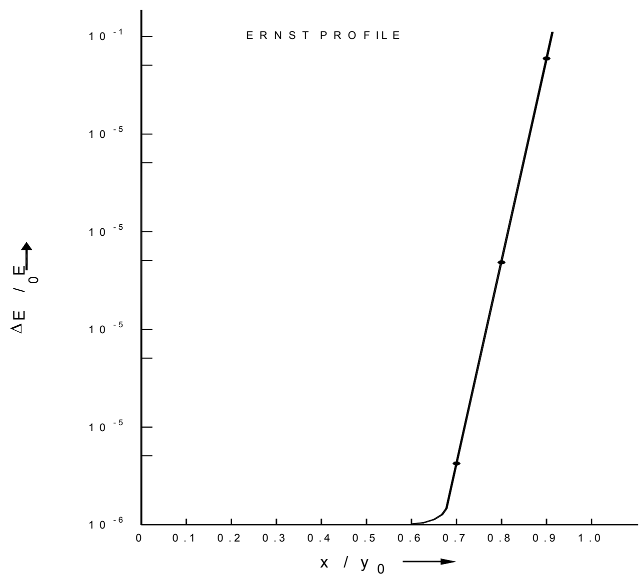

Fig.4. Normalized deviation of Electric Field strength at the surface of the electrode from the central value. E_0 is 100%, and y_0 is 7.5 mm.

K_2= 3.47 X 10^{-6} and V= 1.370799. Using this profile we get 15 mm X 15 mm discharges for CO_2 Lasers and 15 mm X 6 mm discharges in Excimer Lasers.
Fig. 4 shows the variation of the field on the surface of the electrode , in terms of deviation from the central field , which is 100%.

Till now we have solved the Laplace Equation (Equation 1), which is really valid for a charge free region. An electric discharge is not charge free , but has finite electron density . What is the effect on the Electric field distribution. Let us analyse this problem for a simple case. There are two electrodes of infinite extent, separated by distance h meters and having a potential difference of V volts. The space between the electrodes has an electron density of ρ (C / m ^{3}), and the dielectric constant of the medium is ε.

The Poisson's equation for this medium is

$$\frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon} \qquad \text{.......................} \ (11)$$

Integration of the above equation twice gives the following expression for V

$$V = -\frac{\rho}{\varepsilon} x^2 + C_1 x + C_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Since V=0 at x = 0, C_2 is 0. Hence

$$V = -\frac{\rho}{\varepsilon} x^2 + C_1 x \quad , \text{ and}$$

$$E = \frac{\partial V}{\partial x} = C_1 - 2\frac{\rho}{\varepsilon} x \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Since at x=0, E = V/ h, C_1= V/ h, and E is given by

$$E = V/h - 2\frac{\rho}{\varepsilon} x$$

Thus the field is reduced in the presence of the electron density. As the discharge is a strong function of the electric field, the reduction of the field affects the discharge width. In both the discharges for the CO_2 or the Excimer Lasers, we utilize an uniform pre ionization electron density of 10^7/ cc. We are applying a voltage of 15 KV across electrodes separated by 15 mm, and thus dealing with fields of the order of 10^6 V/m. The discharge is a pulsed one, and the initial electron density of 10^7/ cc, is too low, to affect the field distribution across the electrodes. However as the electron density rises to values of the order of 10^{10}/ cc, the electric field will modify the initial field distribution . The peak electron densities for CO_2 Laser discharges are ~ 10^{13}/ cc, and for the Excimer Laser discharges are ~ 10^{15}/ cc. Practical experience with the electrodes fabricated by us shows that we get approximately square discharges in the case of CO_2 Lasers, and much narrower discharges in case of Excimer Lasers.

We have formulated methodologies for design and fabrication of profiled electrodes for gas laser discharges. Programs have been developed to calculate and design all the three types of Profiles dealt in this article. Milling cutters with edges as per the calculated profiles are used for the fabrication of these electrodes. The electrodes have played important part in the development of the pulsed CO_2 and Excimer Lasers in our laboratory.

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References:

1. A novel analytical design method for discharge electrode profiles- E.A.Stapperts, Appl.Phys. Letters 40(12),15 June (1982), pp- 1018-1019.

   In this paper Stapperts describes two electrode profiles. For each he draws the field distribution and relative electron density profile in case of Excimer laser discharge. The 50% width of the electron density profile is approximately at 97% of the central field for both type of electrodes.

2. There is no fixed prescription for the rounding off point. Our experience in operation of the CO_2 and Excimer Laser discharges show that the rounding off should be attempted at the point on the profile, where the field is less than or equal to 70% of the central field.

3. H.J.Seguin, K.Manes & J.Tulip, Rev. Sci. Instrum, 43,1134 (1972)

4. a. W.Rogowski, Arch. Electrotech., 12 (1923), 1

   b. J.D.Cobine, "Gaseous conductors- Theory and Engineering Applications", Dover Publications, pp 177-181,(1958)

5. Improved Uniform-Field Electrode Profiles for TEA Laser and High -Voltage Applications – T.Y.Chang, Rev. of Scientific Instrum., 44(4), April (1973), 405-407

6. Uniform – Field Electrodes with minimum width – Gerard J Ernst, Optics Communications, 49(4), 15 March (1984), 275-277