Polarization-Resolved Measurements using Mueller Matrix Analysis

Application Note

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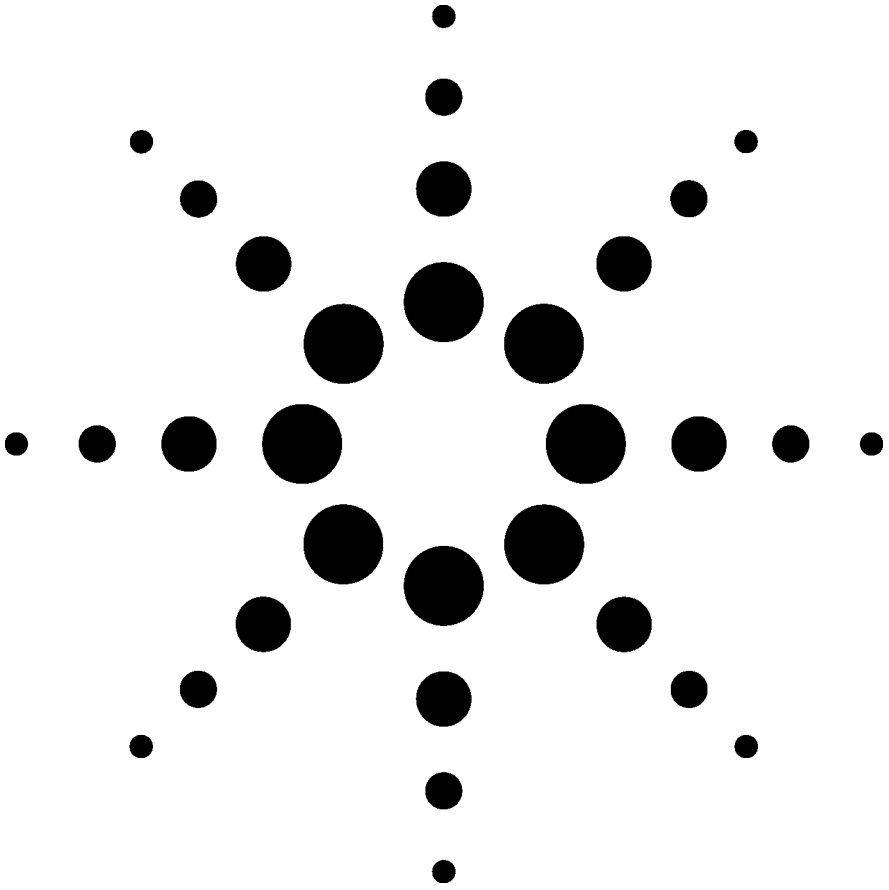

The performance of optical components can have significant dependence on polarization, as with the difference between TE and TM polarization in planar devices like AWG and SOA. Determination of the optical properties for the principal states of polarization can be needed, for instance to determine the polarization dependent wavelength of a bandpass filter. It can be complicated to adjust instrumentation to provide these polarization states. A fast and direct method to do this based on Mueller matrix analysis is described here.

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Introduction

The performance of optical components can depend significantly on input signal polarization as a result of the structure or cross-section of the optical path in the device. In components based on planar waveguides in particular, different optical properties apply to light polarized parallel or perpendicular to the substrate, labeled TE or TM respectively. Arrayed waveguides (AWG) and semiconductor optical amplifiers (SOA) are two examples of such devices. Determination of the optical properties at these principal states of polarization is valuable, for example to determine the polarization dependent wavelength (PDW) of the passband for an AWG channel and to aid in the design and refinement of the component.

Often it is not known ahead of time how to align the polarization of a test source in order to produce light in the principal states of polarization of the optical device under test (DUT), because these states may be unknown and there is often a fiber that changes the polarization in an unknown way between the polarization controller and the DUT. A possible approach of iteratively optimizing the alignment of the polarization controller to yield maximum or minimum insertion loss from the device itself, or from a reference polarizer in its place, is effective but slow.

Instead of such a search method, both the desired states of polarization and the corresponding optical properties can be determined by analysis of Jones-matrix or Mueller-matrix data from the device. In particular, the latter data can be obtained from optical power measurements performed at a predetermined set of input polarization states. It is not necessary to know before the measurement the relation between the chosen set of states and the principal polarization states of the device. This is the same type of measured data obtained for determination of polarization dependent loss (PDL) by the Mueller matrix method. However the analysis is extended to not only find the maximum polarization dependent difference in the loss or other optical property, but to determine the values at the maximum and minimum and to assign the corresponding polarization states to these values.

This Mueller matrix analysis method, described below, provides accurate and fast polarization-resolved measurements and is especially well adapted to swept-wavelength spectral measurements. From the polarization-resolved spectra, parameters such as PDW can be further derived.

Fundamentals

The method described here is used to identify the input polarization states of a DUT associated with the maximum and minimum values of a transmission or reflection parameter, typically insertion loss, as well as those parameter values. Other device response parameters, such as the photocurrent from a detector device could also be determined.

These input states of polarization associated with the maximum and minimum values correspond to what are called principal axes for crystalline optical materials. Since these input states may not be linear in a fiberoptic component where the polarization can change along the fiber after the input connector, the more general label principal states of polarization (PSP) is used here. Any other state of polarization is composed of a combination of the two PSP and will then be associated with an intermediate value of the optical parameter, such as loss.

Determination of the optical parameters for the PSP is particularly useful when these states are directly related to the structure and design of the component. As mentioned above, this is the case for planar waveguide devices. There are generally clear physical reasons for a difference in the optical parameters for light polarized parallel to the plane of the waveguide vs. perpendicular. Keeping the same nomenclature that is used for microwave guides, these are called transverse electric (TE) and transverse magnetic (TM) respectively. Differences between TE and TM parameters can be influenced by the design and the manufacturing process of the waveguide device.

An important example for such a device is the arrayed waveguide (AWG), which provides a wavelength-selective bandpass filter function between input and output fibers. The device is typically fabricated in a multilayer film that is deposited on a flat substrate and is therefore planar. Due to differences in the waveguide properties for TE and TM light, there can be a difference in the wavelength of the passband. Knowledge of this polarization-dependent wavelength (PDW) is important, both for refining the device design and for characterizing the device performance.

Although the TE and TM states are predominantly linear polarized states that are aligned with the waveguide axes, the polarization states at the input to a fiber connecting to the device are not easily known and may even be partly circular due to changes induced by the fiber.

In fact, the PSP can vary along the length of a device, between the input and output, but light entering with an input PSP will exit with the corresponding output PSP. Light entering as a mixture of the two input PSP will generally exit as a different mixture of the output PSP, depending on the different optical parameters of the two PSP.

As an example, consider the following case of a bandpass filter. The minimum and maximum losses define the envelope of the insertion loss vs. wavelength curve, as shown by the dotted lines in Fig. 1. The loss curve is located within this envelope for all polarization states. This includes the effects of possible wavelength shifts of the spectral filter transmission dependent on polarization.

The minimum and maximum loss curves never cross, since the maximum must stay above the minimum. However, at some points the curves may touch each other. These correspond to the points where the spectral loss curves of the TE and TM modes may cross.

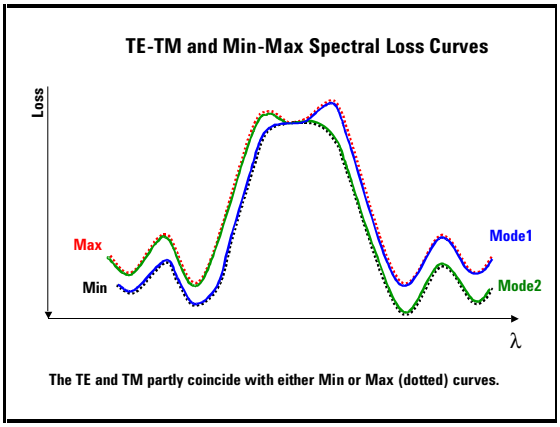

Figure 1. Schematic illustration of the relationship between the maximum and minimum spectral loss curves (dotted) and the loss curves for the TE and TM polarization modes (solid).

Thus, although the maximum and minimum curves can either be determined directly or determined from IL and PDL data, the identification of the TE and TM spectra involves assigning the maximum and minimum for each wavelength to either of the two polarization modes. The solid lines in Fig. 1 illustrate this process.

Algorithm for isolating TE and TM curves

This is an algorithm for obtaining the insertion loss spectra for light polarized along the two PSP of the device under test, based on the Mueller matrix determined for the device by a 4-state measurement. With these two spectra, the polarization dependence of spectral features, like the passband of a filter can be determined. The equations here are taken or adapted from Ref. 1.

Mueller Matrix analysis

1) Standard Mueller-Matrix-method PDL measurement

The measurements for this determination are made at 4 pre-defined polarization states that sufficiently sample all possible states. Here we use the same set of states as in Ref. 1 , which includes (as set at the output of the polarization controller) linear horizontal, linear vertical, linear diagonal, and right-hand circular polarization. To obtain these states with the Agilent 8169A polarization controller, the linear polarizer angle α_p is first aligned with the incoming source for maximum output power and then the quarter-wave and half-wave plates are set according to Table 1. Note that the linear polarizer is used to define the orientation of the "horizontal" axis. The transmission spectra are measured for each polarization state. Refer to the Product Note [1] for more detailed information about the Mueller-Matrix method and the operation of the 8169A.

	Polarizer	Q-plate	H-plate
LH, 0°	α_p	α_p	α_p
LV, 90°	α_p	α_p	$\alpha_p + 45°$
LD, +45°	α_p	α_p	$\alpha_p + 22.5°$
RHC	α_p	$\alpha_p + 45°$	α_p

Table 1. Relationship between the four polarization states and the angles that need to be set for the polarization controller.

2) Calculate Mueller matrix

The result of the above measurements produces four wavelength-dependent arrays of reference data $[P_a(\lambda), P_b(\lambda), P_c(\lambda), P_d(\lambda)]$ and four arrays of DUT data $[P_1(\lambda), P_2(\lambda), P_3(\lambda), P_4(\lambda)]$. The top-row elements of the Mueller matrix are calculated based on these eight arrays of data according to Eqn. 1.

$$\begin{bmatrix} m_{11}(\lambda) \\ m_{12}(\lambda) \\ m_{13}(\lambda) \\ m_{14}(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\frac{P_1(\lambda)}{P_a(\lambda)} + \frac{P_2(\lambda)}{P_b(\lambda)}\right) \\ \frac{1}{2}\left(\frac{P_1(\lambda)}{P_a(\lambda)} - \frac{P_2(\lambda)}{P_b(\lambda)}\right) \\ \frac{P_3(\lambda)}{P_c(\lambda)} - m_{11}(\lambda) \\ \frac{P_4(\lambda)}{P_d(\lambda)} - m_{11}(\lambda) \end{bmatrix} \quad \text{Eqn. 1}$$

3) Calculate insertion loss and PDL

Average insertion loss (IL) and PDL are calculated based on the maximum and minimum transmission data derived from the Mueller matrix. IL is an array of loss data over wavelength.

$$IL_{ave}(\lambda) = 1 - \left(\frac{T_{Max}(\lambda) + T_{Min}(\lambda)}{2} \right) \qquad \text{Eqn. 2}$$

and

$$PDL(\lambda) = \left(\frac{T_{Max}(\lambda)}{T_{Min}(\lambda)} \right) \qquad \text{Eqn. 3}$$

with

$$T_{\max}(\lambda) = m_{11} + \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}$$
$$T_{\min}(\lambda) = m_{11} - \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \qquad \text{Eqn. 4}$$

Note that IL and PDL are typically then converted to be expressed logarithmically in units of dB.

4) Choose reference wavelength with "high" PDL and "low" IL for determining principal polarization states

The algorithm described here determines the PSP at one or more reference points chosen at wavelengths where significant PDL and low IL (e.g. within 3 dB of the bandpass peak) enable accurate resolution of the polarization dependence.

5) Solve for Stokes parameters at "reference λ"

Now the Mueller matrix data is analyzed to solve for the two PSP of the DUT, labeled J and K as in Ref. 1, at the reference wavelength according to Eqns. 5 and 6. The reference λ chosen in the previous step are identified with the index (i) in the following equations, i.e. $m(\lambda_{ref})=m(i)$. The equations produce two normalized Stokes vectors that represent the PSP, $X_J = (x_{J1}, x_{J2}, x_{J3})$ and $X_K = (x_{K1}, x_{K2}, x_{K3})$. For the planar device case, these two polarization states are TE and TM. These vectors have been determined from the data for a certain reference wavelength but are typically good representations for the PSP over an extended wavelength range.

$$x_{J1} = \frac{S1_J}{S0_J} = +\left(\frac{m(i)_{12}}{\sqrt{m(i)_{12}^2 + m(i)_{13}^2 + m(i)_{14}^2}} \right)$$
$$x_{J2} = \frac{S2_J}{S0_J} = +\left(\frac{m(i)_{13}}{\sqrt{m(i)_{12}^2 + m(i)_{13}^2 + m(i)_{14}^2}} \right) \qquad \text{Eqn. 5}$$
$$x_{J3} = \frac{S3_J}{S0_J} = +\left(\frac{m(i)_{14}}{\sqrt{m(i)_{12}^2 + m(i)_{13}^2 + m(i)_{14}^2}} \right)$$

$$x_{K1} = \frac{S1_K}{S0_K} = -\left(\frac{m_{12}(i)}{\sqrt{m_{12}^2(i) + m_{13}^2(i) + m_{14}^2(i)}} \right)$$
$$x_{K2} = \frac{S2_K}{S0_K} = -\left(\frac{m_{13}(i)}{\sqrt{m_{12}^2(i) + m_{13}^2(i) + m_{14}^2(i)}} \right) \qquad \text{Eqn. 6}$$
$$x_{K3} = \frac{S3_K}{S0_K} = -\left(\frac{m_{14}(i)}{\sqrt{m_{12}^2(i) + m_{13}^2(i) + m_{14}^2(i)}} \right)$$

If the PSP indicated by the Stokes vectors do have significant dependence on wavelength, more reference points should be used that are more densely spaced and each will be the reference for a smaller wavelength range. If necessary, the Stokes vectors can be determined separately for every wavelength value of the Mueller matrix arrays. The wavelength dependence of these PSP can then be used to assess 2nd-order PMD (polarization mode dispersion), if required.

6) Solve for transmission over λ

The wavelength-dependent arrays T_J and T_K, as calculated according to Eqn. 7, are the transmission spectra for the two principal polarization states. (This algorithm does not distinguish which spectrum is from TE or TM.) Note that the wavelength dependence in T results from the wavelength dependence of the Mueller matrix values, while the normalized Stokes parameters for the reference wavelength are used over the wavelength range for which the reference Stokes parameters are valid. Parameters like the polarization-dependent wavelength shift of a passband can then be determined from the spectra.

$$T_J(\lambda) = m_{11} + m_{12}x_{J1} + m_{13}x_{J2} + m_{14}x_{J3}$$
$$T_K(\lambda) = m_{11} + m_{12}x_{K1} + m_{13}x_{K2} + m_{14}x_{K3} \qquad \text{Eqn. 7}$$

In this way, the Stokes vector representation of the PSP and the corresponding spectra are determined, based on analysis of data measured at other states of polarization. This allows the measurement sequence to be set without prior knowledge of the PSP in the device. However it is also possible to use the calculated Stokes vectors to set the polarization controller so that the PSP states may be measured directly. This is described in the next section.

Direct PSP Measurements

To confirm the results obtained with the Mueller matrix analysis, or to make further DUT measurements at the PSP without repeating the 4-state measurements, the 8169A polarization controller can be set to produce the PSP determined according to the method above. The following equations from the Appendix of Ref. 1 describe the method to calculate the Q-plate and H-plate positions of the polarization controller when the Stokes vector is known.

7) Find Q-plate and H-plate angles of polarization controller for the PSP (J and K states)

Two sets of waveplate settings for the polarization states are calculated: α_Q and α_H for both the J and K states. This relation can be solved with the following equation. In the following x_1, x_2, $x_3 = x_{J1}$, x_{J2}, x_{J3} from the equations above.

If $x_1>0$, the J state is produced by

$$\alpha_Q = \frac{\arcsin(x_3)}{2} \quad \text{and}$$

$$\alpha_H = \frac{\theta}{2} + \frac{\arcsin(x_3)}{4}, \quad \text{where} \qquad \text{Eqn. 8}$$

$$2\theta = \arctan\left(\frac{x_2}{x_1}\right) + 0^\circ,$$

and the K state is produced by

$$\alpha_Q = -\frac{\arcsin(x_3)}{2} \quad \text{and}$$

$$\alpha_H = \frac{\theta}{2} - \frac{\arcsin(x_3)}{4}, \quad \text{where} \qquad \text{Eqn. 9}$$

$$2\theta = \arctan\left(\frac{x_2}{x_1}\right) + 180^\circ.$$

If $x_1<0$, the J state is produced by

$$\alpha_Q = \frac{\arcsin(x_3)}{2} \quad \text{and}$$

$$\alpha_H = \frac{\theta}{2} + \frac{\arcsin(x_3)}{4}, \quad \text{where} \qquad \text{Eqn. 10}$$

$$2\theta = \arctan\left(\frac{x_2}{x_1}\right) + 180^\circ,$$

and the K state is produced by

$$\alpha_Q = -\frac{\arcsin(x_3)}{2} \quad \text{and}$$

$$\alpha_H = \frac{\theta}{2} - \frac{\arcsin(x_3)}{4}, \quad \text{where} \qquad \text{Eqn. 11}$$

$$2\theta = \arctan\left(\frac{x_2}{x_1}\right) + 0^\circ.$$

8) Measure J-state and K-state

Set the polarization controller according to the above calculations, remembering to offset the values according to the linear polarizer position. (For example, set the quarter-wave plate position to $\alpha_Q+\alpha_P$) Execute reference and DUT measurements over wavelength with the standard λ-scan procedure.

Examples

AWG-multiplexer bandpass filter

Measurements from a prototype-stage AWG are illustrated in Fig. 2. The TE and TM spectra calculated from the Mueller matrix according to Eqn. 7 are shown together with the corresponding direct PSP measurements (J and K) obtained as in Step 8. The good agreement between the two methods provides a strong confirmation of the validity for both approaches. Note that the TE and TM curves have crossing points. Similar good agreement between these two sets of spectra and the minimum and maximum envelopes obtained from PDL analysis of the Mueller matrix confirms that the Stokes vectors obtained at the reference wavelength accurately determine the PSP over the entire measured wavelength range. The results clearly show a shift in wavelength of the passband, depending on the polarization. Thus the two PSP spectra could be analyzed for the center or peak wavelength of the passband and the difference between these for TE vs. TM gives the PDCW or PDW.

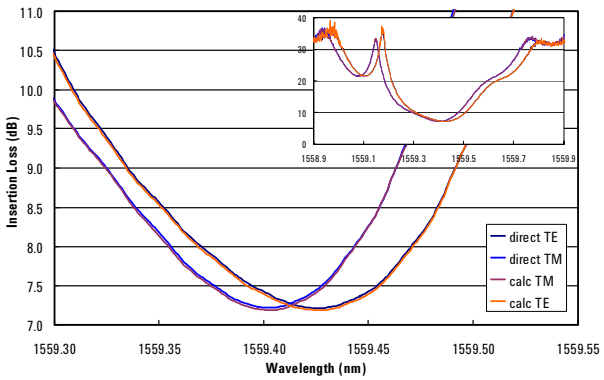

Figure 2. Insertion loss spectra for TE and TM modes of an AWG passband, determined both by Mueller matrix analysis and by direct measurement. The inset shows a wider wavelength range with several crossing points.

Semiconductor optical amplifier (SOA)

Since SOAs are produced on a semiconductor chip, they also have a planar structure with possible differences between TE and TM polarization. The results for polarization-resolved gain measurements on such a device are shown for one wavelength in Fig. 3. Here the gain has been determined as a function of signal power for the two polarization directions. An optical spectrum analyzer, rather than a power meter was used to measure signal power in order to filter out the amplified spontaneous emission (ASE) from the amplifier. Also in this case, good agreement is seen between the results calculated from the Mueller matrix and the results of direct measurement using the calculated Stokes vectors. Note that also in this case, a crossing point of the two spectra could be identified.

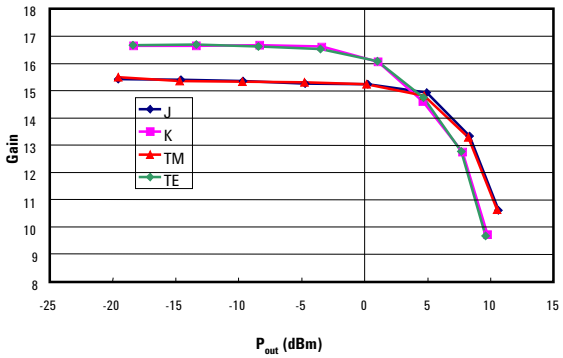

Figure 3. Comparison of SOA gain spectra for TE and TM modes using both calculated results (TE/TM) and directly measured results (J/K).

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[1] Agilent Product Note 5964-9937E: "PDL Measurement using the Agilent 8169A Polarization Controller".

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