General Methods for Optimized Design of Mueller Polarimeters

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Outline

• The Stokes formalism: What ellipsometers can do with?
• The use of the condition number as an objective criteria for optical design.
• Calibration of Mueller polarimeters.

Jones and Stokes Formalisms

<table>
<thead>
<tr>
<th>Jones vector (2 complex components)</th>
<th>Stokes vector (4 real components)</th>
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<td>$E_z \left( \begin{array}{l} \Delta_0 e^{i\phi} \ \Delta_0 e^{i\phi} \end{array} \right)$</td>
<td>$S \left( \begin{array}{l} t_1 \cos \theta_1 - t_2 \sin \theta_1 \ t_1 \sin \theta_1 + t_2 \cos \theta_1 \ 0 \end{array} \right)$</td>
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For non depolarizing samples both, the Mueller matrix and the Jones matrix have the same physical meaning.

Reflection of transmission by a planar isotropic surface can be expressed in terms of ellipsometric angles $\Psi$ and $\Delta$.

Measurement Principle and Experimental set-up

- Modulation matrix $W$: Stokes vectors generated by the polarization state generator (PSG)
- Analysis matrix $A$: signal vector on the polarization state analyser (PSA)
- Complete measurement $B = A*M$.
- The choice of the calibration set is not unique.
- Sample calibration set can be optimized to current working conditions.
- A and W must be as much « non-singular » as possible.

Singular value decomposition of any square matrix $A$

$A = U \Sigma D V^T$ with $U, V$ unitary, $D$ diagonal (singular values)

Conditioning number $C(A) = \Sigma_{\text{min}} / \Sigma_{\text{max}}$ $C(A)$ goes from 0 (A singular) to 1 (A unitary)

Error propagation in a linear transformation $X$ and $Y$: vectors related by $Y = A X$, with errors $\delta X$ and $\delta Y$ Error propagation is proportional to the inverse of $C(A)$:

$$\frac{\delta Y}{Y} = \frac{1}{C(A)} \frac{\delta X}{X}$$

To minimize error propagation, $C(A)$ must be as much close to 0 as possible.

The optical elements and configuration of the PSG and the PSA must be chosen in order to maximize the condition number.

Calibration: Theoretical principle

Ideal matrices

$A M^0 W = (a m w) (a m w)^*$

Experimental matrices

$A M W = (a m w) (a m w)^*$

Calibration: Practical determination of $H^W$ and $H^A$

Calibration of the PSG and the PSA requires a set of calibration samples. The choice of the calibration set is not unique.

Example of calibration set:

1. Measurement of $B_j$ from the set of reference samples: $B_j = A M W$
2. Measurement of $B_1, B_2, B_3$ from the set of reference samples:

$B_1 = A M P_1 W d W$
$B_2 = A M P_2 W d W$
$B_3 = A M D_1 W r W d W$

3. Determination of the Mueller matrices of the ref. samples using

Eigenvalue Method:

4. Calculation of the linear mappings

5. Calculation of the matrix

6. Solution of the 2 linear systems:

Main advantages:

- Simplicity:
  - Only 3 measurements allow to find 16 unknown coefficients.
  - No need to model the properties of the optical elements of the PSG and the PSA.

- Robustness:
  - Some parasitic artifacts (multiple reflections, diverging beams) are automatically accounted for.

- Flexibility:
  - The sample calibration set is not unique.
  - Sample calibration set can be optimized to current working conditions.

Conclusions:

- The design of a polarimeter is based on an objective criterion: conditioning optimization in order to minimize error propagation.
- Multiple optical configurations optimizing the condition number can be considered.
- A robust calibration procedure is applied that does not require modeling of the PSG or PSA optical elements. Easy implementation!