

Generalized Bandpass Sampling Receivers for Software Defined Radio

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Abstract

Based on different sampling theorem, for example classic Shannon's sampling theorem and Papoulis' generalized sampling theorem, signals are processed by the sampling devices without loss of information. As an interface between radio receiver front-ends and digital signal processing blocks, sampling devices play a dominant role in digital radio communications.

Under the concept of Software Defined Radio (SDR), radio systems are going through the second evolution that mixes analog, digital and software technologies in modern radio designs. One design goal of SDR is to put the A/D converter as close as possible to the antenna. BandPass Sampling (BPS) enables one to have an interface between the RF or the higher IF signal and the A/D converter, and it might be a solution to SDR. However, three sources of performance degradation present in BPS systems, harmful signal spectral overlapping, noise aliasing and sampling timing jitter, hinder the conventional BPS theory from practical circuit implementations.

In this thesis work, Generalized Quadrature BandPass Sampling (GQBPS) is first invented and comprehensively studied with focus on the noise aliasing problem. GQBPS consists of both BPS and FIR filtering that can use either real or complex coefficients. By well-designed FIR filtering, GQBPS can also perform frequency down-conversion in addition to noise aliasing reduction. GQBPS is a nonuniform sampling method in most cases. With respect to real circuit implementations, uniform sampling is easier to be realized compared to nonuniform sampling. GQBPS has been also extended to Generalized Uniform BandPass Sampling (GUBPS). GUBPS shares the same property of noise aliasing suppression as GQBPS besides that the samples are uniformly spaced. Due to the moving average operation of FIR filtering, the effect of sampling jitter is also reduced to a certain degree in GQBPS and GUBPS. By choosing a suitable sampling rate, harmful signal spectral overlapping can be avoided. Due to the property of quadrature sampling, the "self-image" problem caused by I/Q mismatches is eliminated. Comprehensive theoretical analyses and program simulations on GQBPS and GUBPS have been done based on a general mathematic model. A circuit architecture to implementing GUBPS in Switched-Capacitor circuit technique has been proposed and analyzed. To improve the selectivity at the sampling output, FIR filtering is extended by adding a 1st order complex IIR filter in the implementation.

GQBPS and GUBPS operate in voltage-mode. Besides voltage sampling, BPS can also be realized by charge sampling in current-mode. Most other research groups in this area are focusing on bandpass charge sampling. However, the theoretical analysis shows that our GQBPS and GUBPS in voltage mode is more efficient to suppress noise aliasing as compared to bandpass charge sampling with embedded filtering. The aliasing bands of sampled-data spectrum are always weighted by continuous-frequency factors for bandpass charge sampling with embedded filtering while discrete-frequency factors for GQBPS and GUBPS. The transmission zeros of intrinsic filtering will eliminate the corresponding whole aliasing bands of both signal and noise in GQBPS and GUBPS, while it will only cause notches at a limited set of frequencies in bandpass charge sampling. In addition, charge sampling performs an intrinsic continuous-time sinc function that always includes lowpass filtering. This is a drawback for a bandpass input signal.

Publications list:

- 1. Yi-Ran Sun and Svante Signell, "The Theory of Generalized Bandpass Sampling in Subsampling Receivers", submitted to IEEE Trans. of CAS-I: Regular paper, December 2005.
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- Yi-Ran Sun, "Bandpass Sampling for Radio Receiver", SocTRic demonstrator project, Acreo AB, Norrköping, Sweden, April 2002.

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List of Abbreviations

$2/2.5/3/4~{ m G}$	the second/second and half/third/fourth Generation
AA	Anti-Aliasing
A/D	Analog-to-Digital
AM	Amplitude Modulated
ARS	Additive Random Sampling
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BK	Basis-Kernel
BPF	BandPass Filter
BPS	BandPass Sampling
BS	cellular system Base Station
CDMA	Code Division Multiple Access
CF	Continuous-Frequency
CMOS	Complementary Metal Oxide Semiconductor
CSMA-CA	Carrier Sense Multiple Access/Collision Avoidance
СТ	Continuous-Time
CT-scan	Computerized Tomography-scan
D/A	Digital-to-Analog
DC	Direct Current
DCF	Density Compensation Factor
DCS	Digital Cellular System
DECT	Digital European Cordless Telephone
DF	Discrete-Frequency
DL	Down-Link
DPD	Digital Product Detector
DT	Discrete-Time
DTFT	Discrete-Time Fourier Transform
DFT	Discrete Fourier Transform
DSB	Double-SideBand
DSSS	Direct Sequence Spread Spectrum
EDS	Energy Density Spectrum
EDGE	Enhanced Data rates for Global/GSM Evolution
FHSS	Frequency-Hopping Spread Spectrum

FIR	Finite Impulse Response
FLOP	Floating Point Operation
GSM	Global System for Mobile communications
GQBPS	Generalized Quadrature BandPass Sampling
GUBPS	Generalized Uniform BandPass Sampling
iid	independent, identically distributed
I/Q	In-phase/Quadrature
IEEE	Institute of Electrical and Electronics Engineers
IF	Intermediate Frequency
IIR	Infinite Impulse Response
IRF	Image-Rejection Filter
IS-95	Interim Standard 95
ISI	Inter-Sample Interval
ISM	Industrial, Scientific and Medical
ITU	International Telecommunication Union
JS	Jitter Sampling
LAN	Local Area Network
LNA	Low Noise Amplifier
LO	Local Oscillator
LPF	LowPass Filter
LPS	LowPass Sampling
LSR	Least Square Reconstruction
LTI	Linear Time-Invariant
MRI	Magnetic Resonance Imaging
NB	Narrow Band
NRR	Noise Reduction Ratio
NUS	NonUniform Sampling
OFDM	Orthogonal Frequency Division Multiplexing
opamp	Operational Amplifier
PCS	Personal Communications Service
PDC	Personal Digital Cellular
PDF	Probability Density Function
PSD	Power Spectral Density
\mathbf{RF}	Radio Frequency
RA	Reconstruction Algorithm
\mathbf{SC}	Switched-Capacitor
SDR	Software Defined Radio
S/H	Sample-and-Hold
SNDR	Signal-to-Noise-and-Distortion Ratio
SNR	Signal-to-Noise Ratio
SSB	Single-SideBand
SSB-SC	Single-SideBand Suppressed Carrier
SVD	Singular-Value Decomposition
TDMA	Time Division Multiple Access

UE	User Equipment for cellular terminal
UL	Up-Link
UMTC	Universal Mobile Telecommunication System
US	Uniform Sampling
WB	Wide Band
W-CDMA	Wideband Code-Division Multiple-Access
WSS	Wide-Sense Stationary

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List of Notations

B	The bandwidth of lowpass information signal
f_c	The carrier frequency
f_{in}	The frequency of input signal
$\delta(t)$	The Dirac delta function
$\delta[m-n]$	The Kronecker delta function
f_s, F_s	The sampling rate
T_s	The sampling interval and $T_s = 1/f_s$
$k(t, t_n)$	The Basis Kernel (BK) of Reconstruction Algorithm (RA)
$Re\{\bullet\}$	The real part of complex signal
< a, b >	The inner product operator
$\hat{x}(t)$	The reconstructed result of $x(t)$
i(t), q(t)	The quadrature (I/Q) components of bandpass signal
*	The complex conjugate operator
*	The convolution operator
\in	An element of
$\sigma_{ au}$	The standard deviation of sampling jitter
$E[\bullet]$	The expectation operator
$F\{\bullet\}$	The Fourier transform operator
$Mean[\bullet]$	The mean value
$\lim_{x \to a} f(x)$	The limit of function $f(x)$
$\operatorname{rect}(ullet)$	The rectangular function
$\operatorname{sinc}(ullet)$	The sinc function
$\mu(t)$	The Heaviside's step function
$p(\tau)$	Probability Density Function (PDF)
$p(\tau_n, \tau_m)$	Joint PDF
\otimes	The ideal sampling operator, i.e., the process of multiplying
B_{eff}	The effective bandwidth of noise
$t_s(n)$	The sampling time instants
$x(t_n)$	The sampled-data of NUS
$x(nT_s)$	The sampled-data of US
$r_{xx}(\gamma)$	The autocorrelation function of $x(t)$
$R_{xx}(f)$	The Fourier transform of $r_{xx}(\gamma)$
f_l	The lower frequency of bandpass signal

f_u	The upper frequency of bandpass signal
U[a,b]	Uniform distribution of a random variable
П	The symbol of product
$\overline{\sum}$	The symbol of sum
$\overline{\int}$	The symbol of integral
Ŭ	The symbol of union
	The symbol of norm
\neq	The symbol of inequality
\approx	The symbol of approximately equal
∞	The symbol of infinity
[●], [●]	The symbol of floor and ceiling function

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Chapter 1

Introduction

After the prediction by British scientist J. C. Maxwell in 1860s that electromagnetic waves would travel at the speed of light, people started to knock on the door of wireless communications. Following the successful experiment with wireless telegraphy by Italian scientist G. Marconi in the 1890s, wireless communication systems were established initially consisting of a transmitter and a receiver [1]. Many scientists have made contributions to the development of the systems since then. The development of wireless communication has experienced many milestones for more than 100 years.

In the beginning, receivers were passive consisting only of a tuned bandpass filter to the dominant frequency. Later, amplifiers were introduced such that the receivers could get an amount of gain and compensate for the path loss. The communication systems of the days were completely in the analog domain, i.e. the information signal was processed by bandpass filtering, frequency translation and analog (de)modulation. With the development of digital computers and signal processing, digital techniques were also introduced in communication systems. Today, digital communication still dominates wireless communications [2].

Shannon's sampling theorem from 1948, which arose from the work of Nyquist in 1924 [3], established the mathematical foundations for information transmission in digital communication systems. The sampling theorem [4] states that a signal s(t) of bandwidth W can be reconstructed from samples s(n) taken at the Nyquist rate of 2W samples per second using the interpolation formula

$$\hat{s}(t) = \sum_{n} s\left(\frac{n}{2W}\right) \frac{\sin[2\pi W(t - n/2W)]}{2\pi W(t - n/2W)}$$
(1.1)

The sampled-data signal is further quantized by an Analog-to-Digital (A/D) converter in the receiver to obtain a pure digital signal (a binary signal consisting of bit '1' or '0') or a Digital-to-Analog (D/A) converter in the transmitter to transform the signal back to the analog domain for transmission.

Standard	Mobile Frequency Range (MHz)	Access Method	Frequency Band (MHz)	Carrier Spacing (MHz)	$egin{array}{c} { m Data} \\ { m Rate} \\ { m (Mbps)} \end{array}$
IS-54/-136	UL:824-849 DL:869-894	TDMA	25	0.03	0.048
IS-95	UL:824-849 DL:869-894	CDMA	25	1.25	1.228
GSM	UL:890-915 DL:935-960	TDMA	25	0.2	0.2708
DCS 1800 (EDGE)	UL:1710-1785 DL:1805-1880	TDMA	75	0.2	0.2708
PCS 1900 (EDGE)	UL:1850-1915 DL:1930-1990	TDMA	60	0.2	0.2708
PDC	UL:940-956 DL:810-826	TDMA	16	0.025	0.042
DECT	1880-1900	TDMA	20	1.728	1.152
IEEE 802.11a	5150-5350	CSMA-CA	200	OFDM: 20	6-54
IEEE 802.11b	ISM:2400-2483.5	CSMA-CA	83.5	FHSS:1 DSSS:25	1-2 5.5-11
IEEE 802.11 g	ISM:2400-2483.5	CSMA-CA	83.5	OFDM:20	6-54
$Bluetooth^{TM}$	ISM:2400-2483.5	TDMA	83.5	FHSS:1	1
DCS 1800 (W-CDMA)	UE:1710-1785 BS:1805-1880	CDMA	75	5	3.84
PCS 1900 (W-CDMA)	UE:1850-1915 BS:1930-1990	CDMA	60	5	3.84
UMTS(3G)	UL:1920-1980 DL:2110-2170	CDMA /TDMA	60	5	3.84

Table 1.1: Wireless Communication Standards

Signal transmission in wireless communication systems makes use of electromagnetic waves located at a certain frequency band. One natural challenge in wireless communication is the shared and limited radio spectrum. An international organization, International Telecommunication Union (ITU) [5], was established to allocate frequency bands to the various radio services and to launch new generations of communication standards globally. In the recent 30 years, wireless communications have emerged from the first generation (1G) in the 1980s to the second generation (2G) in the 1990s and the early 2000s, and from the third generation (3G) in the 2000s to the fourth generation (4G) which is still under discussion. The modulation technology emerged correspondingly from analog modulation in 1G to digital modulation in 2G and thereafter. With the rapid growth in the number of wireless subscribers and increasing demand for high data-rate multimedia applications, digital modulation does not use the channel bandwidth efficiently, which motivates the use of channel coding techniques. Information on wireless communication standards in 2G and 3G are contained in Table 1.1.

In the recent past, the concept of multi-mode and multi-standard radio communication has become more and more attractive, especially with the introduction of 4G which promises to integrate different modes of wireless communication – from indoor networks such as wireless LANs and Bluetooth, to cellular phone, radio and TV broadcasting, and to satellite communications. Short distance wireless provides cheap local access, interacting with long distance wireless to realize a seam-

1.1. CONVENTIONAL RECEIVER ARCHITECTURES

less merger. Under the concept of 4G, users of mobile devices can roam freely from one standard to another. A smart design of transceiver (consisting of transmitter and receiver) is necessitated for the mobile terminals. In contrast to the variety of radio frequency (RF) receiver approaches, transmitter designs are relatively plain performing modulation, up-conversion and power amplification. The performance requirement on each building block in RF receivers is more stringent because a weak RF signal is always received in the presence of strong images and interfering signals. More efforts should be paid on the design of radio receiver front-ends from the system architecture and circuit integration perspective.

1.1 Conventional Receiver Architectures

Superheterodyne Receivers

The conventional radio receiver architecture, superheterodyne, has existed for almost one century and was proposed by Edwin H. Armstrong in the 1910s. In the literature there is usually no distinction between heterodyne and superheterodyne architectures. To "heterodyne" means to mix two frequencies and produce a beat frequency defined by either the difference or the sum of the two. To "superheterodyne" is only to produce a beat frequency defined by the difference of the two frequencies.

Two stages of down-conversion (dual-IF, IF stands for intermediate frequency) based on the theme of superheterodyne is mostly used in today's RF receivers, see Figure 1.1. This receiver translates the signal to a low frequency band by two



Figure 1.1: Conventional dual-IF superheterodyne receiver architecture

stages of down-conversion mixing. If the second IF of a dual-IF receiver is equal to zero, the second down-conversion normally separates the signal to I (in-phase) and Q (quadrature) components for Single-SideBand (SSB) communication systems or frequency-/phase-modulated signals, and the corresponding demodulation and detection are performed at baseband, see Figure 1.2. This down-conversion is realized



Figure 1.2: Superheterodyne receiver architecture with the second IF being equal to zero

by quadrature mixers, which have a 90° phase shift between two Local Oscillators (LOs) signals. Any offset from the nominal 90° phase shift and amplitude mismatches between I and Q components will raise the Bit Error Rate (BER). If the second IF is not equal to zero, the receiver becomes a digital-IF receiver. The IF bandpass signal is processed by an A/D converter, and the I/Q mismatches can be avoided by signal processing in the digital domain.

The choice of IFs influences the trade-off between the image rejection (or sensitivity) and channel-selection (or selectivity). If the IF is high, the image band appears far away from the information band such that the image can be easily suppressed by an Image-Reject Filter (IRF). However, the channel selection filter will require a high Q-factor to select a narrow channel at a high IF. On the contrary, if the IF is low, the design of the channel selection filter becomes easier but the image band is so close to the information band that it becomes difficult to achieve a proper image suppression by a BandPass Filter (BPF). More than one stage of down-conversion makes the trade-off easily achievable. In a dual-IF superheterodyne receiver, the first IF is selected high enough to efficiently suppress the image, and the second IF is selected low enough to relax the requirement on the channel selection filter. The selectivity and sensitivity of the superheterodyne makes it a dominant candidate in RF receiver architectures. Unfortunately, the high Q-factors of the discrete-components in the superheterodyne receiver make it difficult to fully integrate the complete front-end on a single chip.

Homodyne Receivers

In a homodyne receiver, no IF stage exists between RF and baseband. The input of the A/D converter is located at baseband, see Figure 1.3. The channel selection filter is just a LowPass Filter (LPF) prior to the A/D converter. The homodyne receiver has two advantages compared to the superheterodyne receiver. First, the architecture is more simple. Second, the image problem can be avoided due to zero IF such that no IRF is needed.



Figure 1.3: Homodyne receiver architecture

The homodyne receiver allows a higher level of integration than the superheterodyne receiver as the number of discrete components is reduced. However, this receiver inevitably suffers from the problems of LO leakage and DC-offset. The output of the LO may leak to the signal input port of the mixer or the Low Noise Amplifier (LNA) due to improper isolation. The leaked signal will be mixed with the output of the LO (i.e., the origin of the leaked signal) and produce a DC component at the output of the mixer. This is called self-mixing. LO leakage to the antenna may result in a time-varying DC offset due to self-mixing. The undesired DC component and DC offset will corrupt the information signal that is present in the baseband. I/Q mismatches are other associated problems due to the quadrature down-conversion in homodyne receivers. Because the down-converted signal is located at zero frequency, flicker noise or 1/f noise of devices will also corrupt the information signal.

IF Receivers

IF receivers combine the advantage of both superheterodyne and homodyne receivers. The received RF signal is down-converted to IF by an LO, where the IF could be either one or two times the information bandwidth (in low-IF receivers) or arbitrary (in wideband-IF receivers) depending on the system specifications in terms of sensitivity and selectivity [6].

Low-IF receivers

In the low-IF receiver, a low IF stage is present between RF and baseband, and the low IF signal is directly digitized in the A/D converter [6, 7], see Figure 1.4. This architecture is also simple and promising for a higher level integration. As



Figure 1.4: Low-IF receiver architecture

compared to homodyne receivers, low-IF receivers have no DC-offset problem since the signal after the first down-conversion is not around DC. The IF is very low (one or two times the information bandwidth), and it is hard to reject the image signal

in the RF BPF. Both image and wanted signal will be sampled and quantized in the A/D converter. Further frequency down-conversion from the low IF to baseband is realized in the digital domain to avoid problems such as I/Q mismatches in the analog domain.

Wideband IF receivers

A superheterodyne receiver with an RF channel-select frequency synthesizer and an IF or baseband channel-select filter is a narrowband receiver. An alternative architecture based on IF receiver architectures is the wideband IF receiver, see Figure 1.5. In the wideband IF receiver, the entire band containing the information



Figure 1.5: Wideband IF receiver architecture with double down-conversion

signal located at RF is translated to IF by multiplying the output of the LO with a fixed frequency. The IF signal passes through an LPF such that the frequency components above the IF band are removed. One channel out of the entire band is then translated to DC by a tunable LO and fed into an LPF [6, 8]. The selected lowpass channel signal is processed further by an A/D converter which is the same as in the superheterodyne and homodyne receivers.

As compared to the low-IF receiver, this receiver architecture with dual conversion possesses the same properties of high sensitivity and selectivity as the conventional superheterodyne receiver. Image signals can be well rejected by filters prior to the A/D converters, and the resolution requirement on the A/D converters can be reduced. However, I/Q mismatches of image-rejection down-conversion mixer in the analog domain can never be avoided. This architecture is also well-suited for full

integration, and it has the potential to be implemented for multi-band multi-mode radio communications.

It is of advantage to use conventional receiver architectures for single-mode narrowband radio communications since the technologies are mature and it is also easy to fulfill the system performance requirements. Nevertheless, driven by the increased processing power of microprocessors and modern high performance A/D converters, a wideband radio architecture has drawn more and more attention for the support of multi-band multi-mode radio communications. From a general aspect, the wideband IF receiver is not a real wideband receiver since the input to the A/D converter is still narrowband.

A generic wideband receiver evolved from the conventional receiver architecture shown in Figure 1.2 has been used in cellular phone base stations to support multiple wireless communication standards and to meet the rapidly increasing demands of cellular services [9]. In this application, multiple channels at the RF are selected by the first tunable LO instead of a single channel in superheterodyne receivers. All the selected channels are translated to baseband by the second LO with a fixed frequency and then digitized in a wideband A/D converter for I and Q components respectively.

1.2 Subsampling Receiver Architecture

The concept of Software Defined Radio (SDR) was originally conceived for military applications. It consists of a single radio receiver to communicate with different types of military radios using different frequency bands and modulation schemes [10]. This concept is starting to be introduced into commercial applications. SDR means a radio where functionality is extensively defined in software, and it supports multi-band multi-mode radio communication. It establishes the second radio evolution since the migration from analog to digital in 1970s and 1980s. Modern radio designs mix analog hardware, digital hardware and software technologies.

Two key issues of SDR are placement of the A/D converters and DSP performance to cope with the large number of samples [11]. One goal of SDR is to place the A/D converter as close as possible to the antenna. The ideal case is doing sampling and digitization directly on an RF signal. Due to the presence of strong interferers around weak RF information signal, an A/D converter with a higher dynamic range up to around 100 dB might be needed. However, it is hard to achieve by current A/D converter technology. It is more reasonable to do an IF digitization as shown in Figure 1.6. The sampling function can be either classic LowPass Sampling (LPS) or BandPass Sampling (BPS). In LPS, the IF bandpass signal is directly sampled and digitized by an A/D converter, and I/Q mismatches are avoided. However, the LPS rate on an IF bandpass signal is high based on Shannon's sampling theorem (see eq. (1.1)), and the performance requirements, e.g., linearity and dynamic range, on the A/D converter are stringent. In BPS with



Figure 1.6: Subsampling receiver architecture by BPS

a low sampling rate that is only slightly larger than twice the information bandwidth, the requirements on the following A/D converter are alleviated. In addition, BPS can realize down-conversion through the intentional signal spectral folding, which is similar as down-conversion mixing.

1.3 Multistandard Receiver Architectures

A multistandard receiver might be realized by stacking different receivers for different standards into a single receiver. However, the area and power consumption would be extremely high. Instead, a well-designed architecture of a multistandard receiver should optimally share the available hardware resources and make use of the tunable and programmable devices. A proper system specification should be defined for each of the involved standards. Moreover, for battery powered devices it is more important that a highly integrated solution is used so that the area and power consumption are considerably reduced.

From the view of multi-band multi-mode radio communications and the placement of the A/D converter, both the homodyne receiver and the subsampling receiver are candidates for SDR implementation because the A/D converter directly has an interface to RF or higher IF signals. From the view of high level integration, the homodyne receiver, low-IF receiver and the subsampling receiver are most suitable.

Multistandard Zero-IF Receivers

A single-chip multimode receiver for four standards, GSM900, DCS1800, PCS1900 and W-CDMA, was designed in zero-IF (or homodyne) [12]. All the problems associated with the homodyne receiver, e.g., LO leakage, DC-offset, I/Q mismatches and flicker noise, inevitably happen and are treated in many different ways. The corresponding block diagram of the receiver is shown in Figure 1.7. The selection



Figure 1.7: Multi-standard receiver architecture by zero-IF

among different standards is realized by an external digital controller and the hardware is shared as much as possible by different standards. Four different standards use two different channel selection filters. A divide-by-two circuit is used to provide quadrature LO signals for the mixers. The LO signal is generated on-chip such that the LO leakage on the PCB is eliminated and the LO leakage to the RF input is better suppressed. The baseband circuit has two operation modes, one for WCDMA and the other for DCS1800/PCS1900/GSM900.

Multistandard Low-IF Receivers

Another fully integrated multistandard radio receiver was designed in low-IF toward mobile terminals that support five wireless communication standards, Bluetooth, two GSM standards (DCS1800 for Europe, PCS1900 for USA), UMTS, 802.11a/b/g [13]. The corresponding block diagram of the radio receiver is shown in Figure 1.8. Bluetooth provides a wireless link between the radio terminal and other peripherals (e.g., headphone), and it should be active all the time, while the other four standards covering five frequency bands are activated by an RF switch.

1.4 Motivations and Thesis Outline

According to the literature study, there is so far no design for fully integrated multistandard subsampling receivers because of the well known noise aliasing problem



Figure 1.8: Multi-standard receiver architecture by low-IF

in the subsampling system [14, 15], although some single chip RF subsampling receivers have been designed for GSM [16], Bluetooth [16, 17] and 802.11b [18, 19]. Bandpass charge sampling with intrinsic FIR moving average operation and a 1st order IIR filter were used to treat the noise aliasing problem in [16, 17]. Direct RF sampling by quadrature BPS in voltage mode was used in [18, 19] without any specific treatment to noise aliasing. A more general solution compared to the work in [16, 17] using quadrature bandpass charge sampling with composite FIR and IIR filtering was proposed in [20, 21, 22]. Both simulation results and circuit implementations on an IF signal have shown that this solution is promising to suppress noise aliasing in subsampling or bandpass sampling receivers. However, it is known that other processing blocks in radio receiver front-ends operate in voltage mode. Before using charge sampling, an analog voltage signal needs to be first converted to a current signal by a transconductance cell, but it is not necessary for using bandpass voltage sampling and the corresponding front-end receiver architecture is simpler.

In this thesis, bandpass voltage sampling is mainly discussed and also compared with bandpass charge sampling. Three sources of performance degradation in bandpass sampling systems, harmful signal spectral overlapping, noise aliasing and sampling timing jitter, are comprehensively studied. With respect to noise aliasing problem, the theory of generalized BPS in voltage mode is proposed, including the examples of Generalized Quadrature BandPass Sampling (GQBPS) and Generalized Uniform BandPass Sampling (GUBPS). GQBPS and GUBPS perform also FIR filtering that can use either real or complex coefficients besides sampling. The input signal of GQBPS and GUBPS is an RF or a higher IF bandpass signal. The interesting folded information band after GQBPS and GUBPS is located at baseband or lower IF. Higher order intrinsic FIR filtering provides immunity to noise aliasing and suppresses sampling jitter effects to a certain degree. Generalized bandpass sampling receivers based on the theory of generalized BPS may be a good candidate for SDR applications. By using Switched-Capacitor (SC) circuit technique, generalized bandpass sampling with intrinsic FIR filtering is designed and analyzed at circuit level. To improve the selectivity at the sampling output, FIR filtering is extended by adding a 1st order complex IIR filter in the GUBPS implementation. It is a strong motivation that a multistandard bandpass sampling receiver will be seen in a near future based on the contribution of the thesis.

This thesis will be organized as following. In chapter 2, Uniform Sampling (US) and NonUniform Sampling (NUS), voltage sampling and charge sampling, deterministic sampling and random sampling are discussed and compared, respectively. With respect to NUS, nine reconstruction algorithms are evaluated and compared in terms of reconstruction performance and computational complexity. In chapter 3, the basic theory of BPS is reviewed regarding the associated technical problems. The state-of-the-art in BPS applications is presented. In chapter 4, Generalized BPS is proposed based on the Papoulis' generalized sampling theorem with focus on suppressing noise aliasing in BPS systems, although it is also noticed that generalized BPS provides certain improvement on the jitter performance. In chapter 5, eight papers included in this thesis are summarized. Finally in chapter 6, the thesis is concluded and future work is proposed.

Through the thesis, all the signals involved in the theoretical analysis are assumed ideal band-limited, which means that their Fourier transforms are zero for |f| > B, although this is not ideally realizable in practice. For a more general purpose, all the frequency variables shown in the thesis use absolute frequency with no specific unit, which means the value of frequency can be scaled up or scaled down by any value of factor without any influence on the conclusions.

Chapter 2

Sampling and Reconstruction

With the launch of digital radio communications, A/D and D/A converters become important devices as the interface between RF conversions and digital signal processing. Natural signals such as speech, music, images and electromagnetic waves, are generally analogue signals in the continuous-time (CT) domain. To process a signal digitally, it has to be represented in digital format in the discrete-time (DT) domain. It is required that this digital format is fixed, and uniquely represents all the features of the original analogue signal. The reconstructed CT signal from this digital format may not be exactly the same as the original analogue signal, but it is a goal to minimize the difference as much as possible.

The two basic operations during A/D conversion are sampling and quantization. Sampling is to convert a CT analogue information signal into a DT representation by measuring the value of the analogue signal at uniform or nonuniform intervals. Quantization is to convert a value or range of values into a digital value. The number of quantization levels determines the resolution of the A/D converter (in bits per sample). In this chapter, Uniform Sampling (US) and Nonuniform Sampling (NUS), voltage sampling and charge sampling, deterministic sampling and random sampling are introduced and compared. A filter generalized by a Reconstruction Algorithm (RA) is proposed and studied in terms of a Basis-Kernel (BK). Nine RAs are evaluated and compared based on their performance and computational complexity.

2.1 Sampling

Nowadays the sampling theorem plays a crucial role in signal processing and communications. Obtaining the DT sequence $x(t_n)$ to represent a CT function x(t) is known as sampling. An ideal sampling process can be modeled as an input CT signal x(t) multiplied by an infinite sequence of Dirac delta functions, see Figure 2.1. The CT sampled-data signal $x_s(t)$ is given by

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Figure 2.1: Ideal sampling process

$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - t_s(n)), \qquad (2.1)$$

where $t_s(n)$ represents the sampling time instants.

Whether the sampled-data signal uniquely represents the original signal or not depends on the sampling patterns and their implementations. Referring to the sampling period (or interval), sampling can be ideally divided into two categories, Uniform Sampling (US) and NonUniform Sampling (NUS). It is justified to assume that the sampling set is uniformly distributed in many applications, i.e., the samples are acquired at the regular time instants. However, in many realistic situations, the data is known only in a irregularly spaced sampled set. This irregularity is a fact of life and prevents the standard methods of Fourier analysis. For example in communication systems, when data from uniformly distributed samples are lost, the obtained samples are generally nonuniformly distributed, the so-called missing data problem. Scratching a CD is also such kind of a problem. On the contrary, it may be of advantage to use NUS patterns for some special cases (e.g., an aliasing-free sampling) [23, 24]. For NUS, there are four general sampling scenarios: Generalized nonuniform sampling [25], Jitter sampling [26], Additive random sampling [23], and Predetermined nonuniform sampling. Without any specifications, the NUS mentioned in this chapter is predetermined and each sampling instant is known with high precision.

Sampling methods in electrical circuits include voltage sampling and charge sampling. Voltage sampling is a conventional method that is realized by the sampleand-hold (S/H) circuit. It tracks an analog signal and stores its value as a voltage across a sampling capacitor for some length of time. Charge sampling does not track the signal voltage but integrates the signal current within a given time window [27]. An analog signal in voltage mode is first converted to current mode by a transconductance cell before charge sampling. As compared to voltage sampling device only relies on the sampling duration but not on the switch-on resistance so that a wide-band sampler design is more feasible [28]. BPS can also be performed by both charge sampling [17, 29, 30] and voltage sampling.

Sampling is ideally deterministic. Under the effects of jitter, deterministic sampling becomes random sampling, including *jitter sampling* (JS) and *additive random sampling* (ARS) [23]. The main property of random sampling is that the sampling time is not predetermined but is defined by the stochastic process of random jitter.

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JS is a common form in real life since the intentional US is generally used. ARS is equivalent to a nominal ideal NUS under the effects of jitter.

2.1.1 Uniform sampling and Nonuniform sampling

Uniform Sampling (US) For an ideal US process, $t_s(n) = nT_s$. Starting from eq. (2.1), the Fourier transform of $x_s(t)$ can be expressed as

$$X_{s}(f) = \int_{-\infty}^{\infty} x_{s}(t)e^{-j2\pi ft}dt$$
$$= \sum_{n=-\infty}^{\infty} x(nT_{s})e^{-j2\pi fnT_{s}}.$$
(2.2)

This is the well-known Discrete-Time Fourier Transform (DTFT). The Discrete Fourier Transform (DFT) is a special case of DTFT, which is defined to be the DTFT evaluated at equally spaced frequencies over the Nyquist interval $[0, 2\pi)$. The N-point DFT of a length M signal is defined as

$$X(k) = \sum_{m=0}^{M-1} x(m) e^{-j2\pi km/N}, \qquad k = 0, 1, \cdots, N$$
(2.3)

under the assumption that the signal x(m) is periodic with period M. By using Poisson summation formula [31], eq. (2.2) can be written as

$$X_s(f) = f_s \sum_{m=-\infty}^{\infty} X(f - mf_s), \qquad (2.4)$$

where $f_s = 1/T_s$. Obviously, the frequency spectrum of a sampled-data signal is a series of copies of the original CT signal and $X_s(f)$ is a periodic function with period f_s , see Figure 2.2.

A band-limited signal can be completely determined by a US sequence with the sampling rate of at least twice the maximum frequency B (critical- or oversampling) according to the Shannon's sampling theorem [32]:

Theorem 1: If a function f(t) contains no frequencies higher than W cps (in cycles per second), it is completely determined by giving its ordinates at a series of points spaced 1/2W seconds apart.

For $f_s < 2B$ (undersampling), the frequency components above B will be aliased back into the Nyquist band $[-f_s/2, f_s/2]$ such that the original signal cannot be uniquely reconstructed from the sampled-data signal. For an LPS process, the input signal is regarded as a baseband signal or lowpass signal with a bandwidth consisting of the maximum frequency component, and critical- or over-sampling



Figure 2.2: a) Original CT band-limited signal, B = 50 and samples by US, $f_s = 200$; b) The corresponding frequency spectrum of CT signal; c) Corresponding frequency spectrum of the sampled-data signal.

is normally used to avoid the harmful signal spectrum aliasing. The input signal of BPS is, however, always a passband signal or bandpass signal such that BPS can make use of a harmless signal spectrum aliasing by tactically selecting the undersampling rate. More discussion about BPS will be presented in chapter 3.

Nonuniform Sampling (NUS) For an ideal NUS process, $t_s(n) = t_n \neq nT_s$. Starting from eq. (2.1), the frequency spectrum of $x_s(t)$ is not necessarily periodic and the Fourier transform becomes

$$X_{s}(f) = \sum_{n=-\infty}^{\infty} x(t_{n})e^{-j2\pi f t_{n}}.$$
 (2.5)

The corresponding Energy Density Spectrum (EDS) of an ideal NUS is the magni-

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tude squared of the Fourier transform [33], i.e.,

$$P_{s}(f) = |X_{s}(f)|^{2}$$

$$= [\dots + x(t_{0}) \cos 2\pi f t_{0} + x(t_{1}) \cos 2\pi f t_{1}$$

$$+ x(t_{2}) \cos 2\pi f t_{2} + \dots + x(t_{n}) \cos 2\pi f t_{n} + \dots]^{2}$$

$$+ [\dots + x(t_{0}) \sin 2\pi f t_{0} + x(t_{1}) \sin 2\pi f t_{1}$$

$$+ x(t_{2}) \sin 2\pi f t_{2} + \dots + x(t_{n}) \sin 2\pi f t_{n} + \dots]^{2}.$$
(2.6)

NUS technique is well used for obtaining oscillograms in oscilloscopes and spectrograms for spectral analysis [34]. In Jerri's tutorial review [24], it was mentioned that unless the signal is ideally band-limited, there will always be an aliasing error when we sample at the required Nyquist rate. So if there is any alias free sampling it must be based on a rate different from the Nyquist rate or in other words sampling at unequally spaced instants of time. The aperiodic property of the frequency spectrum enables NUS to suppress harmful signal spectrum aliasing.

As shown in Figure 2.3, given a wanted signal $s(t) = \cos(2\pi \cdot 2t)$ (solid line) and a feigned signal $i(t) = \cos(2\pi \cdot 3t)$ (dashed line), when $f_s = 5$, the component at f = 3 is larger than $f_s/2 = 2.5$ and will be folded back to f = 2 (see a)). The samples obtained by US can not uniquely determine the wanted signal. By intentionally introducing a random shift with a uniform distribution $U(-\alpha T_s, \alpha T_s)$ (α is a scale factor) on the equidistant US time instants, for instance $\alpha = 0.3$ for b) and 0.7 for c) in Figure 2.3, the aliasing effect is reduced to a certain degree and the wanted signal can be clearly identified. Obviously, NUS relaxes the requirements on the anti-aliasing (AA) filter.

2.1.2 Voltage sampling and Charge sampling

Voltage sampling Ideally, a voltage sampling process can be directly modeled as the input voltage signal sampled by ideal sampling (see Figure 2.1). In real implementations, voltage sampling is simply modeled in an Sample-and-Hold (S/H) circuit as shown in Figure 2.4 together with the appropriate clock scheme. Assume that $t_n = nT_s$ ($f_s = 1/T_s$), τ is the duty cycle of sampling clock ϕ ($0 < \tau < 1$) and the output voltage level in track mode is neglected (or equal to zero), the sampling function is not a series of Dirac delta functions but the convolution of the Dirac delta functions with a pulse shape, normally a zero-order S/H function. The output voltage by S/H is then given by

$$V_{out}(t) = x_s(t) \star h(t), \qquad (2.7)$$

where $x_s(t)$ is the sampled-data signal by ideal voltage sampling that is defined by eq. (2.1) and h(t) is given by

$$h(t) = \begin{cases} 1, & nT_s \le t \le (n+\tau)T_s \\ 0, & \text{otherwise.} \end{cases}$$
(2.8)

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Figure 2.3: US and NUS sequences (left) and corresponding normalized EDS spectrum (right) based on eq. (2.6) (normalized to $\int |X_s(f)|^2 df = 1$) [35].



Figure 2.4: Voltage sampling in Switched-Capacitor (SC) circuit with the appropriate clock scheme.

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The convolution in the time domain is equivalent to the multiplication in the frequency domain and

$$V_{out}(f) = X_s(f)H(f),$$

= $\tau \cdot e^{-j\pi \frac{f}{f_s}\tau} \cdot \operatorname{sinc}(\frac{f}{f_s}\cdot\tau) \sum_{k=-\infty}^{\infty} X(f-kf_s),$ (2.9)

where $X_s(f)$ is given by eq. (2.4).

Charge sampling Charge sampling integrates charge within a time window $[t_n, t_n + \Delta t]$ instead of storing the voltage value across a sampling capacitor. It can be modeled as shown in Figure 2.5, where $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - t_n - \Delta t)$ represents



Figure 2.5: Charge sampling process

an ideal sampling process and t_n represents the set of starting instants to integrate. The sampled-data signal $x_s(t)$ is given by

$$x_s(t) = x'(t)s(t) = \sum_{n=-\infty}^{\infty} \left(\int_{t_n}^{t_n + \Delta t} x(\xi) d\xi \right) \delta(t - t_n - \Delta t).$$
(2.10)

In real implementation, charge sampling can also be modeled as an SC circuit with an input current signal, see Figure 2.6 together with the appropriate clock scheme. Assuming that $t_n = nT_s$ ($f_s = 1/T_s$) and starting from eq. (2.10), the output



Figure 2.6: Charge sampling in SC circuit with the appropriate clock scheme, where ϕ_1 , ϕ_2 and ϕ_o represent the phase of charge integral, reseting and holding/readout, respectively.

voltage of charge sampling and the corresponding Fourier transform spectrum are given as (see Appendix A)

$$V_{out}(t) = \frac{1}{C_L} \left(\int_{t-\Delta t}^t I_{in}(\xi) d\xi \right) \sum_{n=-\infty}^\infty \delta(t-nT_s - \Delta t)$$
(2.11)

and

$$\tilde{V}_{out}(f) = \frac{\Delta t}{C_L \cdot T_s} \sum_{k=-\infty}^{\infty} I_{in}(f - kf_s) \operatorname{sinc}[(f - kf_s)\Delta t] e^{-j\pi(f + kf_s)\Delta t} (2.12)$$

respectively, without considering the zero-order S/H function at the hold phase, where $\operatorname{sin}(x) = \frac{\sin(\pi x)}{(\pi x)}$.

S/H circuit implementations using voltage sampling has been extensively applied in common data acquisition systems (e.g., speech communication, music, image processing). As compared to charge sampling, voltage sampling is easier to be realized with a simple clock scheme and connected to other front-end conventional voltage processing parts. However, with respect to the frequency performance, the 3dB bandwidth is controlled by the width of time window Δt in charge sampling but $\tau = R_{on}C$ (R_{on} is on resistance of sampling switch) in voltage sampling and the accuracy of R_{on} and C is always limited by current silicon technology. Additionally, further down-scaling of CMOS technologies introduces new problems. A reduction of the power supply voltage, for example, might have an influence on the dynamic range as the random variation of noise does not scale down with process technology and supply voltage. The current mode circuits provide a better alternative [36], although many researchers are also working on low-voltage CMOS circuit designs in voltage mode. Both sampling methods can be used in bandpass sampling receivers under the concept of SDR.

2.1.3 Random sampling

All sampling processes discussed above are deterministic and the Fourier transform is normally used to analyze corresponding properties in the frequency domain. A stochastic process $x_s(t)$ by random sampling is not absolutely integrable, i.e., $\int_{-\infty}^{\infty} |x_s(t)| \not\leq \infty$, and the Fourier transform of $x_s(t)$ does not exist [33]. The study of Power Spectral Density (PSD) is a normal way to analyze random sampling.

Jitter sampling (JS) JS is also called jittered periodic sampling [23] which is an ideal periodic sampling affected by timing jitter. The set of sampling time instants is in the form

$$t_s(n) = nT_s + \tau_n, \qquad n = 0, \pm 1, \pm 2, \cdots,$$
 (2.13)

where T_s is an ideal US interval, τ_n are a family of independent, identically distributed (iid) Gaussian random variables with a zero-mean and a standard deviation
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 σ_{τ} . Normally $\sigma_{\tau} \ll T_s$. In [37], timing jitter is classified into readin jitter and readout jitter depending on the way to be introduced in the system. Readin jitter is introduced when the analog signal is being sampled, whereas readout jitter when the samples of the output of the digital filter are being read out for reconstruction back to an analog signal. In the present thesis work, only the case of readin jitters is considered. It is assumed that the effects of jitter are unknown. If they were known, the sampling theory for deterministic NUS could be used [38, 39].

The PSD spectrum of JS is given by (see Appendix B)

$$R_{\tilde{x}\tilde{x}}(f) = \frac{1}{T_s^2} \sum_{k=-\infty}^{\infty} R_{pp}(kf_s) R_{xx}(f-kf_s) + \frac{1}{T_s} R_{xx}(f) \star (1-R_{pp}(f)), \quad (2.14)$$

where \star denotes the convolution operator, $R_{pp}(f)$ is the PSD of the input signal x(t), $R_{pp}(f)$ is the Fourier transform of $r_{pp}(\gamma)$ and $r_{pp}(\gamma)$ represents the Probability Density Function (PDF) of the sum of two independent random jitter processes. It can be observed that the PSD of JS consists of a discrete-frequency (DF) component (first term in eq. (2.14) and a continuous-frequency (CF) component (second term in eq. (2.14), and it is equivalent to the weighted power spectrum of the original signal plus "additive uncorrelated noise". The DF component is a weighted sum of periodically shifted copies of the input spectrum $R_{xx}(f)$ in the period of average sampling rate f_s . $R_{\tilde{x}\tilde{x}}(f)$ is not necessarily a periodic function except when $R_{pp}(kf_s)$ is periodic. When the jitter is small, $R_{pp}(kf_s)$ decreases slowly and the DF component is almost periodic. For a special case where jitter τ_n is zero, $R_{pp}(f) = 1$, eq. (2.14) reduces to the average PSD of US:

$$R_{\tilde{x}\tilde{x}}(f) = \frac{1}{T_s^2} \sum_{k=-\infty}^{\infty} R_{xx}(f - kf_s).$$
(2.15)

The PSD of JS on a sinusoidal input signal with a random phase is shown in Figure 2.7 (dashed line) for different jitter. The corresponding theoretical weights $R_{pp}(f)$ in CF (dash-dot line), $R_{pp}(kf_s)$ in DF (symbol box) and theoretical PSD evaluation based on eq. (2.14) (solid line) are superimposed. The input frequency is 2 and the average sampling rate is 5. The jitter is assumed to have a uniform distribution $U(-\alpha T_s, \alpha T_s)$ where $\alpha = 0, 0.1, 0.3, 0.5$ is a scale factor defined by jitter and $1/T_s$ is the average sampling rate. The simulation result matches with the theoretical estimation very well. When $\alpha = 0$ (or ideal US), image spectra appear at higher order Nyquist bands (2nd order [2.5, 7.5], 3rd order [7.5, 12.5], etc.). The corresponding weight is a flat straight line since the PDF $r_{pp}(\gamma) = \delta(\gamma)$ in the time domain. With the increase of α , the amplitude of image spectra decreases for increasing frequency, and the peak level is shaped by the DF weight factor $R_{pp}(kf_s)$. When α is increased to 0.5, all image spectra in higher order Nyquist bands disappear and the spectrum uniquely identifies the input signal. Wojtiuk [40] highlighted that alias terms can be suppressed by increasing jitter variance, and also showed that a jitter with a uniform distribution over $[-0.5T_s, 0.5T_s]$ has the



Figure 2.7: The PSD of JS on a sinusoid input signal with f = 2 for different jitter and $f_s = 5$ (in dashed line). The CF function $R_{pp}(f)$ (in dash-dot line), the DF weight factor $R_{pp}(kf_s)$ in different order of Nyquist band (symbol box) and the theoretical PSD evaluation based on eq. (2.14) (in solid line) is superimposed. [a] $\alpha = 0$; [b] $\alpha = 0.1T_s$; [c] $\alpha = 0.3T_s$; [d] $\alpha = 0.5T_s$.

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potential to eliminate the discrete frequency components of the sampled-data signal except for the information signal. This simulation result is consistent with the conclusion given by Wojtiuk [40]. The corresponding sampling scenario with $\alpha = 0.5$ is a kind of alias-free sampling. The PSD analysis of JS shown above was obtained independently, although more simulation examples for the jitter with both a Gaussian distribution and a uniform distribution were shown in [41].

Additive random sampling (ARS) Due to the contribution of nT_s , the PSD of JS at $t_s(n) = nT_s + \tau_n$ still retains the periodic property in the period of $1/T_s$ such that the aliasing is still presented in JS for most cases. It is also observed that when jitter has a uniform distribution over $[-0.5T_s, 0.5T_s]$, i.e., the samples of JS get rid of the characteristics of US and distribute completely irregularly, aliasing from higher order Nyquist bands are significantly suppressed.

Shapiro [23] first introduced ARS and noticed that it breaks up the regular property from JS. It was defined that the samples by ARS are located at

$$t_n = t_{n-1} + \gamma_n, \tag{2.16}$$

where t_{n-1} and t_n are two successive sampling time instants, γ_n is an iid stochastic process with a certain distribution. There exists an average T_s such that $E[\gamma_n] = T_s$ but $t_n - t_{n-1} \neq T_s$. The PDF of $\{\gamma_n\}$ is equal to zero (i.e., $p(\gamma_n) = 0$) for $\gamma_n < 0$. This condition corresponds to the requirement that a set of samples in a given set of indices should come in the order of time.

This is equivalent to a nominal ideal NUS under the effects of jitter, since

$$t_s(n) = t_n + \tau_n = t_{n-1} + \gamma_{n-1} + \tau_n = t_{n-1} + \gamma'_n, \qquad (2.17)$$

where $\{t_n\}$ is the set of sampling time instants of nominal ideal NUS, $E[\tau_n] = 0$ and $E[\gamma'_n] = E[\gamma_n] = T_s$.

Alias-free sampling It was discussed in section 2.1.1 that NUS has the potential to suppress harmful spectrum aliasing. However, it is still a challenge to select the nonuniformly distributed sampling scheme such that the input nonideal bandlimited signal is uniquely determined by the samples without the harmful effects of aliasing (i.e., alias-free sampling). Shapiro and Silverman [23], Beutler [42] and Marsy [43] successively gave or extended the definition and conditions for alias-free sampling. Shapiro [23] also showed that some random sampling schemes (e.g., Poisson sampling) can eliminate aliasing and lead to an unambiguous determination of the PSD.

For the given Poisson process $\{\gamma_n\}$, the corresponding Poisson distribution in terms of the average rate ρ is given by [23]

$$p(\gamma) = \rho e^{-\rho\gamma}.\tag{2.18}$$

The same sinusoidal input signal with a random phase that is used for presenting the PSD of JS is also used for simulating the PSD of ARS. The input frequency is 2 and the average sampling rate is 5. The inter-sample intervals (ISI) $\{\gamma_n\}$ satisfy the Poisson process defined by eq. (2.18). The corresponding simulated PSD is shown in Figure 2.8 (*Left*). The set of $\{\gamma_n\}$ used in the simulation has a Poisson distribution, see Figure 2.8 (*Right*). It is observed that only the frequency component of f = 2 in the PSD exists and aliasing effects from other Nyquist bands are completely avoided. However, the noise floor is significantly increased such that SNR is degraded. Compared to Figure 2.7 [d], the in-band noise power by such ARS is higher than that by JS with the jitter distribution $U[-0.5T_s, 0.5T_s]$.



Figure 2.8: (*Left*:) The PSD of ARS with Poisson process. The input frequency is 2 and the average sampling rate is 5. (*Right*:) The practical $p(\gamma)$ (in vertical bar) used for the ARS simulation as compared to the theoretical $p(\gamma)$.

Most JS cases have cyclical jitter errors, i.e., the errors are repeating and correlated to the input signal. By making use of complete random sampling, i.e., alias-free sampling, the correlated jitter errors are turned into uncorrelated errors (or noise). Signal distortions are reduced such that the signal may be easily identified by a signal detector, e.g., human ear. Random sampling may also provide some degree of analog dithering for eliminating the quantization distortions in the following A/D converter [31]. However, it is inevitable that random sampling causes SNR performance degradation and the signal reconstruction becomes hardly achievable.

2.2 Reconstruction

Depending on the context, "reconstruction" has different definitions. Image reconstruction is defined in imaging technology wherein data is gathered through methods such as computerized tomography-scan (CT-scan) and magnetic resonance imaging (MRI), and then reconstructed into viewable images [44]. In analog signal

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processing, reconstruction mostly means that a CT signal is obtained from the DT data by an interpolation filter or some other filtering processes.

Although modern data processing always use a DT version of the original signal, obtained by a certain sampling pattern on a discrete set, reconstruction to a continuous version of sampled data is also needed for some specific applications. In Hi-Fi applications such as digital audio, to maintain high quality in the resulting reconstructed analog signal, a very high quality analog reconstruction filter (postfilter) is required. Reconstruction from one discrete set to another is also useful in the nonfractional sampling rate alternation in digital signal processing. Additionally, in radio receiver front-ends, if the output of the sampling process is not uniformly distributed, a reconstruction process is needed to reconstruct the nonuniform samples to uniform distributed samples prior to the quantizer.

According to Shannon's sampling theorem [4], a band-limited signal can be exactly reconstructed from its samples by US. The perfect reconstruction formula derived by Whittaker [45] for critical uniform sampling is given by

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \operatorname{sinc}[2B(t-nT_s)], \qquad (2.19)$$

where $x(nT_s)$ represents samples at the series of equidistant sample instants, $T_s = 1/(2B)$, and $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{(\pi x)}$. The reconstruction of the input signal is realized by a convolution summation of uniform distributed samples $x(nT_s)$ with a sinc function equivalent to ideal low-pass filtering.

In practice, the CT signal reconstruction is enhanced by first passing the sampleddata signal $x_s(t)$ through a holding circuit with the function of eq. (2.8), and then feeding it into an LPF or other RAs. The reconstruction discussed in the thesis is only realized by a certain RA without any enhancement from the zero-order holding.

For NUS, even if there is a large number of samples, only few of them possess a uniform distribution property with respect to the average sampling rate. The expansion of X(f) does not consist of periodic replicas of the fundamental spectrum. Consequently, the signal cannot be determined uniquely by the samples with only a lowpass filter. Based on the Fourier series expansion, X(f) can be generally expanded as

$$X(f) = \sum_{n = -\infty}^{\infty} c_n e^{-j2\pi f t_s(n)},$$
(2.20)

where $t_s(n)$ is the set of sampling instants either uniformly or nonuniformly distributed. Using inverse Fourier transform, we obtain the general reconstruction formula:

$$x(t) = \int_{-B}^{B} \left(\sum_{n=-\infty}^{\infty} c_n e^{-j2\pi f t_s(n)} \right) e^{j2\pi f t} df$$

$$= 2B \sum_{n=-\infty}^{\infty} c_n \operatorname{sinc}[2B(t-t_s(n))]. \qquad (2.21)$$

For US $t_s(n) = nT_s$, $c_n = x(nT_s)/2B$ and eq. (2.21) is exactly the same as eq. (2.19). However, for NUS, since $t_s(n) = t_n$ and $c_n \neq x(t_n)/2B$ except when $t_n = nT_s$, the reconstruction formula of eq. (2.21) cannot directly represent the original signal x(t) unless c_n is determined. RAs are expected to accurately predict the original signal x(t) from the nonuniform samples $x(t_n)$.

In biomedical image processing, CT-scan and MRI frequently use the NUS pattern in the frequency domain. Four sampling patterns are shown in Figure 2.9. The sampled data of CT-scan and MRI are measured in the Fourier frequency do-



Figure 2.9: Sampling patterns of nonuniform sampling [46]. (*Top-left*): Polar sampling grid; (*Top-right*): Spiral sampling grid; (*Bottom-left*): variable-density nonuniform sampling grid; (*Bottom-right*): general nonuniform sampling grid.

2.3. BASIS-KERNEL (BK)

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main. The RA is needed to derive the Cartesian US grid (see Figure 2.10) from the acquired data prior to the inverse Fourier transform operation. Inspired by the

Figure 2.10: Cartesian uniform sampling grid.

applications in biomedical image processing, some RAs extensively used in image reconstructions are proposed for the applications of radio communications.

However, the reconstruction process in radio communications is different from that in biomedical image processing. In radio communications, both sampling and reconstruction are in the time domain while they are in the frequency domain in image processing. Additionally, in radio communications, the RA can be used to reconstruct a set of unknown data at a regular time set from the NUS sequence. Then the reconstructed result can be directly fed into the following digital signal processing block (e.g., A/D converter). It is also possible to convert the samples by NUS to a CT signal when an analog signal is needed (e.g., in Hi-Fi) in the processing steam, which is different from the reconstruction in image processing.

2.3 Basis-Kernel (BK)

It is known that $\{e^{j2\pi ft_n}\}\$ is a complete basis for X(f) within the bandwidth [-B, B] and that $\{\operatorname{sinc}[2B(t-t_n)]\}\$ forms a complete basis for x(t) in $t \in (-\infty, \infty)$, given in eq. (2.19) and eq. (2.21). In [47], another sampling basis $k(t, t_n)$ which is the unique reciprocal basis of $\{g(t, t_n)\} = \operatorname{sinc}[2B(t-t_n)]\$ was introduced by Higgins. An expression in terms of Kronecker delta function δ_{mn} is given by

$$\langle k(t, t_m), g(t, t_n) \rangle = \delta_{mn}, \qquad (2.22)$$

where $\langle a, b \rangle$ denotes the inner product of a and b which is given by $\langle a, b \rangle = \int_{-\infty}^{\infty} a(t)b(t)dt$. In $t \in (-\infty, \infty)$, $\{k(t, t_n)\}$ is also a complete basis-kernel for x(t).

CHAPTER 2. SAMPLING AND RECONSTRUCTION

Therefore, x(t) can be given either by

$$x(t) = \sum_{n=-\infty}^{\infty} \langle x(\bullet), k(\bullet, t_n) \rangle g(t, t_n)$$

= $2B \sum_{n=-\infty}^{\infty} c_n g(t, t_n)$ (2.23)

in terms of c_n or by

$$x(t) = \sum_{n=-\infty}^{\infty} \langle x(\bullet), g(\bullet, t_n) \rangle k(t, t_n)$$
$$= \sum_{n=-\infty}^{\infty} x(t_n) k(t, t_n)$$
(2.24)

in terms of the nonuniform samples $x(t_n)$. It was also mentioned in [47] that this method is appropriate in the case of L^2 signals only. In other words, the CT function x(t) has to satisfy $\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$ [48], i.e., having finite energy. According to Parseval's equation [31], $\int_{-B}^{B} |X(f)|^2 df < \infty$. This CT function has a finite energy, in other words, this method is only suitable for a band-limited signal.

However, the only complete orthonormal sampling basis for χ are of the form $\{g(t,t_n)\} = \{g(t,nT_s)\}$ (where χ is a subspace of L^2 -space in the time domain). Obviously, Higgins sampling theorem includes the Shannon's sampling theorem as a special case: For US $t_n = nT_s$,

$$k(t, mT_s) = g(t, mT_s) = \operatorname{sinc}[2B(t - mT_s)],$$

$$\langle k(t, mT_s), g(t, nT_s) \rangle = \operatorname{sinc}[2B(n - m)T_s] = \delta_{mn}.$$
(2.25)

A close form of the basis kernel (BK) $k(t, t_n)$ is needed for the reconstruction of NUS, and $k(t, t_n) \neq g(t, t_n)$.

2.4 Reconstruction Algorithms (RAs)

A filter generalized by a certain RA is expected to reconstruct the signal as close as possible to the original from the nonuniformly distributed samples. The selection of the BK $k(t, t_n)$ or the determination of the coefficient c_n determines the reconstruction performance. The reconstruction filter can be in either CT or DT. Based on eq. (2.23) and eq. (2.24), a new sampling paradigm with RAs is proposed as shown in Figure 2.11 and Figure 2.12.

Frequently used RAs for NUS include

- Low-pass Filtering (LPF) [interpolation],
- Lagrange Interpolating Polynomial [interpolation] [47],

2.4. RECONSTRUCTION ALGORITHMS (RAS)



Figure 2.11: Identity elements of (a) interpolation reconstruction with a CT filter; (b) interpolation reconstruction with a DT filter; (c) iterative reconstruction [49]



Figure 2.12: Identity elements of (a) SVD reconstruction with a CT filter; (b) SVD reconstruction with a DT filter.

- Spline Interpolating [*interpolation*],
- Gridding Algorithm [interpolation] [46],
- Least Square Reconstruction (LSR) Algorithm [SVD],
- Iterative Algorithms [*iterative*] [50, 51],
- Yen's Interpolations [*interpolation*] [52],
- Coefficient c_n Determination Reconstruction Algorithm [SVD],

• "Minimum-Energy" Signals [SVD] [52].

These methods can be simply classified into three types: *interpolation*, *iterative* and SVD methods. *Interpolation* is a common way to reconstruction and directly related to eq. (2.24) with a well-defined BK $k(t, t_n)$. *Iterative* methods are extensively used in image processing. They consist of three steps: orthogonal projection, iteration and procedure convergence. SVD is an important element of many numerical matrix algorithms. If the matrix of eigenvectors of a given matrix is not a square matrix, the matrix of eigenvectors has no matrix inverse, and the given matrix does not have an eigen decomposition. The standard definition for the matrix inverse fails. By SVD, it is possible to obtain a pseudoinverse which is defined as

$$A^{-1} = (A^*A)^{-1}A^* = VDU^T, (2.26)$$

where $A = UDV^T$ is a given $m \times n$ real matrix, U and V are $m \times m$ and $n \times n$ unitary matrices (i.e., $U^* = U^{-1}$, $V^* = V^{-1}$), D is a $m \times n$ diagonal matrix and the elements in the diagonal consist of the singular values of A and zeros, $\{\bullet\}^T$ denotes the matrix transpose operator. All the RAs involving matrix inverse operations are classified within the family of SVD methods, e.g., LSR algorithm and coefficient c_n determination. Four BKs which are based on *interpolation* are shown and compared in Figure 2.13. It is observed that they are symmetric around the origin for US but asymmetric for NUS.



Figure 2.13: The basis-kernels of interpolations in time domain.

2.5. PERFORMANCE EVALUATION OF RAS

2.5 Performance Evaluation of RAs

A reconstruction error curve e(t) can characterize the reconstruction performance, where

$$e(t) = x(t) - \hat{x}(t), \qquad (2.27)$$

x(t) and $\hat{x}(t)$ are the original input signal and the reconstructed signal, respectively. The Signal-to-Noise-and-Distortion-Ratio (SNDR) is normally used to numerically evaluate the accuracy of reconstruction which is defined as [53]

$$SNDR = \frac{\sum_{i=1}^{L} x_i^2}{\sum_{i=1}^{L} (x_i - \hat{x}_i)^2},$$
(2.28)

where i = [1, L] denotes the evaluated points, normally L > N, x_i and \hat{x}_i represent the points from the original and reconstructed signal, respectively. The SNDR in dB is evaluated for the reconstruction performance of sampled and interpolated points of NUS respectively by different RAs, see Table 2.1.

Algorithm	Nonuniform sampling				
	Sampled points	Interpolated poin ts			
Lowpass filtering (LPF)	17.33	17.97			
Lagrange interpolating polynomial	∞	∞^\dagger			
Spline interpolation	∞^{\dagger}	69.22			
Gridding algorithm	18.49	19.22			
LSR algorithm	∞^{\dagger}	39.54			
Iterative algorithm	18.63	13.89			
Yen's interpolation	∞^{\dagger}	40.22			
c_n determination	∞^{\dagger}	39.54			
"Minimum-energy" signals	∞^{\dagger}	37.30			

Table 2.1: SNDR (in dB) comparison of different algorithms (N=34, L=201) for the NUS pattern shown in Figure 2.14

[†] It is a reasonable assumption that SNDR is approximated by ∞ when SNDR> 100.

The computational complexity of a reconstruction filter depends on the accuracy requirement of the simulation model. For the interpolation reconstruction filter, the approximation error between x(t) and $\hat{x}(t)$ can be decreased by increasing the order of the filters or the degree of the interpolation. For the iterative reconstruction, increasing the number of iterations is also helpful for reducing the error when the repeating procedure is convergent. All the RAs are divided into two groups, *sinc-based* and *nonsinc-based*, and the number of floating point operations (FLOP) is evaluated by using Matlab 5.3 and compared in Table 2.2 for different RAs maintaining the performances shown in Table 2.1.

Algorithm	Sinc-based	Nonsinc-based
LPF	$55,\!141$	_
Lagrange	—	$697,\!072$
Spline	_	8.088×10^9
Gridding	64,307	_
LSR	_	$1,\!131,\!518$
Iterative	1,160,311	_
Yen's	_	$1,\!376,\!353$
"Minimum-energy"	610,676	
Coefficient	996,532	

Table 2.2: Computational complexity comparison of different algorithms by evaluating the number of FLOP (floating point operations) [49]



Figure 2.14: Sample distributions by US (*Top*) and NUS (*Bottom*), "+" shows the sampling location and " \circ " the sampled value.

It is noticed that *Lagrange interpolating polynomial* shows the best reconstruction performance for both nonuniformly sampled points and interpolated points.

2.6. RAS IMPLEMENTATION

However, it is observed from eq. (2.24) that the input samples by NUS intercept the interpolating filter impulse response at different time instants. This implies that it has a time-varying characteristic. Currently, Lagrange fractional delay filtering [54, 55, 56] and time-invariant filterbank [57] with synthesis filters generalized to Lagrange interpolating polynomial are two feasible methods for the implementation of Lagrange interpolating polynomial. *Lowpass filtering* using sinc kernel cannot pick up the correct values at the nonuniformly sampled points, and have bad reconstruction performance at both sampled points and interpolated points, although it provides good reconstruction performance for US except at the ends due to the truncation error of the sinc function [49]. *Yen's interpolation* is a modified interpolation function based on LPF [52], and it does improve the reconstruction performance of NUS to a certain degree. *Spline* has also a rather good reconstruction performance of NUS, although it is the most expensive technique. Repeating the procedure many times causes a large number of FLOP for the iterative algorithm.

Those RAs (i.e., gridding and iterative algorithm) which are extensively used in image processing cannot immediately be used for radio communications. One important difference between radio communication and image processing is that the former always requires real-time processing but the latter does not. These RAs together with those based on SVD (i.e., LSR algorithm, coefficient determination and "minimum-energy" signals) have to be applied to blocks of data while the other methods (i.e., Lagrange interpolating polynomial, spline interpolation, Yen's interpolation) can be applied on a sample-by-sample basis.

2.6 RAs Implementation

In general, filters can be implemented with passive components, active components or switched-capacitor (SC) circuit [58].

- **Passive filters** consist of only resistors, capacitors and inductors without amplifying elements e.g., transistors, operational amplifiers (opamps), etc. Passive filters are very simple and do not need power supplies. They can be used at very high frequency since they are not limited by the finite bandwidth of opamps. Passive filters do not provide any signal gain.
- Active filters use opamps with resistors and capacitors in their feedback loops. They have high input impedance, low output impedance and arbitrary gains. Moreover, they avoid use of bulky and nonlinear inductors. The performance at high frequency is limited by the gain-bandwidth products of opamps.
- Switched-capacitor filters are sampled-data filters consisting of only transistor switches and capacitors along with clocks. The operation of SC filters is based on the ability of on-chip capacitors and MOS switches to simulate resistors [59]. SC filters provide signal gain but also introduce more noise,

both clock feed through and thermal noise. Obviously, SC filters are DT filters which is different from passive and active filters. A CT anti-aliasing filter is needed prior to the SC filters.

Due to the discrete-time property of sampled-data signal, the SC filter is a well-suited candidate for the implementation of RAs.

Chapter 3

Classic Bandpass Sampling

Signals can be categorized as lowpass versus bandpass in terms of the center frequency. The lowpass information signal is carrier-modulated in the transmitter such that the transmitted signal is centered at a non-zero frequency, i.e., a bandpass signal. In the transmission of an information signal over a communication channel, bandpass signals are always encountered. The demodulation at the receiver recovers the information signal from the bandpass signal through frequency down-conversion. The carrier-modulated bandpass signal can be represented either as a Double-SideBand (DSB) signal or a Single-SideBand (SSB) signal depending on the definition of the equivalent lowpass information signal [60]. A DSB signal requires twice the channel bandwidth of the equivalent lowpass signal for transmission. For saving transmission bandwidth, an SSB signal is generally used in radio communications, at the cost of a more complicated modulation approach and complex signal processing. A Single-SideBand Suppressed Carrier (SSB-SC) Amplitude Modulated (AM) bandpass signal is expressed as

$$y(t) = Re\{a(t)e^{j2\pi f_c t}\} = i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t),$$
(3.1)

where f_c is the carrier frequency, a(t) is the equivalent complex lowpass signal and given by

$$a(t) = i(t) + jq(t),$$
 (3.2)

i(t), q(t) are called the quadrature (I/Q) components of the bandpass signal, and q(t) is the Hilbert transform of i(t) [60]. The Fourier transforms of the equivalent complex lowpass signal a(t) and its complex conjugate $a^*(t)$ are illustrated in Figure 3.1 a)-d), where I(f) and Q(f) are the Fourier transform of i(t) and q(t), respectively. The spectrum of the corresponding bandpass signal y(t) is shown in Figure 3.1 e).

The bandpass signal defined by eq. (3.1) could be sampled by using either Low-Pass Sampling (LPS) or BandPass Sampling (BPS). Assume that the bandwidth of the equivalent lowpass information signal a(t) is B and $B \ll f_c$. Based on Shannon's sampling theorem, $f_s \ge 2(f_c + B)$ for LPS. BPS is a technique for undersam-



Figure 3.1: Illustration of SSB signal spectra (a-d) and SSB-SC AM bandpass signal spectrum (e) based on the definition in eq. (3.1).

pling a bandpass signal to realize frequency down conversion through intentional aliasing with the sampling rate of being slightly larger than twice the information bandwidth, i.e., $F_s \ge 2B$. Obviously $f_s >> F_s$. An example of sampled-data signal spectrum by LPS and BPS is shown in Figure 3.2. The randomly generated SSB-SC AM bandpass signal is sampled by LPS ($f_s = 2.5(f_c + B) = 1375$) and BPS ($F_s = 2.5B = 125$), respectively, where $f_c = 500$ and B = 50. To avoid harmful signal spectral folding, the minimum sampling rate 2B is not used. Obviously, the output signal spectrum of LPS is a periodic replica of the original modulated bandpass signal spectrum with period of f_s , while the output spectrum of BPS is equivalent to a periodic replica of the equivalent lowpass signal spectrum in the period of F_s , see Figure 3.2 b) and c). The sampled-data signal at the output by BPS is located at a lower frequency, centered at ± 25 in Figure 3.2 c). The BPS technique shrinks the fundamental Nyquist interval $[-f_s/2, f_s/2] = [-687.5, 687.5]$ based on the first Nyquist criterion to the new narrow one $[-F_s/2, F_s/2] = [-62.5, 62.5]$ and realizes a frequency down-conversion at the same time. Different orders of Nyquist bands can be selected and filtered out after BPS in consideration of different implementations [61, 62]. With a goal of low power consumption, BPS becomes more attractive to mixed-signal system design. It has been extensively studied in optics, radar, sonar, communications and general instrumentation, etc.

As discussed in chapter 1, the concept of Software Defined Radio (SDR) has gained more and more attention for its support of multi-mode wideband radio



Figure 3.2: a) Original SSB-SC AM bandpass signal with a bandwidth of B = 50 and $f_c = 500$; b) The corresponding frequency spectrum of sampled-data signal by LPS, $f_s = 2.5(f_c + B)$; c) The corresponding frequency spectrum of sampled-data signal by BPS, $F_s = 2.5B$. Symbol "." and "×" in a) represents the samples from LPS and BPS, respectively.

communications. One key technology of SDR is the placement and design technique of A/D converter in the channel processing stream, and the goal is to put the A/D converter as close as possible to the antenna. By using conventional LPS, the sampling rate would be too high to be realizable by current design technology. BPS may be a solution for SDR by using a much lower sampling rate, see Figure 1.6.

Besides the advantage of lower sampling rate, BPS has also limitations in real implementations. The BPS rate has to be carefully chosen in order to avoid harmful signal spectral aliasing [63]. Noise aliasing is a direct consequence of lower sampling rate as compared to the highest frequency component of the input bandpass signal [15, 64]. The input signal frequency of BPS is still high even though the sampling rate is low. It was shown that the jitter effects depend on both the variance of the random jitter and the input frequency. The performance at the output of BPS is degraded as compared to the equivalent LPS system, an ideal image-rejecting mixer followed by an ideal lowpass sampler [15, 64]. Finally before this chapter is concluded, some existing implementation examples using BPS will be shown and

compared with regards to the noise aliasing performance.

3.1 Sampling Rate Selection

The classic bandpass sampling theory states that for Uniform Sampling (US) the signal can be reconstructed if the sampling rate is at least twice the information bandwidth, i.e., $F_s^{min} = 2f_u/n$, where $n = \lfloor f_u/B \rfloor$ denoting the largest integer less than or equal to f_u/B , f_u is the upper boundary frequency of the bandpass signal [14]. Feldman & Bennett [65] and Kohlenberg [66] also showed that for US, the minimum BPS rate is only valid for integer band position [14, 67], i.e., $\lfloor f_u/B \rfloor = f_u/B$. The sampling rate selection of BPS depends significantly on the band position which represents the distance between DC and the information band.

Assume that a real SSB bandpass signal is located at $[-f_u, -f_l] \cup [f_l, f_u]$ with a fractional band position (i.e., it is not necessary that $\lfloor f_u/B \rfloor = f_u/B$), where f_l and f_u are given by $f_l = f_0 - B/2$, $f_u = f_0 + B/2$, B is the equivalent lowpass information bandwidth, and f_0 is the center frequency of the information band (see Figure 3.3). By using BPS with the sampling rate of F_s , there exist many Nyquist bands represented by dashed-line triangles in Figure 3.3. The value of x and $F_s/2$ determines the center frequency of the corresponding sampled-data spectrum at the output. Then we can easily get

$$(f_0 - x) - (n - 1)\frac{F_s}{2} = 0,$$

$$F_s = \frac{2(f_0 - x)}{n - 1}.$$
 (3.3)

To avoid harmful signal spectral folding, the following constraints should be satisfied:

$$\left\{ \begin{array}{ll} x > B/2 & (3.4.\mathrm{a}) \\ \frac{B}{2} + x < \frac{F_{\mathrm{s}}}{2} & (3.4.\mathrm{b}) \end{array} \right.$$

Substituting eq. (3.4.a) and eq. (3.4.b) into eq. (3.3), the acceptable sampling rate should be in the range of

$$\frac{2f_u}{n} < F_s < \frac{2f_l}{n-1}.$$
(3.4)

This is consistent with the conclusions presented in [14, 68]. For a special case when $x = F_s/2$, the process of BPS is equivalent to homodyne or direct down-conversion. Starting from eq. (3.3), the corresponding sampling rate is given by

$$F_s = \frac{2f_0}{n}.\tag{3.5}$$

This special case causes loss of information for a non-symmetric SSB signal since the spectra down-converted from the positive and negative frequency band are folded

3.2. NOISE SPECTRUM ALIASING



Figure 3.3: An example of SSB bandpass signal with a fractional band position located at $[f_l, f_u] \cup [-f_u, -f_l]$

over at DC and can never be separated later. This sampling rate is only suitable for sampling a symmetric DSB signal, e.g. a sinusoidal signal or a carrier signal modulated by a real lowpass signal.

Based on eq. (3.4), the constraints of acceptable sampling rates for SSB signals can be depicted graphically by Figure 3.4 [14, 65]. The sampling rates indicated by the dashed line within the restricted area in Figure 3.4 are defined by eq. (3.5) [69]. However, it is difficult to adjust the BPS rate exactly to be equal to one value. Any small sampling rate variation will cause F_s move into the disallowed area, resulting in an incorrect folding of the signal spectrum.

It is also observed from Figure 3.4 that the set of allowable BPS rates consists of n disconnected segments within $[2B, \infty)$. To do sampling efficiently, a lower sampling rate is more attractive. With the increase of f_u/B , n is increased and consequently the gap between any two disallowed segments in the area of lower F_s becomes narrower and narrower. Even a small error in the sampling rate might cause F_s to fall into a disallowed area. Selection of sampling rate becomes more and more difficult.

3.2 Noise Spectrum Aliasing

It is known that a resistor charging a capacitor gives rise to a total thermal noise with power kT/C [70], where k is Boltzmann constant, T is the absolute temperature and C is the capacitance. The on-resistance of the switch will introduce thermal noise at the output. The noise is stored on the capacitor along with the instantaneous value of the input voltage when the switch turns off. As shown in Figure 3.5, the resistor R_{on} and sampling capacitor C construct an LPF with a transfer function of

$$H(f) = \frac{1}{1 + j2\pi f R_{on}C},$$
(3.6)



Figure 3.4: The allowed and the disallowed (shaded area) uniform sampling rates versus the band position, F_s is BPS rate, B is the bandwidth, and the information band is located at $[f_l, f_u] \cup [-f_u, -f_l]$ [14].

with the 3dB bandwidth of $f_{3dB} = 1/(2\pi R_{on}C)$. Thermal noise is known as Additive White Gaussian Noise (AWGN) in communication theory, i.e. having a delta-function autocorrelation with a flat Power Spectral Density (PSD). The PSD of thermal noise introduced by the resistor R_{on} can be given as $S_{in}(f) = 4kTR_{on}$ with a one-sided representation, or $S_{in}(f) = 2kTR_{on}$ with a two-sided representation. The corresponding noise PSD at the output of LPF is given by

$$S_{out}(f) = S_{in}(f)|H(f)|^2 = 2kTR_{on}\frac{1}{1+4\pi^2 f^2 R_{on}^2 C^2}$$
(3.7)

by a two-sided representation, and the total noise power is obtained as

$$P_{out} = \int_{-\infty}^{\infty} S_{out}(f) df = \frac{kT}{C}.$$
(3.8)

For modeling purposes, the output noise of LPF performed by the RC network can be made equivalent to AWGN with a constant PSD within an effective noise bandwidth B_{eff} . Both noise sources share the same noise power kT/C, see Figure 3.5 (b), i.e.

$$P_{out} = \frac{kT}{C} = 2kTR_{on} \cdot (2B_{eff}), \qquad (3.9)$$

and

$$B_{eff} = \frac{1}{4R_{on}C} = \frac{\pi}{2}f_{3dB}.$$
(3.10)

3.2. NOISE SPECTRUM ALIASING



Figure 3.5: (a) A switched-capacitor (SC) sampling device. (b) Illustration of the effective noise bandwidth, where $B_{eff} = \frac{\pi}{2} f_{3dB}$.

The effective noise bandwidth of the sampling device B_{eff} depends on the onresistance in the switch and the sampling capacitance, and it is normally larger than the maximum frequency of the input signal. Besides the capacitor switching noise (kT/C noise), opamp wide-band noise and opamp 1/f noise are two other less dominant noise sources in practical SC circuits [71]. To simplify the following analysis, the capacitor switching noise is regarded as the only noise source in the sampling device, and the RC-filtered white noise is replaced by band-limited white noise with the same power within an effective bandwidth B_{eff} .

By using BPS, the wideband kT/C noise will be folded due to the effect of subsampling such that the resulting SNR by BPS is lower than the equivalent LPS system (i.e. an ideal image reject mixer followed by LPS) in the presence of the same noise source, see Figure 3.6. The SNR degradation in dB is given as [15]

$$SNR_{deg} \approx 10 \log_{10} \frac{B_{eff}}{B} \cdot \frac{B}{F_s/2} = 10 \log_{10} \frac{2B_{eff}}{F_s}.$$
 (3.11)

An anti-aliasing filter (either LPF or BPF) is normally used to reduce the out-ofband noise prior to the sampler.



Figure 3.6: Illustration of (a) BPS system and (b) the equivalent LPS system.

The effect of noise aliasing can be graphically interpreted by the PSD. It is known that an ideal uniform BPS is identical to an ideal uniform LPS followed by a decimation operation [64] (see Figure 3.7) provided that the BPS rate $F_s \geq 2B$



Figure 3.7: Bandpass sampling equivalence

and the LPS rate $f_s \geq 2(f_c + B)$ according to the Nyquist rate, where M is the decimation factor and $M = f_s/F_s$. To avoid the noise aliasing in LPS, it is assumed that $f_s \geq 2B_{eff}$, although the thermal noise with a constant PSD of $2kTR_{on}$ is still present. The PSD of noise by LPS in the fundamental Nyquist band is illustrated in Figure 3.8 (a). Assume that the minimum sampling rate 2B is used for BPS



Figure 3.8: Illustration of noise aliasing. (a) The PSD of noise by LPS in the fundamental Nyquist band; (b) The PSD of noise by BPS, where the shaded area represents the fundamental positive Nyquist band.

and B_{eff} is an integer multiple M of B. By doing an M-fold decimation on the output of LPS, the sampling rate will be reduced to the rate of BPS. The PSD of the decimation output $Y_d(f/f_s)$ in terms of the input PSD $Y(f/f_s)$ is given by [72]

$$Y_d(f/f_s) = \frac{1}{M} \sum_{k=0}^{M-1} Y((f/f_s - k)/M), \qquad (3.12)$$

3.3. JITTER EFFECT

where

$$Y(f) = \begin{cases} 2kTR_{on}, & -B_{eff} \le f \le B_{eff} \\ 0, & \text{otherwise.} \end{cases}$$
(3.13)

The decimation operation is equivalent to first stretching $Y(f/f_s)$ by a factor M to obtain $Y(f/(M \cdot f_s))$, then create M-1 copies of this stretched version by shifting it uniformly in successive amounts of 1, and finally add all these shifted stretched versions to the unshifted stretched version $Y(f/f_s)$ and then divided by M. The final PSD after the M-fold decimation is illustrated in Figure 3.8 (b). The PSD of noise from $-F_s/2$ to $F_s/2$ is increased by M such that the output SNR by BPS is degraded as compared to the equivalent LPS system shown in Figure 3.6 (b).

The effects of noise aliasing was also demonstrated by simulations using the MATLAB *psd* function, see Figure 3 in attached **Paper III**. If the sampled-data signal of LPS is first fed into a BPF and then decimated, the out-of-band noise is suppressed by the BPF and hence SNR is increased as compared to that of BPS. Note that when using BPS, the out-of-band noise cannot be suppressed by a discrete filter. Some special considerations to the noise aliasing is needed in the process of sampling. Regarding the noise aliasing problem in BPS systems, a new sampling concept, generalized bandpass sampling is proposed specially for treating the noise aliasing problem and will be discussed in chapter 4.

3.3 Jitter Effect

The intention of sampling systems is to obtain a sample value at the corresponding time instant for an input signal. Based on sampling theories, it is expected to uniquely determine the input signal by the sampled data information. The effects of random errors on the nominal sampling time instant are commonly called timing jitter. As shown in Figure 3.9, the random error τ_n which is a time offset from the nominal time instant t_n causes a random error $\varepsilon_{\tau}(n)$ in the amplitude. The effect



Figure 3.9: Illustration of jitter on time and amplitude

of jitter on the spectrum of the signal may give rise to new discrete components and produce frequency selective attenuation [26].

For a sinusoidal input signal $y(t) = A \sin(2\pi f_{in}t)$, the theoretical Signal-to-Noise-and-Distortion ratio (SNDR) in the presence of jitter is given by [15]

$$SNDR = \frac{A^2/2}{\overline{N}_{\tau}} = \begin{cases} 1/(4\pi^2 f_{in}^2 \sigma_{\tau}^2), & 2\pi f_{in} \sigma_{\tau} << 1\\ 1/[2(1-e^{-2\pi^2 f_{in}^2 \sigma_{\tau}^2})], & \text{otherwise}, \end{cases}$$
(3.14)

where \overline{N}_{τ} is the average noise power, σ_{τ} is the standard deviation of jitter and normally defined as a percentage of the sampling period T_s . Moreover, the expression of SNDR for $2\pi f_{in}\sigma_{\tau} \ll 1$ is valid for all jitter distributions while the other SNDR expression only applies to a random jitter with Gaussian distribution $N(0, \sigma_{\tau})$. Note that SNDR depends on both the standard deviation of random jitter σ_{τ} and the input frequency f_{in} , but not the sampling frequency. Small jitter noise can be approximately regarded as sampled AWGN. For large jitter, this assumption is not valid anymore. For the same sampling rate, the input frequency of BPS is normally larger than that in the equivalent LPS system. The noise power of BPS corresponding to the same standard deviation of jitter σ_{τ} is normally larger than for LPS.

Jitter effects for a general input signal was discussed in [73, 74]. The time skew problem in A/D converter system is very similar to the jitter problem and was analyzed and compared in [74].

3.4 Overview of Studies on Bandpass Sampling

BPS theory does not only stand by the theoretical analysis but has also been implemented and measured in various communication systems, e.g., the subsampling mixers [17, 19, 20, 30, 75, 76, 77] and in $\Delta\Sigma$ modulators [78]. Switched-capacitor (SC) circuits are often used in these designs based on either voltage-mode or current-mode sampling.

Conceptual subsampling mixer

1. Starting from the basic concept of a subsampling mixer, Eriksson and Tenhunen [79] performed noise figure analysis and measurement on a conventional subsampling mixer (see Figure 3.10) consisting of only one sampling switch, one sampling capacitor and one output buffer. The measurement result showed that the noise figure of the subsampling mixer can be described by a model containing contributions of aliased thermal noise and the noise from buffer amplifier. The SNR performance of the mixer depends significantly on the sampling rate, which is consistent with the theoretical analysis presented here.

2. Following the basic concept of a subsampling mixer, Pekau and Haslett [77] proposed an alternative implementation of a differential subsampling mixer. Fig-



Figure 3.10: Sampling mixer conceptual model

ure 3.11 only shows single ended version of the circuit. The input signal of the mixer is at an RF frequency of 2.4 GHz, the sampling rate is 100 MHz and the output frequency is 20 MHz. The subsampling mixer is combined with a low-noise amplifier (LNA) such that the signal before sampling has a gain. The parallel resonant LC load of the proposed sampler filters out some of the thermal noise due to the resistance of the signal noise R_{in} . The mixer has better noise figure performance as compared to the work in [19, 20, 79]. However, it doesn't provide any immunity to the problem of noise aliasing.



Figure 3.11: A subsampling mixer incorporating with an LAN

Continuous-time bandpass $\Sigma\Delta$ modulator

 $\Sigma\Delta$ A/D converters have been extensively studied because they are robust against circuit imperfections, have inherent linearity due to the use of single-bit quantizers and can achieve high resolution. It is known that $\Sigma\Delta$ modulators provide noise shaping on the quantization noise through the feedback loop such that the SNR performance at the output is dramatically increased as compared to not using a $\Sigma\Delta$ modulator. Making use of the feedback loop, a $\Sigma\Delta$ modulator provides noise shaping to both the quantization noise and noise aliasing due to subsampling. Hussein [78] proposed a continuous-time (CT) bandpass $\Sigma\Delta$ modulator with a second order bandpass filter. The block diagram of the proposed modulator is shown in Figure 3.12. The center frequency f_{in} is tuned to $(N - 1/4)f_s$ and f_s is



Figure 3.12: Block diagram of a CT bandpass $\Sigma\Delta$ modulator

the subsampling rate with respect to f_{in} . Note that subsampling exists inside the loop such that noise aliasing is attenuated by the CT bandpass loop filter. However, the effective Q-factor of the CT bandpass filter is reduced after the subsampling operation, see eq. (11) in [80]. The requirements on the CT bandpass filter can be very demanding in this solution.

Charge sampling with FIR/IIR filtering

The previous BPS implementations are all based on voltage mode sampling. It has been analyzed that the wideband thermal noise introduced in the sampling device will be folded in the fundamental Nyquist band by voltage mode BPS. As compared to voltage sampling, charge sampling has the unique property that the different Nyquist bands of the sampled data spectrum are weighted by an sinc function $\operatorname{sinc}[(f - kf_s)\Delta t]$, see eq. (A.4) in Appendix A, where f_s is the sampling rate, Δt is the duration to charge integration and k represents different order of Nyquist bands. Obviously such sinc weighting factor provides zeros or notches at $f = N/\Delta t + kf_s$ (N is an arbitrary integer) and improves the SNR performance at the sampling output. BPS in current mode or bandpass charge sampling is realized by integrating the input current at a high rate while sampling or reading-out the output of the integral at a low rate.

1. The simple intrinsic notch filter by the sinc function is not sufficient enough to compensate for the effect from noise aliasing. A bandpass charge sampling combined with an Nth order FIR moving average filter and a 1st order IIR filter (the so called direct sampling mixer) was proposed in a discrete-time (DT) Bluetooth receiver [17, 30]. The corresponding schematic and the appropriate clock scheme are shown in Figure 3.13. The charge sharing among C_H and C_R realizes a single-



Figure 3.13: a) Direct sampling mixer with intrinsic 1st order IIR filtering and b) the appropriate clock scheme.

pole recursive IIR operation. The combination of the sinc function, the FIR moving average filter, and the IIR operation provides a certain noise aliasing suppression.

2. Karvonen *et al.* also noticed the advantage of using the intrinsic CT sinc function in charge sampling and proposed a quadrature charge-domain sampler with embedded FIR and IIR filtering functions [22]. The composite filtering of the CT built-in sinc function in charge sampling and the embedded FIR, IIR filtering functions integrated into the sampling process would improve the noise aliasing suppression. The principles of the proposed charge domain FIR and IIR sampler are shown in Figure 3.14. The function of DT FIR filtering is realized by the multiple accumu-



Figure 3.14: The principle of quadrature charge-domain FIR sampler (*Left*) and IIR sampler (*Right*) [22].

lation of charge into the sampling capacitors. Different integration of input current results in different sequences of weights that correspond to the filtering coefficients. Starting from eq. (A.4), the frequency spectrum of sampled-data signal by charge sampling combined with FIR filtering can be expressed as (see Appendix A)

$$\tilde{V}_{out}'(f) = \frac{\Delta t}{C_L \cdot t} e^{-j2\pi f/f_s} \sum_{k=-\infty}^{\infty} H(f - kf_s) \tilde{I}_{in}(f - kf_s) \text{sinc}[(f - kf_s)\Delta t], \quad (3.15)$$

3.4. OVERVIEW OF STUDIES ON BANDPASS SAMPLING

where the transfer function of FIR filtering is given by

$$H(f - kf_s) = \sum_{n=0}^{N} h_n e^{-j\pi(f - kf_s)(2n-1)\Delta t}$$
(3.16)

and h_n is the filtering coefficient. To improve the selectivity, the charge-domain FIR sampler can be further combined with a 1st order IIR filter by simply adding one additional shared switched capacitor into the feedback loop of the integrator, see Figure 3.14 (*Right*).

Voltage sampling with FIR/IIR filtering

Besides bandpass charge sampling, it is also possible to introduce FIR and IIR filtering into bandpass voltage sampling. For bandpass charge sampling, the frequency down-conversion or mixing is realized by decimating a high rate integrated value. However, it is possible to realize sampling, filtering and frequency down-conversion at the same time in the process of voltage sampling by using complex FIR filtering [81, 82]. In addition, the built-in FIR sinc function and the introduced DT FIR filtering function in bandpass charge sampling only provide noise aliasing suppression at a certain set of frequencies corresponding to the notches of the composite filtering function, see eq. (3.15). Moreover, the CT sinc function in charge sampling always includes a lowpass function, which is a drawback for a bandpass input signal.

Generalized bandpass sampling combined with FIR filtering in voltage mode was proposed in [61, 81, 83, 84]. The corresponding frequency spectrum of sampled-data signal is expressed as

$$X_{s}(f) = \frac{1}{T_{s}} \sum_{k=-\infty}^{\infty} H(kf_{s}) X(f - kf_{s}), \qquad (3.17)$$

where the transfer function of intrinsic FIR filtering is given by

$$H(kf_s) = \sum_{n=0}^{N} h_n e^{-j2\pi k f_s T_D \cdot n}$$
(3.18)

and h_n is the filtering coefficient, T_D represents a unit-time delay of FIR filtering. Comparing eq. (3.15) and eq. (3.17), it is of more advantage to use voltage mode generalized bandpass sampling since the introduced FIR filtering function provides a discrete-frequency magnitude response such that the whole noise aliasing band corresponding to the notches of the filtering function will be cancelled. In the next chapter, generalized bandpass sampling with FIR filtering will be discussed in more detail.

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Chapter 4

Generalized Bandpass Sampling Receivers

In 1949, Shannon stated that any function limited to the bandwidth B and the time interval T can be specified by giving 2BT samples [4]. Besides classic Shannon's sampling theorem, there are actually many other ways to extract the data from the signal and still be able to fully reconstruct the signal. For example, the samples by nonuniform sampling or the samples from the signal and its derivative at half the Nyquist rate at least for each (i.e., derivative sampling) can also uniquely determine the signal without loss of information. Papoulis established the generalized nonuniform sampling theorem in 1977 [25] which is an extension of classic Shannon's sampling theorem. It states that a band-limited signal is uniquely determined by the samples on the outputs from M linear systems with input of the signal at 1/Mtimes the Nyquist rate at least for each. The Papoulis' generalization of sampling theorem extensively summarizes many different sampling cases: (i) derivative sampling [85], (ii) Recurrent nonuniform sampling [52], (iii) quadrature sampling [69], and so on.

In digital communications, the modulated signal is always expressed in terms of I/Q format or in quadrature. The main advantage of I/Q modulation is the symmetric case of combining independent signal components into a single composite signal and later splitting such a composite signal into its independent component parts [86]. It is more attractive to use quadrature mixers or quadrature sampling to separate the signal into I and Q parts before the baseband in radio communications. It is inevitable to encounter signal processing of complex signals in quadrature processing systems. Using quadrature processing has also the advantage of image rejection [70, 81, 87].

Regarding the noise aliasing problem in BPS systems and starting from Papoulis' generalized sampling theorem, a new sampling concept using bandpass voltage sampling, Generalized Quadrature BandPass Sampling (GQBPS) is invented. GQBPS performs intrinsic FIR filtering that uses either real or complex filter coefficients. Both theoretical analysis and simulation results show that GQBPS with inherent FIR filtering is promising to avoid noise aliasing when increasing the filter order. The jitter effect is also suppressed to a certain degree under the moving averaging operation of inherent FIR filtering. However, GQBPS has always limited noise and jitter performance improvement that is determined by the time resolution of the sampling scheme. Alternatively, Generalized Uniform BandPass Sampling (GUBPS) that is an extension of GQBPS is then proposed. GUBPS has the same property to noise aliasing suppression and jitter reduction as GQBPS while it has no any limitation of the performance improvement on noise and jitter. In addition, GUBPS with evenly spaced samples is normally easier to be implemented as compared to GQBPS with nonunformly spaced samples in most cases. Finally, a generalized bandpass sampling receiver based on the concept of GUBPS is implemented at circuit level by Switched-Capacitor (SC) circuit technique. To improve the selectivity at the sampling output, FIR filtering is extended by adding a 1st order complex IIR filter in the implementation.

4.1 Papoulis' Generalized Sampling Theorem

The generalized sampling expansion was first introduced in [25]. As shown in Figure 4.1, given M linear systems with transfer functions of $\{H_k(\omega)\}, k = 1, 2, \cdots, M$,



Figure 4.1: Identity of signal representation by Papoulis' generalized sampling theorem.

the output responses from the linear systems to an input band-limited signal f(t) is given by

$$g_k(t) = \int_{-\omega_0}^{\omega_0} F(\omega) H_k(\omega) e^{j\omega t} d\omega, \qquad (4.1)$$

where $\omega_0 = 2\pi B$ is the bandwidth of the signal, and $F(\omega)$ is the Fourier transform of f(t). Each of the *M* output responses is ideally sampled at time instant nT_s in

4.1. PAPOULIS' GENERALIZED SAMPLING THEOREM

at least 1/M times Nyquist rate. The combination of M linear systems $\{H_k(\omega)\}$ and ideal sampling represents an alternative to classic sampling. For an acceptable sampling case, there always exist M Linear Time-Invariant (LTI) functions $\{y_k(t)\}$ such that the input signal f(t) can be obtained at the final output in terms of $\{g_k(nT_s)\}$ (i.e., the samples of $\{g_k(t)\}$) and $\{y_k(t)\}$:

$$f(t) = \sum_{n=-\infty}^{\infty} [g_1(nT_s)y_1(t-nT_s) + g_2(nT_s)y_2(t-nT_s) + \dots + g_M(nT_s)y_M(t-nT_s)],$$
(4.2)

where $T_s = 1/f_s$, $\{y_k(t)\}$ is given by

$$y_k(t) = \frac{1}{\Delta\omega} \int_{-\omega_0}^{-\omega_0 + \Delta\omega} Y_k(\omega, t) e^{j\omega t} d\omega, \qquad (4.3)$$

and $\Delta \omega = 2\omega_0/M$, $T_s = 2\pi/\Delta \omega$, the *M* unknown functions $\{Y_k(\omega, t)\}$ are determined by *M* linear expressions that can be written in matrix form:

$$\begin{bmatrix} H_1(\omega) & H_2(\omega) & \cdots & H_M(\omega) \\ H_1(\omega + \Delta \omega) & H_2(\omega + \Delta \omega) & \cdots & H_M(\omega + \Delta \omega) \\ \vdots & \vdots & \vdots & \vdots \\ H_1[\omega + (M-1)\Delta \omega] & H_2[\omega + (M-1)\Delta \omega] & \cdots & H_M[\omega + (M-1)\Delta \omega] \end{bmatrix}$$

$$\begin{bmatrix} Y_1(\omega,t) \\ Y_2(\omega,t) \\ \vdots \\ Y_M(\omega,t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j\Delta\omega t} \\ \vdots \\ e^{j(M-1)\Delta\omega t} \end{bmatrix}$$
(4.4)

Generalized sampling with M branches is also called Mth-order sampling. It could be either US or NUS depending on the distribution of sampling time instants from all the branches. For the special case of M = 1, the generalized sampling theorem is reduced to the classic Shannon's sampling theorem.

4.1.1 Example 1 – Recurrent nonuniform sampling

Starting from the generalized sampling theorem and Figure 4.1 and assuming that the sampling time instant of one branch lags behind the previous one by α_k ($|\alpha_k| < T_s/2$), we have

$$g_k(t) = f(t + \alpha_k)$$

and

$$H_k(\omega) = e^{j\alpha_k\omega}$$



Figure 4.2: Identity of signal representation of recurrent nonuniform sampling.

Such sampling is also referred to as bunched or interlaced sampling [38]. The M unknown $Y_k(\omega, t)$ are given by

$$\begin{bmatrix} 1 & e^{j\alpha_{1}\omega} & \cdots & e^{j\alpha_{M}\omega} \\ 1 & e^{j\alpha_{1}(\omega+\Delta\omega)} & \cdots & e^{j\alpha_{M}(\omega+\Delta\omega)} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & e^{j\alpha_{1}[\omega+(M-1)\Delta\omega]} & \cdots & e^{j\alpha_{M}[\omega+(M-1)\Delta\omega]} \end{bmatrix} \cdot \begin{bmatrix} Y_{1}(\omega,t) \\ Y_{2}(\omega,t) \\ \vdots \\ Y_{M}(\omega,t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j\Delta\omega t} \\ \vdots \\ e^{j(M-1)\Delta\omega t} \end{bmatrix}$$
(4.5)

By using the Vandermonde determinant, a closed form of the reconstructing function $y_k(t)$ is given by eq. (4.47) in [38]. Yen also showed a similar expression of $y_k(t)$ for recurrent nonuniform sampling, see eq. (9) in [52]. For the special case when M = 2, Papoulis gave an exact reconstruction formula, see eq. (16) in [25]. One can always find the M LTI functions $\{y_k(t)\}$ so that the input signal f(t)is uniquely determined by the sampled data $f(nT_s + \alpha_k)$ in recurrent nonuniform sampling. Recurrent nonuniform sampling is applicable in real implementations.

4.1.2 Example 2 – Quadrature lowpass sampling

Starting from Figure 4.1 and assuming M = 2, $H_1(\omega) = 1$ and $H_2(\omega) = H_{tr}(\omega)$, generalized sampling is specialized to the example of quadrature lowpass sampling, see Figure 4.3. The filter $H_{tr}(\omega)$ is called Hilbert transformer. The impulse response and the frequency response of this filter is given by [60]

$$h(t) = \frac{1}{\pi t}, \qquad -\infty < t < \infty, \tag{4.6}$$

4.1. PAPOULIS' GENERALIZED SAMPLING THEOREM



Figure 4.3: Identity of signal representation of quadrature lowpass sampling, where $\tilde{f}(t)$ represents the Hilbert transform of f(t).

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = \begin{cases} -j, & \omega > 0\\ 0, & \omega = 0\\ j, & \omega < 0, \end{cases}$$
(4.7)

This filter realizes a 90° phase shift for all frequencies within the bandwidth of the real input signal. Obviously $\tilde{f}(t)$ is a shifted version of f(t) by 90°. The corresponding sampled-data signal consists of the samples from f(t) and $\tilde{f}(t)$ by lowpass sampling.

Starting from eq. (4.4), the two unknown LTI systems $Y_1(\omega, t)$ and $Y_2(\omega, t)$ are given by

$$\begin{bmatrix} 1 & H_{tr}(\omega) \\ 1 & H_{tr}(\omega + \omega_o) \end{bmatrix} \cdot \begin{bmatrix} Y_1(\omega, t) \\ Y_2(\omega, t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j\omega_0 t} \end{bmatrix},$$
(4.8)

where $\omega_0 = 2\pi B$. Hence,

$$Y_1(\omega, t) = 1 - H_{tr}(\omega) \cdot \frac{e^{j\omega_0 t} - 1}{H_{tr}(\omega + \omega_0) - H_{tr}(\omega)},$$

$$Y_2(\omega, t) = \frac{e^{j\omega_0 t} - 1}{H_{tr}(\omega + \omega_0) - H_{tr}(\omega)}.$$
(4.9)

The corresponding reconstruction function $y_1(t)$ and $y_2(t)$ exist and can easily be obtained as

$$y_1(t) = \frac{\sin(2\pi Bt)}{2\pi Bt}, \qquad y_2(t) = \frac{\cos(2\pi Bt) - 1}{2\pi Bt},$$
 (4.10)

based on eq. (4.3). Quadrature lowpass sampling is also applicable in real implementations.

4.1.3 Example 3 – Quadrature Bandpass Sampling

For an SSB-SC AM bandpass input signal in radio communication front-ends, quadrature mixers controlled by a quadrature LO are normally used to downconvert the bandpass signal and split the bandpass signal into the corresponding modulated I and Q parts before sampling. As we discussed in chapter 3, BPS



Figure 4.4: Identity of signal representation of quadrature bandpass sampling based on Kohlenberg's sampling theorem.

technique realizes frequency down-conversion which is similar to a mixer. A smart design of BPS system can realize both down-conversion and quadrature sampling on a bandpass signal.

Starting from Figure 4.2 for recurrent nonuniform sampling, assuming M = 2, $\alpha_1 = 1/(4f_c) \pm m/(2f_c), m = 0, 1, 2, \cdots$ (f_c is the carrier frequency of f(t)), input signal f(t) is a bandpass signal defined by eq. (3.1), the output responses of two linear systems f(t) and $f(t + \alpha_1)$ are given by

$$\begin{aligned} f(t) &= i(t)\cos(2\pi f_c t) - q(t)\sin(2\pi f_c t) \\ f(t+\alpha_1) &= i(t+\alpha_1)\cos\left[2\pi f_c(\frac{1}{4f_c}\pm\frac{m}{2f_c})\right] - q(t+\alpha_1)\sin\left[2\pi f_c(\frac{1}{4f_c}\pm\frac{m}{2f_c})\right] \\ &= (-1)^{m+1}\left[i(t+\alpha_1)\sin(2\pi f_c t) + q(t+\alpha_1)\cos(2\pi f_c t)\right]. \end{aligned}$$

Without losing generality, it can be assumed that $\alpha_1 = 1/(4f_c)$, i.e. m = 0. Note that there exists a 90° phase shift between f(t) and $f(t + \alpha_1)$. Such special recurrent nonuniform sampling exhibits the properties of quadrature sampling. This is the so called quadrature bandpass sampling. Based on the Kohlenberg's sampling theorem [66] that an SSB signal x(t) located at $(f_l, f_u) \cup (-f_u, -f_l)$ can be exactly represented by

$$\hat{x}(t) = \sum_{n=0}^{N-1} [x(nT_s)s(t-nT_s) + x(nT_s + \alpha)s(nT_s + \alpha - t)], \qquad (4.11)$$

the quadrature bandpass sampling can be equivalently modeled as shown in Figure 4.4, where $T_s \ge 1/B$ is the BPS rate, $\alpha = 1/(4f_c)$ and s(t) are given by eq. (31) in [66]. The samples of quadrature bandpass sampling are sufficient to represent the input SSB bandpass signal as long as the sampling rate on each sampling branch is equal to or larger than half of the acceptable minimum BPS rate (i.e., $\ge 2B$). Note that when $\alpha = T_s/2$, the quadrature bandpass sampling becomes the conventional uniform BPS. The *I* (in-phase) and *Q* (quadrature) component are obtained by alternating between two sampling branches.
Besides the exact interpolating function developed by Kohlenberg for an SSB bandpass signal reconstruction, Ries [88] also suggested a form of general reconstruction function derived from a lowpass reconstruction kernel. An alternative way to represent a bandpass signal by the samples is to use a carrier-modulated sinc function based on THEOREM 4.2 in [88]:

$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x(t_n)s(t-t_n)$$

where

$$s(t, t_n) = Re\{ sinc[2B(t - t_n)]e^{j2\pi f_c t} \}$$
(4.12)

and the set of $\{t_n\}$ consists of the samples from both I and Q branches. In general, eq. (4.12) could be extended to

$$s(t, t_n) = Re\{k(t, t_n)e^{j2\pi f_c t}\}$$
(4.13)

for any BK discussed in section 2.4 provided that the expression of $k(t, t_n)$ could be found, where $k(t, t_n)$ is defined by eq. (2.24).

4.2 Generalized Quadrature Bandpass Sampling

Based on the concept of quadrature bandpass sampling, the algorithm of Generalized Quadrature BandPass Sampling (GQBPS) was first addressed in [61] especially for treating the problem of noise aliasing in BPS systems. Both theoretical analysis and simulation results show that GQBPS is promising to suppress noise aliasing and also increases the jitter performance to a certain degree.

The proposed GQBPS receiver in terms of quadrature bandpass sampling is shown in Figure 4.5, and can be generalized to an arbitrary order of sampling. An



Figure 4.5: The model of GQBPS receiver based on quadrature bandpass sampling.

RF/IF bandpass signal $x(t) = Re\{a(t)e^{j2\pi f_c t}\}$ is sampled by quadrature bandpass sampling, where a(t) represents the upper sideband of the equivalent lowpass signal, see eq. (3.2) and Figure 3.1, although the complex conjugate of a(t), $a^*(t)$ that represents the lower sideband of the equivalent lowpass signal can also be used in the model. The sampled-data signal is fed into a bandpass reconstruction filter s(t) that might be realized by using a carrier-modulated sinc function, see eq. (4.12). Decimation or resampling may be required to realize either the second down-conversion or the data rate reduction prior to A/D conversion.

4.2.1 GQBPS with real FIR Filtering

Starting from the basic concept of sampling theory in chapter 2, the sampled-data signal $x_s(t)$ by 2nd order GQBPS is given by

$$x_s(t) = x(t) \left[\sum_{n = -\infty}^{\infty} \delta(t - nT_s) + \sum_{n = -\infty}^{\infty} \delta(t - nT_s - \alpha) \right], \quad (4.14)$$

and the corresponding Fourier transform of $x_s(t)$ is

$$X_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} (1 + e^{-j2\pi k f_s \alpha}) X(f - k f_s), \qquad (4.15)$$

where the second sampling sequence lags behind the first by α and

$$\alpha = 1/(4f_c) + m/(2f_c) \qquad (m = 0, \pm 1, \pm 2, \cdots)$$
(4.16)

for quadrature bandpass sampling, X(f) is the Fourier transform of the deterministic input SSB bandpass signal x(t) with a bandwidth of B. Note that 2nd order GQBPS performs 1st order FIR filtering and the transfer function is given by

$$H(f) = 1 + e^{-j2\pi f\alpha}$$
(4.17)

sampled at discrete-frequencies $f = kf_s$. The corresponding magnitude response of $H(kf_s)$ is illustrated in the dashed-line in Figure 4.6 for $f_c = 700$, B = 5, $f_s = f_c/4$, $\alpha = 1/(4f_c)$. Note that a single transmission zero is located at $f = f_c - B/2$.

The corresponding frequency spectral analysis is illustrated in Figure 4.7, where $A_l(f)$ and $A_r(f)$ represent respectively the folded spectra of the negative and positive frequency components of the input signal, and $X_s(f) = A_l(f) + A_r(f)$. To avoid signal spectra overlapping, the ratio of f_c to f_s should be an integer or a half integer and $f_s \geq 2B$ [61, 82]. It is observed that the whole folded band located at $f_c - B/2$ ($k = 2f_c/f_s$) that corresponds to the transmission zero of $H(kf_s)$ for $A_l(f)$ is completely eliminated while the band located at the same band position of X(f) (k = 0) is the same as X(f) multiplied by a gain factor. Note that the transmission zero of $H(kf_s)$ causes the whole band being zeroed out due to the discrete frequency property by quadrature sampling. In addition to the whole folded information signal band, the whole noise aliasing band is also eliminated correspondingly. This is in contrast to charge sampling where the aliasing products are a function of the continuous frequency, see eq. (3.15) in chapter 3. The transmission zeros provided



Figure 4.6: Magnitude response of inherent 1st order and 3rd order real FIR filtering in GQBPS, where $k \in [-f_c/f_s, 2f_c/f_s]$, $f \in [-(f_c + B/2), f_c - B/2]$, $f_c = 700$, B = 5, $f_s = f_c/4$, $\alpha = 1/(4f_c)$, H_{max} and H_{zero} represent the expected maximum transmission and the transmission zero, respectively.

by the intrinsic FIR function in charge sampling only cause notches at a limited set of frequencies, but not the whole bands.

The transfer function H(f) given by eq. (4.17) is a special case of 1st order FIR filtering that all the coefficients are one. It can be extended to a more general case with arbitrary order of filtering and arbitrary coefficients. By increasing the order of filtering, more transmission zeros are introduced while the transmission to the interesting folded information band is still the same such that the SNR performance is increased. Increasing the order of filtering is equivalent to increasing the effective sampling rate of GQBPS, i.e., putting more sampling branches in parallel and maintaining the time lag between any two successive sampling branches being equal to α . For 4th order GQBPS with 3rd order FIR filtering that can be modeled in Figure 4.8, one more transmission zero is introduced, see solid-line in Figure 4.6. It is a promising technique to be able to suppress the effect of noise aliasing significantly by GQBPS when the order of FIR filtering is high enough or



Figure 4.7: Illustration of spectral folding for 2nd order GQBPS with 1st order FIR filtering, where $f_s = f_c/4 > 2B$, $\alpha = 1/(4f_c)$. A real bandpass filter S(f) with a bandwidth of f_s is shown here, although it can be a complex filter with a bandwidth as narrow as B.

the effective sampling rate is large enough compared to the effective noise bandwidth B_{eff} . The frequency spectrum of GQBPS combined with FIR filtering can in general be expressed as

$$X_{s}(f) = \frac{1}{T_{s}} \sum_{k=-\infty}^{\infty} H(kf_{s}) X(f - kf_{s}), \qquad (4.18)$$

where the discrete frequency transfer function of intrinsic FIR filtering is given by

$$H(kf_s) = \sum_{n=0}^{N} \beta_n e^{-j2\pi k f_s \alpha n}$$

$$\tag{4.19}$$

and β_n represents the real filtering coefficients.

Based on the above analysis, the interesting folded information band is still located at the same band position as the input signal. The advantage of BPS, realizing the frequency down-conversion by signal spectral folding, is not reflected by this sampling scheme.



Figure 4.8: 4th order GQBPS with 3rd order FIR filtering, where $\alpha = 1/4f_c$.

4.2.2 GQBPS with complex FIR filtering

It is noticed that the interesting folded band corresponding to k = 0 could be shifted to a very low frequency if real FIR filtering is transformed to complex FIR filtering. Starting from the "modulation" property of the z-transform, the new transfer function in the z-domain is given by

$$H(z) = \sum_{n=0}^{N} (z_0 \cdot z)^{-n} |_{z=e^{j2\pi k f_s \cdot \alpha}},$$
(4.20)

assuming that the corresponding real FIR filtering has all-one coefficients. The "modulation" factor z_0^{-1} is equal to $e^{-j2\pi f_c \cdot \alpha}$ such that the interesting folded band is shifted to the left by f_c in the frequency domain (equivalent to a phase shift of $\pi/2$). The illustration of spectral folding in GQBPS with transformed filtering is shown in Figure 4.9, and it is consistent with the magnitude response of transformed FIR filtering, see Figure 4.10. Comparing with eq. (4.19), the coefficient β_n is not real but complex due to the filtering transformation. The coefficient β_n of complex FIR filtering transformed from all-one-coefficient real FIR filtering is given as

$$\beta_n = e^{-j\frac{\pi}{2} \cdot n}.\tag{4.21}$$

Finally, we can easily obtain a general expression of the resulting SNR (in dB) for (N + 1)th order GQBPS in the presence of band-limited white thermal noise



Figure 4.9: Illustration of spectral folding by 2nd order GQBPS with complex FIR filtering for $f_s = f_c/4 > 2B$, $\alpha = 1/(4f_c)$, $\beta_0 = 1$, $\beta_1 = -j$. A real lowpass filter S(f) with a bandwidth of $f_s/2$ is shown here, although it can be a complex upper sideband filter with a bandwidth as narrow as B.

introduced in the process of sampling:

$$SNR_{tot} = SNR_{deg} - SNR_{imp} = 10 \log_{10} \frac{2B_{eff}}{2 \cdot f_s} - 10 \log_{10} \frac{N+1}{2} = 10 \log_{10} \frac{2B_{eff}}{(N+1) f_s},$$
(4.22)

where B_{eff} is the effective noise bandwidth, SNR_{deg} by 2nd order GQBPS is defined by eq. (3.11) with $F_s = 2 \cdot f_s$, f_s is the sampling rate of single sampling branch in GQBPS, N + 1 is the total number of sampling branches of higher order GQBPS and $(N + 1)f_s$ represents the corresponding effective sampling rate. The effect of noise aliasing is completely avoided when $SNR_{tot} = 0$, although it could never happen for GQBPS due to the limited effective sampling rate $4f_c$ that is defined by the time resolution (i.e., $\alpha = 1/(4f_c)$) of the sampling scheme. The SNR improvement by higher order GQBPS is valid for GQBPS with both real and complex FIR filtering.



Figure 4.10: Magnitude response of inherent 1st order and 3rd order complex FIR filtering in GQBPS, where $k \in [-f_c/f_s, 2f_c/f_s]$, $f \in [-(2f_c + B/2), f_c - B/2]$, $f_c = 700, B = 5, f_s = f_c/4, \alpha = 1/(4f_c), H_{max}$ and H_{zero} represent the expected maximum transmission and the transmission zero, respectively. For 1st order FIR filtering, $\beta_0 = 1, \beta_1 = -j$, while for 3rd order, $\beta_0 = 1, \beta_1 = -j, \beta_2 = -1, \beta_3 = j$.

Moreover, it is also observed from Figure 4.7 that the "self-image" folding bands (i.e., $k = \pm 2f_c/f_s$) are always transferred to zero due to the quadrature property of sampling. GQBPS provides the immunity to the "self-image" problem due to I/Qmismatches. However, the performance of "self-image" rejection strongly depends on the phase shift of the sampling clock accuracy [82].

Note that the samples are still nonuniformly spaced in GQBPS except for some special cases, e.g. $(N + 1) \cdot \alpha = T_s$. It is known that conventional uniform BPS has the same noise aliasing performance as GQBPS with the same order of sampling [61]. With respect to the circuit implementation, it is much easier to realize uniform sampling with inherent FIR filtering compared to nonuniform sampling. Under the concept of uniform quadrature bandpass sampling [14], GQBPS strategy is extended to Generalized Uniform BandPass Sampling (GUBPS).

4.3 Generalized Uniform Bandpass Sampling

A special case of 2nd order GQBPS is when the sampling rate for each sampling branch satisfies [14]

$$f_s = \frac{2f_c}{2m-1}, \qquad m = 1, 2, \cdots$$
 (4.23)

which is called Uniform Quadrature BandPass Sampling (UQBPS). UQBPS possesses all the properties of quadrature BPS and the sampling time instants are uniformly distributed. When more sampling branches with the same sampling rate f_s are introduced and the time lag between any two successive sampling sequences is $T_D = 1/[(N+1) \cdot f_s]$ (where N is the order of FIR filtering or N + 1 is the order of sampling), the phase shift between any two samples at the output of sampling is not equal to 90° any more, which is not quadrature sampling and will be referred to as Generalized Uniform BandPass Sampling (GUBPS)¹. UQBPS is a special case of GUBPS with 2nd order sampling and 1st order FIR filtering. Similar to GQBPS, GUBPS also performs FIR filtering besides sampling. The transfer function of intrinsic FIR filtering in GUBPS is in general expressed as [81, 82]

$$H(kf_s) = \sum_{n=0}^{N} \beta_n e^{-j2\pi k f_s \cdot T_D \cdot n},$$
(4.24)

where β_n could be either real or complex. The coefficients of complex FIR filtering transformed from all-one-coefficient real FIR filtering is defined as

$$\beta_n = e^{-j2\pi \lfloor f_c/f_s \rfloor \cdot \frac{n}{(N+1)}},\tag{4.25}$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. The magnitude responses of $H(kf_s)$ with and without filtering transformation are shown in Figure 4.11. For the special case when N = 1 and N = 3, there is no difference between FIR filtering with real coefficients and the corresponding transformed filtering. The illustration of spectral folding for 2nd order GUBPS (or UQBPS) is shown in Figure 4.12.

GUBPS maintains all the same properties to noise aliasing suppression and "selfimage" band rejection as GQBPS, while GUBPS has no limitation on the maximum effective sampling rate. Starting from eq. (4.22) with f_s defined by eq. (4.23), we can easily have

$$SNR_{tot} = 10\log_{10}\frac{B_{eff} \cdot (2m-1)}{(N+1)f_c}$$
(4.26)

for GUBPS. When $B_{eff} \cdot (2m-1)/[(N+1)f_c] = 1$, noise aliasing is completely suppressed, i.e., the required order of FIR filtering is $N = (2m-1) \lceil \frac{B_{eff}}{f_c} \rceil - 1$, where $\lceil x \rceil$ is the smallest integer larger than or equal to x. Note that the interesting output

¹In all the published papers, UQBPS was used as a general name for both N = 1 and N > 1. GUBPS is a more suitable name and will be used instead in the dissertation.



Figure 4.11: Magnitude response of inherent 1st order, 3rd order and 7th order FIR filtering in GUBPS, where $k \in [-f_c/f_s, 2f_c/f_s]$, $f \in [-(2f_c + B/2), f_c - B/2]$, $f_c = 700, B = 5, f_s = 2f_c/9, \alpha = 1/(4f_c), H_{max}$ and H_{zero} represent the maximum transmission and the zero transmission, respectively: (*Top*) Real FIR filtering with all-one coefficients; (*Bottom*) Complex FIR filtering with the coefficients defined by eq. (4.25).

samples by GUBPS are not located at baseband but centered at $f_s/2 + B/2$, and further down-conversion is needed by either resampling or decimation. This is often a small problem since this operation is performed at the low frequency f_s compared to f_c .

4.4 Noise and Jitter Performances

The noise and jitter performances of GQBPS and GUBPS when increasing the order of intrinsic FIR filtering are evaluated and compared by using MATLAB simulations. A randomly generated bandlimited SSB signal with B = 5 is frequency translated to $f_c = 700$ by multiplying with a sinusoidal carrier. Oversampling with respect



Figure 4.12: Illustration of spectral folding by 2nd order UQBPS for $f_s = 2f_c/9 > 2B$ (m = 5). A real bandpass filter S(f) with a bandwidth of f_s is shown here, although it can be a complex filter with a bandwidth as narrow as B.

to the classic BPS theorem is used to show the effect of noise aliasing, where $f_s = f_c/4$ for GQBPS and $f_s = 2f_c/9$ (i.e., m = 5 in eq. (4.23)) for GUBPS. The time shift between any two successive sampling branches is always $\alpha = 1/(4f_c)$ in GQBPS and $T_D = 1/[(N+1)f_s]$ (N is the order of FIR filtering) in GUBPS. The impulse response of real FIR filtering are all one, and all complex FIR filtering is transformed from real FIR filtering with all-one coefficients. The simulation results are also compared with the theoretical expectation.

4.4.1 Noise aliasing performance

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To evaluate noise aliasing performance, bandlimited white Gaussian noise is added such that the in-band noise power is equal to the out-of-band noise power, and the effective noise bandwidth $B_{eff} = 10 f_c$. The sampling time instants are well predetermined, i.e., no sampling jitter is present. The simulation results of SNR (in dB) in the presence of noise aliasing versus the order of FIR filtering for GQBPS and GUBPS are shown in Figure 4.13. The evaluated SNRs based on eq. (4.22) are superimposed.



Figure 4.13: The simulated and theoretically evaluated SNR in the presence of noise aliasing versus the order of intrinsic FIR filtering (a) in GQBPS; (b) in GUBPS.

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As discussed in section 4.2.2, the effective sampling rate in GQBPS cannot be increased any more when it reaches to $4f_c$ that is determined by the time resolution α of the sampling scheme. For current simulation environment, the effective sampling rate becomes $4f_c$ when N = 15, which is consistent with the simulation result, see Figure 4.13. When $N \leq 15$, the simulation and theoretical evaluation results are consistent, namely, the SNR performance is improved by 3dB when the order of sampling is doubled. As stated before, GUBPS has no effective sampling rate limitation. However, the noise power at the sampling output can never be lower than that at the input. In other words, when the effective sampling rate $(N+1)f_s$ is equal to or larger than $2B_{eff}$, noise aliasing is avoided completely, see eq. (4.22) and the SNR performance is not improved anymore by increasing the order of FIR filtering. For the current simulation, this corresponds to N = 63. However, it is observed from Figure 4.13 that the in-band noise aliasing at the interesting band is avoided completely when the order of filtering $N \geq 49$, even though the effective sampling rate is still lower than $2B_{eff}$ and some other folded bands still suffer from noise aliasing. This result is consistent with the simulation result shown in Figure 8 in [81].

4.4.2 Jitter performance

To evaluate jitter performance, the randomly generated input signal is sampled in the presence of small random jitter with a Gaussian distribution $N(0, \sigma_{\tau})$, where the standard deviation of jitter $\sigma_{\tau} = \lambda T_s/3$ and $\lambda = 0.01$, $T_s = 1/f_s$ (f_s is the sampling rate for each sampling branch), i.e., $\sigma_{\tau} = 1.9 \times 10^{-5}$ for GQBPS and $\sigma_{\tau} = 2.1 \times 10^{-5}$ for GUBPS. The simulated SNRs (in dB) in the presence of only jitter versus the order of FIR filtering in GQBPS and GUBPS are shown in Figure 4.14.

It is known that GQBPS performs both sampling and FIR filtering, where the process of FIR filtering performs moving average and provides the function of noise reduction. Assuming that FIR filtering has a unity gain at DC and all the sampling branches have equal weights, the noise reduction ratio (NRR) of length-N FIR filtering is defined as

$$NRR = \sum_{n=0}^{N-1} |\beta_n|^2 = N \cdot (\frac{1}{N})^2 = \frac{1}{N},$$
(4.27)

where β_n are the coefficients that could be either real or complex. Based on the properties of β_n , see eq. (4.21) and eq. (4.25), the definition of NRR is the same for both real and complex FIR filtering. Once the order of sampling is doubled, i.e., $N \to 2N$, the variance of noise at the output of FIR filtering will theoretically be reduced by 3dB, see dashed-line in Figure 4.14.

It is observed that the simulated SNR is very consistent with the theoretical expectation for GQBPS within $1 \le N \le 15$ and GUBPS. For GUBPS, the SNR in the presence of jitter is always improved with increased order of FIR filtering until it approaches the SNR of ideal GUBPS. However, for GQBPS under the current



Figure 4.14: The simulated and theoretically evaluated SNR in the presence of timing jitter versus the order of intrinsic FIR filtering (a) in GQBPS; (b) in GUBPS.

simulation, the effective sampling rate reaches the maximum when N = 15, and the jitter performance is not increased any more when N > 15.

Besides suppressing noise aliasing, GQBPS and GUBPS with FIR filtering also provides certain immunity to jitter noise, which can not be achieved by conventional oversampling. As discussed in [89], the jitter effects in conventional mixers are similar in sampling systems. It is inevitable to encounter jitter in mixers if clock jitter is present in oscillators. It is of more advantage to use GQBPS and GUBPS with complex FIR filtering to realize frequency down-conversion instead of a conventional mixer since one can make use of the FIR filtering process to improve the jitter performance. The moving average operation of FIR filtering can be also used to explain the improvement of noise aliasing in GQBPS and GUBPS.

4.5 Complex Signal Processing

Complex signal processing is extensively used in wireless communication systems, e.g., image-rejecting systems in RF receivers and quadrature modulation systems, although all the physical circuits only deal with a real voltage or current signal. Due to the presence of quadrature sampling and complex filtering in GQBPS and GUBPS, complex signal processing is needed.

Complex signal processing normally includes four basic operations: complex addition, complex multiplication, complex integration for continuous-time (CT) filters or complex delay in discrete-time (DT) filters [90]. A complex signal consists of a pair of real value signals at a certain time instant denoted as the real and imaginary part. All the complex operations can be equivalently realized by doing corresponding real operations on these two real signals. A complex addition is simply to add two real parts and two imaginary parts independently. A complex multiplication operation can be directly defined from the mathematic definition. Assume a complex input signal $x(t) = x^r(t) + jx^i(t)$ multiplied by the impulse response of a complex filter $h(t) = h^r(t) + jh^i(t)$ equals y(t), i.e.,

$$y(t) = [x^{r}(t) + jx^{i}(t)] \cdot [h^{r}(t) + jh^{i}(t)]$$

= $[x^{r}(t)h^{r}(t) - x^{i}(t)h^{i}(t)] + j[x^{r}(t)h^{i}(t) + x^{i}(t)h^{r}(t)]$
= $y^{r}(t) + jy^{i},$ (4.28)

where the superscript r and i represent real and imaginary part of a complex signal, respectively. The signal-flow graphic of the complex multiplication operation is illustrated in Figure 4.15. A complex integration for CT filters or complex delay in DT filters is simply to integrate or delay the input real and imaginary parts, respectively.

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Figure 4.15: Illustration of complex signal multiplication by a) a complex operation; b) the equivalent real operations.

4.6 CMOS Implementations

4.6.1 Passive and active sampling

Passive sampling circuits consisting of only capacitors and switches permit very fast sampling and can handle a large input bandwidth [91]. The circuit bandwidth is determined by the on-resistance of the switch and the sampling capacitance. It has been shown that passive sampling can successfully sample a 900MHz signal [75]. It might be a good candidate to GQBPS and GUBPS implementations.

Passive sampling is extensively used in high-speed time-interleaved A/D converters. To increase the effective sampling rate, several sampling branches can operate in parallel which is similar to the sampling architecture of GQPBS and GUBPS. However, the total number of converters in time-interleaved A/D converters is equal to the total number of parallel sampling branches while only two A/D converters are needed in the proposed GQBPS and GUBPS architecture, one for real and the other for imaginary data processing. The conventional passive sampling architecture is very sensitive to clock skew between parallel sampling channels. An improved passive sampling technique that is insensitive to skew was proposed by Gustavsson and Tan [92]. As shown in Figure 4.16, a global sampling clock ϕ is introduced. When the clock phases ϕ_i ($i = 1, 2, \dots M$) and ϕ are high, the input is charging the *i*th sampling capacitor. When ϕ goes low, the input analog value is sampled by the sampling capacitor since one plate of the sampling capacitor is floating. The advantage of the circuit is that the clock phases ϕ_i always goes low after ϕ goes low. Even if there are large phase skews between successive clock phases

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Figure 4.16: Illustration of improved passive sampling technique that is insensitive to phase skew.

 ϕ_i , they do not have any influence on the sampling instant and the problem with skew is eliminated.

However, passive sampling can inevitably encounter offset or distortion problems in RC sampling network [93]. Active sampling circuits consisting of switches, sampling capacitors and operational amplifiers (opamp) can realize offset cancellation or autozero by using a unit gain feedback [94]. However, active sampling takes more time than passive sampling due to the settling time of the opamp limiting the sampling rate. Additionally, the circuit bandwidth is limited by both the RC sampling network and the opamp bandwidth. Based on the resettable gain circuit architecture [95], an available implementation method for GQBPS and GUBPS in active sampling was proposed [84], and the corresponding unit cell of schematic for 1st order sampling is shown in Figure 4.17 (a). It is obvious that the circuit cancels



Figure 4.17: Unit cell of schematic for 1st order sampling by (a) resettable gain circuit; (b) capacitive-reset gain circuit.

the DC-offset, although the output level is always reset to zero which requires an opamp with a high slew rate. To reduce the slew rate requirement, capacitive-reset gain circuit can be used instead [95], see Figure 4.17 (b). Such a circuit architecture has the same valid output level and DC-offset property as resettable gain circuit, but the output is always reset to the value that is determined by the value of the output at the previous phase and the DC-offset instead of zero. Both two reset

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gain circuits possess the property of highpass response from the opamp's input to the output although the response of the whole circuit does not necessarily have a high-pass response. The 1/f noise introduced in opamp located at the low frequency range is then highpass filtered. One can put multiple unit cells in parallel with appropriate clock scheme to realize higher order sampling with higher order FIR filtering [84].

4.6.2 Implementation of FIR and IIR filtering

Starting from the theoretical analysis on GQBPS with complex FIR filtering (see section 4.2.2), when the complex FIR coefficient β_n is defined by eq. (4.21), the outputs from complex FIR filtering are repeatedly obtained as

$$\begin{cases} i(nT_s + \frac{k}{4f_c}), & 0, & i(nT_s + \frac{k+2}{4f_c}), & 0 & \text{real data} \\ 0, & q(nT_s + \frac{k+1}{4f_c}), & 0, & q(nT_s + \frac{k+3}{4f_c}) & \text{imaginary data}, \end{cases}$$

where k = 4m, $m = 0, 1, 2, \dots, \lfloor N/4 \rfloor$ and N is the order of FIR filtering. The I/Q components corresponding to the equivalent lowpass signal a(t) are directly obtained at the output of GQBPS. The sampled-data signal can be decimated and quantized by the following process step, but a real lowpass anti-aliasing filter with the bandwidth of 2B (B is the SSB information bandwidth) is needed before decimation on each real and imaginary data path.

For GUBPS with complex FIR filtering, a complex signal multiplication is needed, see Figure 4.15. The output after each complex coefficient multiplication in the process of sampling consists of real and imaginary parts which are the inputs to the following digital processing. As shown in Figure 4.12 the interesting information band of sampled-data signal is located at $f_s/2 + B/2$, where f_s is the sampling rate for each sampling branch. The sampled data are modulated at this lower IF. A bandpass filter located at the interesting band is needed prior to the second frequency down-conversion (or decimation) and quantization process. Such bandpass filter is normally realized as a complex filter since either the positive or negative frequency components are needed in real implementations, and it could be either an FIR, IIR filter or a combination of an FIR and an IIR. The corresponding implementation block diagram for GUBPS is shown in Figure 4.18. FIR filters can have linear phase and guaranteed stability, but filter orders are normally large for small transition bands and delays are large as well. However, a small transition band can be achieved by a small IIR filter order. A common implementation of higher order IIR filters is to cascade 2nd order IIR filters (or "biquads"). IIR filters are the digital counterpart to analog filters. They use feedback, and will normally require less computing resources than an FIR filter for similar performance. Nevertheless IIR filters may be unstable and very dependent on the filter architecture.

In the following, GUBPS with complex FIR filtering will be implemented by Switched-Capacitor (SC) circuit technique. The sampling function combined with



Figure 4.18: Block diagram of GUBPS combined with complex FIR filtering and β_n $(n = 0, 1, \dots, N)$ represents the corresponding set of complex coefficients. The interesting band of sampled-data signal is further filtered out by a complex FIR/IIR bandpass filter.

FIR filtering is realized by passive sampling. The digital complex bandpass filter is designed as an IIR filter using complex SC filter design method [96].

Due to the sampled-data nature of SC technique, FIR filtering can be implemented in parallel structure, see Figure 4.19. To simplify it is assumed that the input-offset voltage of opamp is neglected and no offset compensation is considered in the circuit. This compensation circuit can be easily added when it is needed, see Figure 4.17. The corresponding appropriate clock scheme is shown in Figure 4.20. The input voltage is sampled by N + 1 parallel sampling channels, where the sampling clock on each channel lags behind the one on the previous channel by T_D . On each sampling channel, the voltage is sampled by two different capacitors in parallel C_{b_i} , $C_{b_{i+1}}$ ($i = 2n, n = 0, 1, 2, \dots N$). The sampled voltage level must be held by using an opamp with a capacitor and a reset switch in the negative feedback loop. Obviously this implementation use passive sampling since no opamp is used to generate delay resulting in less power consumption. Using charge conservation analysis, one can easily get the transfer function of the sampling circuit (see Appendix C):

$$H_{FIR}(z) = z^{-1/2} \sum_{i=0}^{N} \left[\frac{C_{b_{2i}}}{C_h} + j \frac{C_{b_{2i+1}}}{C_h} \right] z^{-i},$$
(4.29)

where the values of complex impulse response coefficients $\frac{C_{b_{2i}}}{C_h}$, $\frac{C_{b_{2i+1}}}{C_h}$ are determined by eq. (4.25)), and the unit-time delay element z^{-1} is given by $z^{-1} = e^{-j2\pi k f_s T_D}$. It is obvious that this sampling circuit performs Nth order FIR filtering. The extra delay element $z^{-1/2}$ in the numerator only causes a phase shift.

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Figure 4.19: FIR filter design by passive sampling.

The sampled voltage levels determined by the ratios of $\frac{C_{b_{2i}}}{C_h}$ and $\frac{C_{b_{2i+1}}}{C_h}$ represent the real and imaginary voltages, respectively. The equivalent sampling rate of the circuit is $(N + 1)/T_s$. An extra switch operating at a lower rate $1/T_s$ realizes a decimation operation at the output.

In order to improve selectivity, we extend the FIR filtering function by adding a 1st order complex IIR filter. It is easy to see that the circuit shown in Figure 4.21 is a 1st order real IIR lowpass filter, where ϕ and $\overline{\phi}$ are a pair of complementary clocks. The corresponding transfer function is given by

$$H_{IIR-LP}(z) = \frac{C_b z^{-1}}{(C_a + C_h) - C_h z^{-1}}.$$
(4.30)

A 1st order complex bandpass filter can be obtained by frequency shifting or "modulating" a lowpass filter [96]:

$$z \to z_0 z, \tag{4.31}$$

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Figure 4.20: The appropriate clock scheme of the proposed circuit architecture that is shown in Figure 4.19.



Figure 4.21: An example of 1st order lowpass filter.

where

$$z_0 = e^{-j2\pi f_0 T_s} = \cos(2\pi f_0 T_s) - j\sin(2\pi f_0 T_s) = \alpha - j\beta,$$
(4.32)

and f_0 is the center frequency of the bandpass filter, and $1/T_s$ is the operating frequency of the IIR filter. The transfer function of the IIR bandpass filter transformed from $H_{IIR-LP}(z)$ is obtained as

$$H_{IIR-BP}(z) = \frac{C_b(\alpha - j\beta)z^{-1}}{(C_a + C_h) - C_h(\alpha - j\beta)z^{-1}}.$$
(4.33)

The corresponding circuit implementation for a real input signal is shown in Figure 4.22. The "modulation" factor z_0 introduces two cross-coupled capacitors be-



Figure 4.22: Circuit implementation of 1st order complex IIR bandpass filter transformed from the 1st order IIR lowpass filter shown in Figure 4.21.

tween the inputs and outputs of the opamps in the real and imaginary datapaths, and also cross terms from the signal input. Assuming $\phi=0$ and $\overline{\phi}=1$ at $t=nT_s$ and it toggles to $\phi=1$ and $\overline{\phi}=0$ at $t=(n+1)T_s$. The input-output equation can be written as

$$V_{in}(n)C_b\alpha + V_{out}^{Im}(n)C_h\beta = V_{out}^{Re}(n+1)(C_h + C_a) - V_{out}^{Re}(n)C_h\alpha -V_{in}(n)C_b\beta - V_{out}^{Re}(n)C_h\beta = V_{out}^{Im}(n+1)(C_h + C_a) - V_{out}^{Im}(n)C_h\alpha.$$
(4.34)

One can easily get the transfer function of the circuit:

$$\frac{V_{out}^{Re}(z) + jV_{out}^{Im}(z)}{V_{in}(z)} = \frac{C_b(\alpha - j\beta)z^{-1}}{(C_h + C_a) - C_h(\alpha - j\beta)z^{-1}},$$

which is consistent with the expectation, see eq. (4.33). This circuit can be easily extended for a complex input signal $V_{in}(t) = V_{in}^{Re}(t) + jV_{in}^{Im}(t)$ by adding two more cross terms for the imaginary part.

The circuit architectures discussed above are proposed only for single ended output implementations. It is known that fully differential opamps provide a larger output voltage swing than their single ended counterparts, which is important when the input supply voltage is small. Two outputs with complementary signs are obtained at the same time in the differential output. Even-order nonlinearilities can be rejected by a balanced circuit [97]. In addition, it is seen from eq. (4.34)that two cross-coupled capacitors $C_h\beta$ cause opposite signs in charge conservation because the upper one is inverting bottom plate sampling and the lower one toggle switching. So do the cross terms $C_b \alpha$ and $C_b \beta$ at the signal input. Toggle switching is parasitics sensitive and normally unexpected in practice [98, 99]. It can be avoided by using fully differential technique. A fully differential implementation of the 1st order complex IIR bandpass filter with a perfect balance circuit is shown in Figure 4.23 which replaces all the toggle switches by inverting bottom plate sampling. Note that this IIR complex bandpass filtering implementation is only valid for $\alpha, \beta > 0$, but it is easy to be adjusted for $\alpha, \beta < 0$ by changing the components in the opamps' feedback loops [96].

The numerator of $H_{IIR-BP}(z)$ in eq. (4.33) is determined by the passive sampling array in the 1st order complex IIR bandpass filter. It can be easily replaced by the sampling array with intrinsic FIR filtering to achieve a composite FIR/IIR filtering function. A single ended implementation of GUBPS with composite FIR/IIR filtering and the appropriate clock scheme are proposed, see Figure 4.24. As discussed in section 4.2.2, the wanted band is centered at $f_s/2 + B/2$ after GUBPS. The IIR filter is combined with the extra decimation switch so that the operating rate of IIR filtering is $1/T_s$. Based on eq. (4.32), $\alpha, \beta < 0$ for GUBPS implementations. The circuit realization of 1st order complex IIR bandpass filtering shown in Figure 4.22 is used after minor changes. The circuit is analyzed using charge conservation, see Appendix C. The transfer function of the circuit is given by

$$H_{FIR-IIR} = \frac{z^{-1} \sum_{i=0}^{N} [C_{b_{2i}} + jC_{b_{2i+1}}] z^{-i}}{(C_h + C_a) - C_h(\alpha - j\beta) z^{-(N+1)}}.$$
(4.35)

The numerator is determined by the passive sampling array with intrinsic FIR filtering in a high operating rate and the denominator determines the pole of the 1st order complex IIR filter in a 1/N times lower operating rate as compared to FIR filtering. Two more toggles switches are introduced to compensate the negative α that can be avoided in fully differential implementations. A sharper roll-off in the gain response of IIR filter can be obtained by adding more stages to the first order IIR filter, either by cascade coupling or using ladder structures [98].

For the same specification as in Figure 4.11, the magnitude responses of 15th order FIR filtering, 1st order complex bandpass IIR filtering, and the combination of FIR/IIR filtering are shown in Figure 4.25. The coefficients of FIR filtering



Figure 4.23: Fully differential realization of 1st order complex IIR bandpass filter transformed from the 1st order IIR lowpass filter shown in Figure 4.22.

are defined by eq. (4.25) for GUBPS implementations. It is observed that FIR filtering realizes the maximal transformation at the interesting frequency band and IIR filtering functions as a bandpass filter to select the interesting band. Note that the repeated bands from the IIR filter are at the notches of the FIR filter, except for the wanted band. The magnitude response of composite FIR/IIR filtering shows good selectivity. The 3 dB bandwidth and the center frequency of the 1st order



Figure 4.24: The Proposed circuit architecture of GUBPS with composite FIR/IIR filtering by complex SC filter design for single ended implementations together with the appropriate clock scheme.

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Figure 4.25: Magnitude responses of intrinsic 15th order complex FIR filtering in GUBPS (*Top* in solid line), 1st order complex IIR filtering at the output of GUBPS (*Top* in dashed line) and the composite FIR/IIR filtering $H_{FIR/IIR}(f)$ (*Bottom*) for $k \in [-2f_c/f_s, 0]$ in the range of $f \in [-(2f_c + B/2), f_c - B/2]$, where k is the index of folding bands shown in eq. (4.24), $f_c = 700, B = 5, f_s = 2f_c/9, T_D = T_s/16, T_s = 1/f_s$. The complex coefficients of FIR and IIR filtering are determined by eq. (4.25) and eq. (4.32), respectively.

complex IIR bandpass filter determine the selectivity performance. The complex IIR bandpass filter defined by eq. (4.35) has the same 3 dB bandwidth as the corresponding lowpass prototype defined by eq. (4.30):

$$\cos\omega_{-3dB} = 1 - \frac{1}{2(\frac{C_h}{C_a} + 1)\frac{C_h}{C_a}}.$$
(4.36)

Obviously the 3 dB bandwidth only depends on the ratio of C_h to C_a . The center frequency of the composite FIR/IIR filter is determined by α and β . Starting from eq. (4.32) with $f_0 = f_s/2 + B/2$,

$$\alpha = -\cos(\pi\xi), \qquad \beta = -\sin(\pi\xi), \qquad (4.37)$$

CHAPTER 4. GENERALIZED BANDPASS SAMPLING RECEIVERS



Figure 4.26: The gain response of the designed 1st order complex IIR bandpass filter (a) with different 3dB bandwidth and the same center frequency (i.e. $\xi = 0.5$); (b) with the same 3dB bandwidth (i.e. $C_h/C_a = 9$) but different center frequency.

where $\xi = B/f_s$. The magnitude response of only $|H_{IIR-BP}(e^{j\omega})|^2$ is shown in Figure 4.26 with respect to the ratio of C_h/C_a and the value of ξ .

Chapter 5

Summary of appended papers

- **Paper I:** Nonuniform sampling and reconstruction theory are studied. Starting from Basis-Kernel (BK), a general reconstruction formula is given for nonuniform sampling (NUS). Six NUS reconstruction algorithms that have been mostly used in image processing are reviewed based on different BK expressions. The corresponding reconstruction performances and hardware implementations in radio receiver applications are compared and discussed.
- **Paper II:** Three sources of performance degradation in BandPass Sampling (BPS) systems, harmful signal spectral folding, noise spectral folding and timing jitter, are reviewed. The performance of reconstruction algorithms for nonuniform BPS in the presence of the noise sources are discussed based on simulations.
- Paper III: This journal paper is a summary of Paper II and part of Paper I.
- **Paper IV:** Regarding the noise aliasing in BPS systems and starting from the Papoulis' generalized sampling theorem, Generalized Quadrature BandPass Sampling (GQBPS) is proposed and studied in the frequency domain for both deterministic and stochastic input signal. The noise aliasing performance of GQBPS and conventional BPS are compared. The theoretical analysis shows that GQBPS might be a potential way to reduce noise aliasing at the cost of a more complicated reconstruction algorithm.
- **Paper V:** GQBPS in voltage-mode with inherent FIR filtering is presented. By using sampling equivalence, this sampling strategy is comparable to another strategy, charge sampling with intrinsic FIR/IIR filtering. The theoretical analysis and simulation results show that this inherent FIR filtering not only has the advantage to reject or attenuate images and interferences, but is also helpful to suppress noise aliasing. GQBPS is NUS except for some special cases. GQBPS with inherent FIR filtering is then extended to a special case

of uniform sampling, Generalized Uniform BandPass Sampling (GUBPS)¹ with inherent FIR filtering, that is easier to be implemented than GQBPS.

- **Paper VI:** GUBPS¹ is analyzed for both ideal sampling and a sample-and-hold. It is shown that the noise aliasing performance of GUBPS¹ with intrinsic FIR filtering is not degraded by the effect of sample-and-hold in real sampling circuits. In addition, one available implementation method to GUBPS¹ by using active sampling based on reset gain circuit is proposed and discussed at circuit level.
- **Paper VII:** The filtering transformation in GQBPS is studied. Inherent FIR filtering in GQBPS and GUBPS¹ with real coefficients is transformed such that the interesting band is shifted to a lower frequency. The main advantage of such transformation is to achieve a frequency down-conversion besides sampling and noise aliasing suppression. The whole subsampling system with GQBPS and GUBPS¹ is simplified by using complex filtering.
- **Paper VIII:** This submitted journal paper is a summary of **Paper IV**, **Paper V** and **Paper VII** together with the analysis on the sensitivity to the accuracy of sampling clock in generalized bandpass sampling systems.

 $^{^{1}}$ In all the published papers, UQBPS was used as a general name for both 2nd order and higher order generalized uniform bandpass sampling. GUBPS is a more suitable name and will be used instead in the dissertation.

Chapter 6

Conclusions and Future Work

The first part of this thesis reviews and compares different radio receiver architectures for conventional and subsampling receivers, single standard and multistandard receivers. The motivation of the thesis is to investigate a novel receiver architecture for Software Defined Radio (SDR). Under the concept of SDR, subsampling receivers in BandPass Sampling (BPS) technique becomes more and more attractive.

After that, sampling is discussed, including Uniform Sampling (US) and NonUniform Sampling (NUS), voltage sampling and charge sampling, deterministic sampling and random sampling. A single ideal lowpass filter based on Shannon's sampling theorem is not good enough to reconstruct the signal from the samples by NUS. A general reconstruction formula in terms of a basis-kernel is proposed and nine reconstruction algorithms (RAs) starting from the formula are evaluated and compared especially for deterministic NUS in terms of reconstruction performance and computational complexity. It is investigated that most of these RAs are extensively used in off-line image processing, but algorithms based on *interpolation* are also possibly used in on-line radio communications. Reconstruction becomes hard for random sampling, but random sampling may be helpful for signal identification and eliminating the quantization distortions in the following A/D converters.

Then the classic BPS theory is reviewed from the aspects of sampling rate selection, noise aliasing and jitter. The existing studies on BPS are presented and compared. It is noticed that noise aliasing plays an important role in the BPS applications.

Starting from the Papoulis' generalized sampling theorem, generalized bandpass sampling including Generalized Quadrature BandPass Sampling (GQBPS) and Generalized Uniform BandPass Sampling (GUBPS) are invented especially for dealing with the performance degradation due to noise aliasing in BPS systems. It is observed that GQBPS and GUBPS perform intrinsic FIR filtering that uses either real or complex filter coefficients. The theoretical analysis and simulation results show that both GQBPS and GUBPS realize sampling, frequency downconversion and noise aliasing suppression by well-designed intrinsic FIR filtering. Both noise and jitter performance will be theoretically increased by 3 dB when the length of FIR filtering is doubled. However, GQBPS has always limited noise and jitter performance improvement that is determined by the time resolution of sampling. Additionally, the samples by GQBPS are nonuniformly spaced for most cases. GUBPS has better noise and aliasing performance and is easier to be implemented as compared to GQBPS. In the final part, a generalized bandpass sampling receiver based on the concept of GUBPS is implemented at circuit level by Switched-Capacitor (SC) circuit technique. To obtain a better selectivity at the sampling output, an extra IIR filter combined with a decimation operation is introduced.

Bandpass voltage sampling and bandpass charge sampling are two promising candidates for subsampling receivers. In this thesis, these two sampling methods are also analyzed and compared in theory. It is shown that generalized bandpass sampling in voltage-mode is more efficient to suppress noise aliasing than bandpass charge sampling with embedded filtering technique. This is determined by the different frequency responses of two sampling techniques. The same noise aliasing suppression may be achieved by lower order FIR filtering in generalized bandpass voltage sampling as compared to bandpass charge sampling.

Regarding the problems in real SC circuit implementations, further studies are still needed for the proposed SC circuit architecture of generalized bandpass sampling receiver, e.g. cross-talk among multiple sampling branches, charge injection and clock feedthrough of switches, parasitic issues in physical level and the trade-off between other performances and power consumption, etc. A final silicon implementation of the proposed SC circuit architecture is expected.

Both GQBPS and GUBPS were invented orienting to single-band RF/IF applications. More deep studies on multi-band RF bandpass sampling have come out [62, 100, 101], and it is of more interest to further investigate GQBPS and GUBPS for multi-band RF applications. It is promising to see multistandard sub-sampling receivers using the concept of generalize bandpass sampling in the near future.

Bibliography

- [1] G. Bussey, *Marconi's Atlantic Leap*. Marconi Communications, 2001.
- [2] L. Ahline and J. Zander, *Principles of Wireless Communications*. Studentlitteratur, Lund, 1997.
- [3] H. Nyquist, "Certain factors affecting telegraph speed," Bell Syst. Tech. J., vol. 3, p. 324, 1924.
- C. E. Shannon, "A mathematical theory of communication," Bell Syst. Tech. J., vol. 27, pp. 623–656, October 1948.
- [5] Online, International Telecommunication Union: http://www.itu.int/, URL.
- [6] S. Mirabbasi and K. Martin, "Classical and modern receiver architectures," *IEEE Communications Magazine*, pp. 132–139, November 2000.
- [7] J. Crols, "Low-IF topologies for high-performance analog front ends of fully integrated receivers," *IEEE Trans. CAS-II:Express Briefs*, vol. 45, pp. 269– 282, March 1998.
- [8] J. C. Rudel et al., "A 1.9-GHz wide-band IF double conversion CMOS receiver for cordless telephone applications," *IEEE Journal of Solid-State Cir*cuits, vol. 32, pp. 2071–2088, Dec. 1997.
- K. C. Zangi and R. D. Koilpillai, "Software radio issues in cellular base stations," *IEEE Journal of Solid-State Circuits*, vol. 17, pp. 561–573, April 1999.
- [10] R. J. Lackey and D. W. Upmal, "Speakeasy: The military software radio," *IEEE Commun. Mag.*, vol. 33, pp. 56–61, May 1995.
- [11] J. Mitola, "The software radio architecture," *IEEE Communications Maga*zine, pp. 26–38, May 1995.
- [12] J. Ryynänen et al., "A single-chip multimode receiver for GSM900, DCS1800, PCS1900, and WCDMA," *IEEE Journal of Solid-State Circuits*, vol. 38, pp. 594–602, April 2003.

- [13] M. Brandolini *et al.*, "Toward multistandard mobile terminals fully integrated receivers requirements and architectures," *IEEE Trans. Microwave Theory and Techniques*, pp. 1026–1038, March 2005.
- [14] R. G. Vaughan, N. L. Scott, and D. R. White, "The theorem of bandpass sampling," *IEEE Trans. on Signal Processing*, vol. 39, pp. 1973–1984, Sept. 1991.
- [15] Y.-R. Sun and S. Signell, "Effects of noise and jitter in bandpass sampling," Journal of Analog Integrated Circuits and Signal Processing –Special Issue of Norchip'03, vol. 42, pp. 85–97, January 2005.
- [16] K. Muhammad et al., "Digital RF processing: Toward low-cost reconfigurable radios," *IEEE Communications Magazine*, pp. 105–113, August 2005.
- [17] K. Muhammad *et al.*, "A discrete-time bluetooth receiver in a 0.13μm digital CMOS process," *Proceeding of ISSCC*, pp. 268–269, 2004.
- [18] D. Jakonis, Direct RF Sampling Receivers for Wireless Systems in CMOS Technology. PhD thesis, Linköping University, Sweden, 2004.
- [19] D. Jakonis *et al.*, "A 2.4-GHz RF sampling receiver front-end in 0.18μm CMOS," *IEEE Journal of Solide-State Circuits*, vol. 40, pp. 1265–1277, June 2005.
- [20] S. Karvonen *et al.*, "A low noise quadrature subsampling mixer," *Proceeding of ISCAS*, vol. 4, pp. 790–793, May 2001.
- [21] S. Karvonen and J. Kostamovaara, "Charge-domain FIR sampler with programmable filtering coefficients," *Proceeding of ISCAS*, vol. 5, pp. 4425–4428, May 2005.
- [22] S. Karvonen *et al.*, "A quadrature charge-domain sampler with embedded FIR and IIR filtering functions," *To appear in IEEE Journal of Solid-State Circuits*, 2006.
- [23] H. S. Shapiro and R. A. Silverman, "Aliasing free sampling of random noise," J. Soc. Indust. Appl. Math., vol. 8, pp. 225–248, June 1960.
- [24] A. J. Jerri, "The shannon sampling theory its various extensions and applications: A tutorial review," *Proc. of the IEEE*, vol. 65, no. 11, pp. 1565–1598, 1977.
- [25] A. Papoulis, "Generalized sampling expansion," IEEE Trans. on Circuits and Systems, vol. CAS-24, pp. 652–654, Nov. 1977.
- [26] A. V. Balakrishnan, "On the problem of time jitter in sampling," IRE Trans. on Information Theory, pp. 226–236, April 1961.

- [27] L. R. Carley and T. Mukherjee, "High-speed low-power integrating CMOS sample-and-hold amplifier architecture," *IEEE Custom Integrated Circuits Conference*, pp. 543–546, 1995.
- [28] G. Xu, Charge Sampling Circuits and A/D Converters Theory and Experiments. PhD thesis, Lund University, Sweden, 2004.
- [29] J. Yuan, "Accurate sampling of radio signals beyond GHz in CMOS," GHz 2000 Symposium, pp. 277–280, March 2000.
- [30] K. Muhammad and R. B. Staszewski, "Direct RF sampling mixer with recursive filtering in charge domain," *Proc. of IEEE International Symposium on Circuits and System (ISCAS)*, vol. I, pp. 577–580, 2004.
- [31] S. J. Orfanidis, *Introduction to signal Processing*. Prentice Hall, 1996.
- [32] C. E. Shannon, "Communication in the presence of noise," Proc. IRE, vol. 37, pp. 10–21, 1949.
- [33] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: principles, algorithms, and applica tions.* Prentice Hall, 1996.
- [34] "Digital alias-free signal processing–application notes: Nonuniform sampling (AN1)," *EURODASP*, 2001.
- [35] Y.-R. Sun and S. Signell, "Effects of noise and jitter in bandpass sampling," Proc. 21st Norchip conference, Nov. 2003.
- [36] S. S. Rajput and S. S. Jamuar, "Low voltage analog circuit design techniques," *IEEE Circuits and Systems Magazine*, vol. 2, pp. 24–42, First Quarter 2002.
- [37] B. Liu, "Timing jitter in digital filtering of analog signals," *IEEE Trans. on Circuits and Systems*, vol. CAS-22, pp. 218–223, March 1975.
- [38] R. J. Marks II, Introduction to Shannon Sampling and Interpolation Theory. Springer-Verlag, 1991.
- [39] F. A. Marvasti, "A unified approach to zero-crossing and nonuniform sampling," Oak Park Ill., 1987.
- [40] J. J. Wojtiuk, Randomised Sampling for Radio Design. PhD thesis, Univ. of South Australia, 2000.
- [41] S. Signell, "Jittered uniform sampling examples," Proc. of ISCAS, pp. 988– 991, May 2004.
- [42] F. J. Beutler, "Alias-free randomly timed sampling of stochastic processes," *IEEE Trans. Info. Theory*, vol. 16, pp. 147–152, 1970.

- [43] E. Marsy, "Alias-free sampling: An alternative conceptualization and its applications," *IEEE Trans. Info. Theory*, vol. 24, pp. 317–324, 1978.
- [44] Online, http://docs.sun.com/db/doc/805-4368, URL.
- [45] E. T. Whittaker, "On the functions which are represented by the expansions of the interpolation theory," *Proc. Royal Soc.*, *Edinburgh*, vol. 35, pp. 181– 194, 1914.
- [46] H. Sedarat and D. G. Nishimura, "On the optimality of the gridding reconstruction algorithm," *IEEE Trans. on Medical Imaging*, vol. 19, pp. 306–317, April 2000.
- [47] J. R. Higgins, "A sampling theorem for irregularly spaced sample points," *IEEE Trans. Info. Theory*, vol. 22, pp. 621–622, 1976.
- [48] L. Mate, Hilbert space methods in science and engineering. Adam Hilger, Bristol and New York, 1989.
- [49] Y.-R. Sun and S. Signell, "Algorithms for nonuniform bandpass sampling in radio receiver," Proc. of International Symposium on Circuits and Systems (ISCAS), vol. I, pp. 1–4, May 2003.
- [50] C. Cenker, H. G. Feichtinger, and M. Herrmann, "Iterative algorithms in irregular sampling a first comparison of methods," *IEEE*, 1991.
- [51] A. Aldroubi and K. Gröchenig, "Nonuniform sampling and reconstruction in shift-invariant spaces," SIAM Rev., vol. 43, no. 4, pp. 585–620, 2001.
- [52] J. L. Yen, "On nonuniform sampling of bandwidth-limited signals," IRE Trans. on circuit theory, vol. CT-3, pp. 251–257, Dec. 1956.
- [53] B. Sankur and L. A. Gerhardt, "Reconstruction of signals from nonuniform samples," *IEEE international conference on communications*, pp. 1513–1518, 1973.
- [54] Online, http://ccrma-www.stanford.edu/jos/Interpolation/Interpolation.html, URL.
- [55] V. Välimäki, Discrete-Time Modeling of Acoustic Tubes Using Fractional Delay Filters. PhD thesis, Helsinki University of Technology, Finland, 1995.
- [56] T. I. Laakso et al., "Splitting the unit delay," IEEE Signal Processing Magazine, vol. 13, pp. 30–60, January 1996.
- [57] Y. C. Eldar and A. V. Oppenheim, "Filterbank reconstruction of bandlimited signals from nonuniform and generalized samples," *IEEE Trans. Sig. Proc.*, vol. 48, no. 10, pp. 2864–2875, 2000.

- [58] K. Lacanette, "A basic introduction to filters active, passive and switchedcapacitor," Application Note 779, National Semiconductor, April 1995.
- [59] B. Razavi, Design of Analog CMOS Integrated Circuits. McGrawHill, 2001.
- [60] J. G. Proakis, Digital Communications, Fourth Edition. McGraw-Hill, 2001.
- [61] Y.-R. Sun and S. Signell, "A generalized quadrature bandpass sampling in radio receivers," in Proceedings of Asia South Pacific Design Automation Conference (ASP-DAC), pp. 1288–1291, January 2005.
- [62] C. H. Tseng and S. C. Chou, "Direct downconversion of multiband RF signals using bandpass sampling," *IEEE Trans. on Wireless Communications*, vol. 5, pp. 72–76, January 2006.
- [63] Y.-R. Sun and S. Signell, "Jitter performance of reconstruction algorithms for nonuniform bandpass sampling," *Proc. European Conference of Circuit Theory and Design (ECCTD)*, vol. II, pp. 353–356, Sept. 2003.
- [64] Y.-R. Sun and S. Signell, "Effects of noise and jitter on algorithms for bandpass sampling in radio receivers," *Proc. of IEEE International Symposium on Circuits and System (ISCAS)*, May 2004.
- [65] C. B. Feldman and W. R. Bennett, "Bandwidth and transmission performance," Bell Syst. Tech. J., vol. 28, pp. 490–595, 1949.
- [66] A. Kohlenberg, "Exact interpolation of band-limited functions," J. Appl. Phys., vol. 24, pp. 1432–1436, Dec. 1953.
- [67] A. J. Coulson, "A generalization of nonuniform bandpass sampling," IEEE Trans. on Signal processing, vol. 43, pp. 694–704, Mar. 1995.
- [68] J. D. Gaskell, "Linear systems, Fourier transforms, and Optics," Wiley, New York, 1978.
- [69] J. L. J. Brown, "On uniform sampling of amplitude modulated signals," IEEE Trans. Aerosp. Electron. Syst., vol. AES-19, July 1983.
- [70] B. Razavi, *RF Microelectronics*. Prentice Hall, 1998.
- [71] J. H. Fischer, "Noise sources and calculation techniques for switched capacito r filters," *IEEE Journal of Solid-state Circuits*, vol. SC-17, pp. 742–752, August 1982.
- [72] P. P. Vaidynanathan, Multirate Systems and Filter Banks. Prentice Hall, 1993.
- [73] S. S. Awad, "Analysis of accumulated timing-jitter in the time domain," *IEEE Trans. on Instrumentation and Measurement*, vol. 47, no. 1, pp. 69–73, 1998.

- [74] H. Kobayashi et al., "Aperture jitter effects in wideband sampling systems," Proc. of 16th IEEE Instrumentation and Measurement Technology Conference, vol. II, pp. 880–885, 1999.
- [75] P. Y. Chan *et al.*, "A highly linear 1-GHz CMOS downconversion mixer," *ESSCIRC*, 1993.
- [76] E. Cijvat, "A 1.8 GHz subsampling CMOS downconversion circuit for integrated radio circuits," *Proc. of ISCAS*, vol. II, pp. 65–68, 1998.
- [77] H. Pekau and J. W. Haslett, "A 2.4 GHz CMOS sub-sampling mixer with integrated filtering," *IEEE Journal of Solide-State Circuits*, vol. 40, pp. 2159– 2166, November 2005.
- [78] A. I. Hussein and W. B. Kuhn, "Bandpass ΣΔ modulator employing undersampling of RF signals for wireless communication," *IEEE Trans. of CAS-II: Analog and digital signal processing*, vol. 47, pp. 614–620, July 2000.
- [79] P. Eriksson and H. Tenhunen, "The noise figure of a sampling mixer: Theory and measurement," *Proc. of ICECS*, pp. 899–902, 1999.
- [80] Y. Chen and K.-T. Tiew, "A sixth-order subsampling continuous-time bandpass Delta-Sigma modulator," Proc. of ISCAS'05, pp. 5589–5592, May 2005.
- [81] Y.-R. Sun and S. Signell, "Filtering transformation in generalized quadrature bandpass sampling," Proc. of IEEE International Conference on Electronics and Circuits, and Systems (ICECS), December 2005.
- [82] Y.-R. Sun and S. Signell, "The theory of generalized quadrature bandpass sampling in subsampling receivers," *submitted to IEEE Trans. of CAS-I: Regular paper*, December 2005.
- [83] Y.-R. Sun and S. Signell, "Generalized quadrature bandpass sampling with FIR filtering," in Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS), pp. 4429–4432, May 2005.
- [84] Y.-R. Sun and S. Signell, "Analysis and implementation of uniform quadrature bandpass sampling," in Proceedings of IEEE Workshop on Signal Processing System (SiPS), pp. 137–142, November 2005.
- [85] D. A. Linden and N. M. Abramson, "A generalization of the sampling theorem," *Inform. Contr.*, vol. 3, pp. 26–31, 1960.
- [86] "Digital modulation in communications systems-an introduction," Agilent Application Notes 1298, 2001.
- [87] M. Valkama and M. Renfors, "A novel image rejection architecture for quadrature radio receivers," *IEEE Trans. on Circuits and Systems-II: Express Briefs*, vol. 51, pp. 61–68, Feb. 2004.
- [88] S. Ries, "Reconstruction of real and analytic band-pass signals from a finite number of samples," *Signal Processing*, vol. 33, no. 3, pp. 237–257, 1993.
- [89] V. J. Arkesteijin *et al.*, "Jitter requirements of the sampling clock in software radio receivers," *IEEE Trans. CAS-II: Express briefs*, vol. 53, pp. 90–94, February 2006.
- [90] K. W. Martin, "Complex signal processing is not complex," IEEE Trans. CAS-I:Regular Paper, vol. 51, pp. 1823–1836, September 2004.
- [91] M. Gustavsson, J. J. Wikner and N. Tan, CMOS data converters for communications. Norwell, MA: Kluwer, 2000.
- [92] M. Gustavsson and N. N. Tan, "A global passive sampling technique for highspeed switched-capacitor time-interleaved ADCs," *IEEE Trans. on Circuits* and Systems – II: Analog and Digital Signal Processing, vol. 47, pp. 821–831, September 2000.
- [93] K. Y. Kim, A 10-bit 100 Msample-per-Second Analog-to-Digital Converter in 1μm CMOS Technology. PhD thesis, University of California, Los Angeles, 1996.
- [94] P. Allen and D. R. Holberg, CMOS Analog Circuit Design (2nd Edition). NY Oxford University Press, 2002.
- [95] D. A. Johns and K. Martin, Analog Integrated Circuit Design, vol. 22. Wiley, 1997.
- [96] T. Shier and S. Signell, "Complex switched capacitor filter and designing method for such a filter," United States Patent, No. 6,266,289 B1, July 2001.
- [97] P. R. Gray et al., Analysis and Design of Analog Integrated Circuits (4th edition). John Wiley & Sons, Inc., 2001.
- [98] S. Signell, Studies of Structures for Switched-Capacitor Filters with Applications. PhD thesis, TRITA-TTT 8706, Royal Institute of Technology (KTH), 1987.
- [99] L. F. Nozkberg, *Filter*. KTH, 1995.
- [100] D. M. Akos et al., "Direct bandpass sampling of multiple distinct RF signals," IEEE Trans. on Communications, vol. 47, pp. 983–988, 1999.
- [101] J. Bae and J. Park, "An efficient algorithm for bandpass sampling of multiple RF signals," *IEEE Signal Processing Letters*, vol. 13, pp. 193–196, April 2006.
- [102] L. W. Couch II, Digital and Analog Communication Systems, (5th Edition). Prentice-Hall International, 1987.

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Appendix A

Fourier Transform Analysis of Conventional Charge Sampling and Charge Sampling with FIR filtering

As shown in Figure 2.5, the ideal sampled-data signal by conventional charge sampling process has been written as eq. (2.10) and is shown here again:

$$x_s(t) = \sum_{n=-\infty}^{\infty} \left(\int_{t_n}^{t_n + \Delta t} x(\xi) d\xi \right) \delta(t - t_n - \Delta t).$$
(A.1)

Assuming that $t_n = nT_s$ $(f_s = 1/T_s)$, the output voltage of charge sampling in Figure 2.6 in the time domain is given by

$$V_{out}(t) = \frac{1}{C_L} \sum_{n=-\infty}^{\infty} \left(\int_{nT_s}^{nT_s + \Delta t} I_{in}(\xi) d\xi \right) \delta(t - nT_s - \Delta t)$$
$$= \frac{1}{C_L} \left(\int_{t-\Delta t}^{t} I_{in}(\xi) d\xi \right) \sum_{n=-\infty}^{\infty} \delta(t - nT_s - \Delta t).$$
(A.2)

The infinite periodic Dirac delta function can be represented as its corresponding Fourier series:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s - \Delta t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{j2\pi k f_s(t - \Delta t)},$$
 (A.3)

the corresponding Fourier transform spectrum of $V_{out}(t)$ is given by

$$\begin{split} \tilde{V}_{out}(f) &= \int_{-\infty}^{\infty} V_{out}(t) e^{-j2\pi ft} dt \\ &= \frac{1}{C_L} \int_{-\infty}^{\infty} \left(\int_{t-\Delta t}^{t} I_{in}(\xi) d\xi \right) \cdot \left(\frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{j2\pi k f_s(t-\Delta t)} \right) e^{-j2\pi ft} dt \\ &= \frac{1}{C_L \cdot T_s} \int_{-\infty}^{\infty} \left(\int_{t-\Delta t}^{t} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) e^{j2\pi \nu \xi} d\nu \right) d\xi \right) \\ &\quad \cdot \sum_{k=-\infty}^{\infty} e^{j2\pi k f_s(t-\Delta t)} e^{-j2\pi ft} dt \\ &= \frac{1}{C_L \cdot T_s} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \left(\int_{t-\Delta t}^{t} e^{j2\pi \nu \xi} d\xi \right) d\nu \right) \\ &\quad \cdot \sum_{k=-\infty}^{\infty} e^{j2\pi k f_s(t-\Delta t)} e^{-j2\pi ft} dt \\ &= \frac{\Delta t}{C_L \cdot T_s} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \mathrm{sinc}(\nu \Delta t) e^{j2\pi \nu (t-\Delta t/2)} d\nu \right) \\ &\quad \cdot e^{j2\pi k f_s(t-\Delta t)} \cdot e^{-j2\pi ft} dt \\ &= \frac{\Delta t}{C_L \cdot T_s} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \mathrm{sinc}(\nu \Delta t) \\ &\quad \cdot \left(\int_{-\infty}^{\infty} e^{-j\pi k f_s \Delta t} \cdot e^{j2\pi (\nu+kf_s)(t-\Delta t/2)} \cdot e^{-j2\pi f(t-\Delta t/2)} \cdot e^{-j\pi f \Delta t} dt \right) d\nu \\ &= \frac{\Delta t}{C_L \cdot T_s} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \mathrm{sinc}(\nu \Delta t) e^{-j\pi (f+kf_s)\Delta t} \delta(f-\nu-kf_s) d\nu \\ &= \frac{\Delta t}{C_L \cdot T_s} \sum_{k=-\infty}^{\infty} \tilde{I}_{in}(f-kf_s) \mathrm{sinc}[(f-kf_s)\Delta t] e^{-j\pi (f+kf_s)\Delta t}, \end{split}$$

where $\tilde{I}_{in}(\nu)$ is the Fourier transform of $I_{in}(\xi)$, and $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$.

With introducing Nth order FIR filtering in the charge sampling process, the output voltage in the time domain is changed to

$$V'_{out}(t) = \frac{1}{C_L} \sum_{n=-\infty}^{\infty} \left[h_0 \int_{t_n}^{t_n + \Delta t} I_{in}(\xi) d\xi + h_1 \int_{t_n + \Delta t}^{t_n + 2\Delta t} I_{in}(\xi) d\xi + \cdots + h_N \int_{t_n + (N-1)\Delta t}^{t_{n+1}} I_{in}(\xi) d\xi \right] \delta(t - t_{n+1}).$$
(A.5)

Assuming $t_n = nT_s$ $(f_s = 1/T_s)$, $t_{n+1} = (n+1)T_s$, i.e., $N \cdot \Delta t = T_s$ and using the Fourier series of infinite periodic Dirac function, the corresponding Fourier transform spectrum becomes:

$$\begin{split} \tilde{V}_{out}'(f) &= \int_{-\infty}^{\infty} V_{out}'(f) e^{-j2\pi f t} df \\ &= \frac{1}{C_L \cdot T_s} \int_{-\infty}^{\infty} \left[h_0 \int_{t-T_s}^{t-T_s + \Delta t} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) e^{j2\pi\nu\xi} d\nu \right) d\xi \right. \\ &\quad + h_1 \int_{t-T_s + \Delta t}^{t-T_s + 2\Delta t} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) e^{j2\pi\nu\xi} d\nu \right) d\xi + \cdots \right. \\ &\quad + h_N \int_{t-T_s + (N-1)\Delta t}^{t} \left(\int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) e^{j2\pi\nu\xi} d\nu \right) d\xi \right] \sum_{k=-\infty}^{\infty} e^{j2\pi k f_s(t-T_s)} e^{-j2\pi f t} dt \\ &= \frac{1}{C_L \cdot T_s} \int_{-\infty}^{\infty} \left[h_0 \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \left(\int_{t-T_s}^{t-T_s + \Delta t} e^{j2\pi\nu\xi} d\xi \right) d\nu \right. \\ &\quad + h_1 \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \left(\int_{t-T_s + \Delta t}^{t} e^{j2\pi\nu\xi} d\xi \right) d\nu + \cdots \right. \\ &\quad + h_N \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \left(\int_{t-T_s + (N-1)\Delta t}^{t} e^{j2\pi\nu\xi} d\xi \right) d\nu + \cdots \right. \\ &\quad + h_N \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \left(\int_{t-T_s + (N-1)\Delta t}^{t} e^{j2\pi\nu\xi} d\xi \right) d\nu + \cdots \right. \\ &\quad + h_N \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \sin(\nu\Delta t) e^{j2\pi\nu(t-T_s + \Delta t/2)} d\nu \\ &\quad + h_1 \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \sin(\nu\Delta t) e^{j2\pi\nu(t-T_s + \Delta t/2)} d\nu + \cdots \right. \\ &\quad + h_N \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \sin(\nu\Delta t) e^{j2\pi\nu[t-T_s + (2N-1)\Delta t/2]} d\nu \\ &= \frac{\Delta t}{C_L \cdot T_s} e^{-j2\pi f T_s} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{I}_{in}(\nu) \sin(\nu\Delta t) \delta(f - \nu - kf_s) \\ &\quad \cdot \sum_{n=0}^{N} h_n e^{-j\pi\nu(2n-1)\Delta t} d\nu \\ &= \frac{\Delta t}{C_L \cdot T_s} e^{-j2\pi f T_s} \sum_{k=-\infty}^{\infty} \tilde{I}_{in}(f - kf_s) \sin[(f - kf_s)\Delta t] H(f - kf_s), \end{split}$$
(A.6)

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where the transfer function of FIR filtering is given by

$$H(f - kf_s) = \sum_{n=0}^{N} h_n e^{-j\pi(f - kf_s)(2n-1)\Delta t},$$
(A.7)

and h_n represents the coefficients of FIR filtering.

Appendix B

PSD Spectrum Analysis of Jitter Sampling

Starting from eq. (2.1) and eq. (2.13), the CT sampled-data signal by JS is given by

$$\tilde{x}_{js}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s - \tau_n)$$
$$= \sum_{n=-\infty}^{\infty} x(t_s(n))\delta(t - nT_s - \tau_n),$$
(B.8)

where the input signal x(t) could be either deterministic or stochastic process, nT_s is the expected uniform sampling time instants, τ_n represents a family of iid random variables, and normally $\tau_n \ll T_s$, $\tilde{x}_{js}(t)$ is a stochastic process. The statistic process τ_n and x(t) are independent. The autocorrelation function of $\tilde{x}_{js}(t)$ is given by

$$\begin{aligned} r_{\tilde{x}\tilde{x}}(\gamma,t) &= E_{x,\gamma} \left[\tilde{x}_{js}(t+\gamma)\tilde{x}_{js}^{*}(t) \right] \\ &= E_{\gamma} \left[\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} E_{x} \left[x(t+\gamma)x^{*}(t) \right] \cdot \delta(t+\gamma - mT_{s} - \tau_{m})\delta(t-nT_{s} - \tau_{n}) \right] \\ &= E_{\gamma} \left[\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} r_{xx}(\gamma) \cdot \delta(t+\gamma - mT_{s} - \tau_{m})\delta(t-nT_{s} - \tau_{n}) \right] \\ &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_{xx}(\gamma) \cdot \delta(t+\gamma - mT_{s} - \tau_{m})\delta(t-nT_{s} - \tau_{n}) \\ p(\tau_{m}, \tau_{n})d\tau_{m}d\tau_{n} \\ &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_{xx}(\gamma) \cdot \delta(t+\gamma - mT_{s} - \tau_{m})\delta(t-nT_{s} - \tau_{n}) \\ p(\tau_{m})p(\tau_{n})d\tau_{m}d\tau_{n} \\ &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} r_{xx}(\gamma)p(t-mT_{s} + \gamma)p(t-nT_{s}), \end{aligned}$$
(B.9)

for $m \neq n$, where $E[\bullet]$ represents an expectation operator, $E_{x,\gamma}$ is the average over the product statistics of τ_n and x(t), E_{γ} is over the statistics of τ_n and E_x over the statistics of x(t), $r_{xx}(\gamma)$ is the autocorrelation function of x(t), γ is a timelag between any two variables of stochastic process $\tilde{x}_{js}(t)$, $p(\tau_m, \tau_n)$ is the joint probability density function (PDF) of $\{\tau_n\}$ and $\{\tau_m\}$. The random variables τ_n and τ_m are assumed independent such that $p(\tau_m, \tau_n) = p(\tau_n)p(\tau_m)$, where p(x) is the PDF of stochastic process x. When m = n, $\tau_m = \tau_n$,

$$r_{\tilde{x}\tilde{x}}(\gamma) = r_{xx}(0)\delta(\gamma),\tag{B.10}$$

where $r_{xx}(0)$ corresponds to the total input signal power. Assuming that $\tilde{x}_{js}(t)$ is a wide-sense stationary (WSS) process and $\tilde{x}_{js}(t)$, $\tilde{x}_{js}(t+\gamma)$ are jointly ergodic, the time average may be used to replace the ensemble average. The autocorrelation function of $\tilde{x}_{js}(t)$ is simplified by time-average over a single sampling period [102]:

$$r_{\tilde{x}\tilde{x}}(\gamma) = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} r_{\tilde{x}\tilde{x}}(\gamma, t) dt$$
$$= \frac{1}{T_s} r_{xx}(\gamma) \left(\sum_{l=-\infty}^{\infty} r_{pp}(lT_s + \gamma) - r_{pp}(\gamma) + \delta(\gamma) \right), \quad (B.11)$$

where $r_{pp}(lT_s + \gamma)$ is the convolution of two PDF functions. Based on Wiener-Khintchine Theorem, the PSD of the WSS process $\tilde{x}_{js}(t)$ can be obtained from the Fourier transform of the autocorrelation function $r_{\tilde{x}\tilde{x}}(\gamma)$ [102],

$$\begin{aligned} R_{\tilde{x}\tilde{x}}(f) &= \frac{1}{T_s} \int_{-\infty}^{\infty} \left[r_{xx}(\gamma) \left(\sum_{l=-\infty}^{\infty} r_{pp}(lT_s + \gamma) - r_{pp}(\gamma) + \delta(\gamma) \right) e^{-j2\pi f\gamma} \right] d\gamma \\ &= \frac{1}{T_s} \left(\int_{-\infty}^{\infty} r_{xx}(\gamma) e^{-j2\pi f\gamma} d\gamma \right) \\ &\quad \star \left(\sum_{l=-\infty}^{\infty} \left[r_{pp}(lT_s + \gamma) - r_{pp}(\gamma) + \delta(\gamma) \right] e^{-j2\pi f\gamma} d\gamma \right) \\ &= \frac{1}{T_s} R_{xx}(f) \star \left(\sum_{l=-\infty}^{\infty} R_{pp}(f) e^{j2\pi lT_s} - R_{pp}(f) + 1 \right) \\ &= \frac{1}{T_s} R_{xx}(f) \star \left(\frac{1}{T_s} R_{pp}(f) \sum_{k=-\infty}^{\infty} \delta(f - kf_s) + [1 - R_{pp}(f)] \right) \\ &= \frac{1}{T_s^2} \sum_{k=-\infty}^{\infty} R_{pp}(kf_s) R_{xx}(f - kf_s) + \frac{1}{T_s} R_{xx}(f) \star (1 - R_{pp}(f)) , (B.12) \end{aligned}$$

where \star denotes the convolution operator, $R_{xx}(f)$ and $R_{pp}(f)$ is the Fourier transform of $r_{xx}(\gamma)$ and $r_{pp}(\gamma)$, respectively and $f_s = 1/T_s$.

Appendix C

Charge Conservation Analysis of the SC FIR Filter and the Composite FIR/IIR Filter in the GUBPS Implementation

1. FIR filter:

Starting from Figure 4.19 with the corresponding appropriate clock scheme shown in Figure 4.20, the circuit can be analyzed by charge conservation [98]. The clock skew problem is not considered in the analysis, and it is assumed that clock $\phi = 1^{\circ}$ when $\phi_{2i} = 1^{\circ}$ ($i = 0, 1, 2, \dots, N$). The input-offset voltages of the opamps are neglected. The charges stored on different capacitors at different clock phases are listed below:

$$t = nT_s: \phi_0 = 1, \phi_m = 0 (m \neq 0) \text{ and } \phi_a = 1$$
$$Q_{Cb_0}^{(0)}(n) = C_{b_0} V_{in}^{(0)}(n) \quad Q_{Cb_1}^{(0)}(n) = C_{b_1} V_{in}^{(0)}(n)$$
$$Q_{Ch_1}^{(0)}(n) = 0 \qquad \qquad Q_{Ch_2}^{(0)}(n) = 0$$

 $t=nT_s:~\phi_1{=`1'},~\phi_m{=`0'}~(m\neq 1)$ and $\phi_a{=`0'}$

$$\begin{aligned} Q_{C_{b_0}}^{(1)}(n) &= 0 & Q_{C_{b_1}}^{(1)}(n) = 0 \\ Q_{C_{h_1}}^{(1)}(n) &= V_{out}^{Re(1)}(n)C_h & Q_{C_{h_2}}^{(1)}(n) = V_{out}^{Im(1)}(n)C_h \\ \vdots & \vdots & \vdots \end{aligned}$$

 $t = nT_s$: ϕ_{2N} ='1', ϕ_m ='0' ($m \neq 2N$) and ϕ_a ='0'

$$Q_{C_{b_0}}^{(2N)}(n) = C_{b_0} V_{in}^{(2N)}(n) \quad Q_{C_{b_1}}^{(2N)}(n) = C_{b_1} V_{in}^{(2N)}(n)$$
$$Q_{C_{h_1}}^{(2N)}(n) = Q_{C_{h_1}}^{(2N-1)}(n) \quad Q_{C_{h_2}}^{(2N)}(n) = Q_{C_{h_2}}^{(2N-1)}(n)$$

 $t = nT_s$: $\phi_{2N+1} = 1$, $\phi_m = 0$ $(m \neq 2N + 1)$ and $\phi_a = 0$

$$Q_{C_{b_0}}^{(1)}(n) = 0 \qquad \qquad Q_{C_{b_1}}^{(1)}(n) = 0 Q_{C_{h_1}}^{(2N+1)}(n) = V_{out}^{Re(2N+1)}(n)C_h \qquad Q_{C_{h_2}}^{(2N+1)}(n) = V_{out}^{Im(2N+1)}(n)C_h$$

$$t = (n+1)T_s: \ \phi_0 = `1', \ \phi_m = `0' \ (m \neq 0) \text{ and } \phi_a = `1'$$

$$Q_{C_{b_0}}^{(0)}(n+1) = C_{b_0}V_{in}^{(0)}(n+1) \qquad Q_{C_{b_1}}^{(0)}(n+1) = C_{b_1}V_{in}^{(0)}(n+1)$$

$$Q_{C_{b_1}}^{(0)}(n+1) = 0 \qquad \qquad Q_{C_{b_2}}^{(0)}(n+1) = 0$$

where C_{h1} and C_{h2} represent the integrating capacitor in the feedback loop of opamp for the real and imaginary datapath, respectively. It is observed that the charges stored in the sampling capacitors are conserved by C_h at each holding phase. General charge conservation equations are given by

$$Q_{C_{h1}}^{(2i+1)}(n) = Q_{C_{h1}}^{(2i)}(n) + Q_{C_{b_{2i}}}^{(2i)}(n) \qquad Q_{C_{h1}}^{(2i)}(n) = Q_{C_{h1}}^{(2i-1)}(n)$$
$$Q_{C_{h2}}^{(2i+1)}(n) = Q_{C_{h2}}^{(2i)}(n) + Q_{C_{b_{2i+1}}}^{(2i)}(n) \qquad Q_{C_{h2}}^{(2i)}(n) = Q_{C_{h2}}^{(2i-1)}(n)$$

The charges stored on C_h before C_h is reset is given by

$$Q_{C_{h1}}^{(2N+1)}(n) = \sum_{i=0}^{N} Q_{C_{b_{2i}}}^{(2i)}(n) + Q_{C_{h1}}^{(0)}(n)$$
$$Q_{C_{h2}}^{(2N+1)}(n) = \sum_{i=0}^{N} Q_{C_{b_{2i+1}}}^{(2i)}(n) + Q_{C_{h2}}^{(0)}(n)$$

The corresponding input-output relation is written as

$$\left(V_{out}^{Re(2N+1)}(n) + jV_{out}^{Im(2N+1)}(n)\right)C_h = \sum_{i=0}^N C_{b_{2i}}V_{in}^{(2i)}(n) + j\sum_{i=0}^N C_{b_{2i+1}}V_{in}^{(2i)}(n)$$

and the transfer function is given by

$$H_{FIR}(z) = \frac{V_{out}(z)}{V_{in}(z)} = z^{-1/2} \sum_{i=0}^{N} \left[\frac{C_{b_{2i}}}{C_h} + j \frac{C_{b_{2i+1}}}{C_h} \right] z^{-i}.$$
 (C.13)

where $z^{-1} = e^{-j2\pi k f_s T_D}$ and T_D is the time delay between any two successive sampling branches.

2. Composite FIR/IIR filter:

Starting from Figure 4.24 with the appropriate clock scheme, it is assumed that

$$C_{pi} = C_h(1-\alpha), \qquad C_{ai} = C_a, \qquad C_{hi} = C_h, \qquad C_{qi} = C_h|\beta|,$$

where subscripts i = 1, 2 are used to distinguish that the capacitor is the upper one or the lower one, and the clock ϕ is going '1' when ϕ_{2i} $(i = 0, 1, 2, \dots, N)$ is going to '1'. Because both α and β are negative in the GUBPS implementation, an absolute value of β is used here. The duty cycle of ϕ determines the charging time. The charges stored on different capacitors at different clock phases are listed below: $t=nT_s:\;\phi_{2N+1}=\phi_0{=`1'},\,\phi_m{=`0'}\;(m\neq 2N+1,0)$ and $\phi_b{=`1'}$

$$\begin{array}{ll} Q^{(0)}_{C_{b_{2N}}}(n) = 0 & Q^{(0)}_{C_{b_{2N+1}}}(n) = 0 \\ Q^{(0)}_{C_{b_0}}(n) = C_{b_0}V^{(0)}_{in}(n) & Q^{(0)}_{C_{b_1}}(n) = C_{b_1}V^{(0)}_{in}(n) \\ Q^{(0)}_{C_{p_1}}(n) = V^{Im(0)}_{out}(n)C_{p_1} & Q^{(0)}_{C_{p_2}}(n) = V^{Re(0)}_{out}(n)C_{p_2} \\ Q^{(0)}_{C_{a_1}}(n) = 0 & Q^{(0)}_{C_{a_2}}(n) = 0 \\ Q^{(0)}_{C_{h_1}}(n) = V^{Re(0)}_{out}(n)C_{h_1} & Q^{(0)}_{C_{h_2}}(n) = V^{Im(0)}_{out}(n)C_{h_2} \\ Q^{(0)}_{C_{q_1}}(n) = V^{Re(0)}_{out}(n)C_{q_1} & Q^{(0)}_{C_{q_2}}(n) = V^{Im(0)}_{out}(n)C_{q_2} \end{array}$$

$$t = nT_s + T_D$$
: $\phi_1 = \phi_2 = 1, \phi_m = 0$ $(m \neq 1, 2)$ and $\phi_b = 1$

$$\begin{array}{ll} Q^{(1)}_{C_{b_0}}(n) = 0 & Q^{(1)}_{C_{b_1}}(n) = 0 \\ Q^{(1)}_{C_{b_2}}(n) = C_{b_2} V^{(1)}_{in}(n) & Q^{(1)}_{C_{b_3}}(n) = C_{b_3} V^{(1)}_{in}(n) \\ Q^{(1)}_{C_{p_1}}(n) = V^{Im(1)}_{out}(n) C_{p_1} & Q^{(1)}_{C_{p_2}}(n) = V^{Re(1)}_{out}(n) C_{p_2} \\ Q^{(1)}_{C_{a_1}}(n) = 0 & Q^{(1)}_{C_{a_2}}(n) = 0 \\ Q^{(1)}_{C_{h_1}}(n) = V^{Re(1)}_{out}(n) C_{h_1} & Q^{(1)}_{C_{h_2}}(n) = V^{Im(1)}_{out}(n) C_{h_2} \\ Q^{(1)}_{C_{q_1}}(n) = V^{Re(1)}_{out}(n) C_{q_1} & Q^{(1)}_{C_{q_2}}(n) = V^{Im(1)}_{out}(n) C_{q_2} \\ \vdots & \vdots \end{array}$$

$$t = nT_s + NT_D$$
: $\phi_{2N-1} = \phi_{2N} = 0$ ($m \neq 2N - 1, 2N$) and $\phi_b = 1$

$$\begin{aligned} &Q_{C_{b_{2N-2}}}^{(N)}(n) = 0 & Q_{C_{b_{2N-1}}}^{(r)}(n) = 0 \\ &Q_{C_{b_{2N-2}}}^{(N)}(n) = C_{b_{2N}}V_{in}^{(N)}(n) & Q_{C_{b_{2N+1}}}^{(N)}(n) = C_{b_{2N+1}}V_{in}^{(N)}(n) \\ &Q_{C_{p_1}}^{(N)}(n) = V_{out}^{Im(N)}(n)C_{p_1} & Q_{C_{p_2}}^{(N)}(n) = V_{out}^{Re(N)}(n)C_{p_2} \\ &Q_{C_{a_1}}^{(N)}(n) = 0 & Q_{C_{a_2}}^{(N)}(n) = 0 \\ &Q_{C_{b_1}}^{(N)}(n) = V_{out}^{Re(N)}(n)C_{h_1} & Q_{C_{h_2}}^{(N)}(n) = V_{out}^{Im(N)}(n)C_{h_2} \\ &Q_{C_{q_1}}^{(N)}(n) = V_{out}^{Re(N)}(n)C_{q_1} & Q_{C_{q_2}}^{(N)}(n) = V_{out}^{Im(N)}(n)C_{q_2} \end{aligned}$$

 $t=(n+1)T_s:\;\phi_{2N}=\phi_1{=`1"},\phi_m{=`0"}\;(m\neq 2N,1)$ and $\phi_b{=`0"}$

$$\begin{aligned} &Q^{(0)}_{C_{b_{2N}}}\left(n+1\right)=0 & Q^{(0)}_{C_{b_{2N+1}}}\left(n+1\right)=0 \\ &Q^{(0)}_{C_{b_0}}\left(n+1\right)=C_{b_0}V^{(0)}_{in}\left(n+1\right) & Q^{(0)}_{C_{b_1}}\left(n+1\right)=C_{b_1}V^{(0)}_{in}\left(n+1\right) \\ &Q^{(0)}_{C_{p_1}}\left(n+1\right)=0 & Q^{(0)}_{C_{p_2}}\left(n+1\right)=0 \\ &Q^{(0)}_{C_{a_1}}\left(n\right)=V^{Re(0)}_{out}\left(n+1\right)C_{a_1} & Q^{(0)}_{C_{a_2}}\left(n\right)=V^{Im(0)}_{out}\left(n+1\right)C_{a_2} \\ &Q^{(0)}_{C_{h_1}}\left(n\right)=V^{Re(0)}_{out}\left(n+1\right)C_{h_1} & Q^{(0)}_{C_{h_2}}\left(n\right)=V^{Im(0)}_{out}\left(n+1\right)C_{h_2} \\ &Q^{(0)}_{C_{q_1}}\left(n+1\right)=0 & Q^{(0)}_{C_{q_2}}\left(n+1\right)=0 \end{aligned}$$

It is assumed that the input-offset voltages of the opamps are neglected. The charge conservation equations at the switching-off activity of ϕ_b ('1' \rightarrow '0') are obtained as

$$\begin{split} & Q_{C_{b_{2N}}}^{(N)}(n) - Q_{C_{p1}}^{(N)}(n) + Q_{C_{a1}}^{(N)}(n) + Q_{C_{h1}}^{(N)}(n) - Q_{C_{q1}}^{(N)}(n) \\ &= Q_{C_{b_{2N}}}^{(0)}(n+1) - Q_{C_{p1}}^{(0)}(n+1) + Q_{C_{a1}}^{(0)}(n+1) + Q_{C_{h1}}^{(0)}(n+1) - Q_{C_{q1}}^{(0)}(n+1) \\ & Q_{C_{b_{2N+1}}}^{(N)}(n) - Q_{C_{p2}}^{(N)}(n) + Q_{C_{a2}}^{(N)}(n) + Q_{C_{h2}}^{(N)}(n) + Q_{C_{q2}}^{(N)}(n) \\ &= Q_{C_{b_{2N+1}}}^{(0)}(n+1) - Q_{C_{p2}}^{(0)}(n+1) + Q_{C_{a2}}^{(0)}(n+1) + Q_{C_{h2}}^{(0)}(n+1) + Q_{C_{q2}}^{(0)}(n+1) \\ \end{split}$$

Starting from the basic integrating equations:

$$V_{out}^{Re(N)}(n) \cdot C^{Re} = \sum_{i=0}^{N-1} V_{in}^{(i)}(n) C_{b_{2i}} + V_{out}^{Re(0)}(n) \cdot C^{Re}$$
$$V_{out}^{Im(N)}(n) \cdot C^{Im} = \sum_{i=0}^{N-1} V_{in}^{(i)}(n) C_{b_{2i+1}} + V_{out}^{Im(0)}(n) \cdot C^{Im},$$

where C^{Re} and C^{Im} represent the total integrating capacitance at the output of real and imaginary datapath, and substituting charge equations of different capacitors for different clock phases into the above charge conservation equations, we have

$$\begin{split} \sum_{i=0}^{N} V_{in}^{(i)}(n) C_{b_{2i}} &+ V_{out}^{Re(0)}(n) C_h - V_{out}^{Re(0)}(n) C_h(1-\alpha) \\ &- V_{out}^{Im(0)}(n) C_h |\beta| = V_{out}^{Re(0)}(n+1) (C_h + C_a) \\ \sum_{i=0}^{N} V_{in}^{(i)}(n) C_{b_{2i+1}} &+ V_{out}^{Im(0)}(n) C_h - V_{out}^{Im(0)}(n) C_h(1-\alpha) \\ &+ V_{out}^{Re(0)}(n) C_h |\beta| = V_{out}^{Im(0)}(n+1) (C_h + C_a). \end{split}$$

The input-output equation is then obtained as

$$\sum_{i=0}^{N} (C_{b_{2i}} + jC_{b_{2i+1}}) V_{in}^{(i)}(n) = V_{out}^{(0)}(n+1)(C_h + C_a) - V_{out}^{(0)}(n)C_h(\alpha - j\beta),$$

where $V_{out}(n) = V_{out}^{Re}(n) + jV_{out}^{Im}(n)$, and the corresponding transfer function of the composite FIR/IIR filter is given by

$$H_{FIR-IIR}(z) = \frac{V_{out}(z)}{V_{in}(z)} = \frac{z^{-1} \sum_{i=0}^{N} (C_{b_{2i}} + jC_{b_{2i+1}}) z^{-i}}{(C_h + C_a) - C_h(\alpha - j\beta) z^{-(N+1)}}, \quad (C.14)$$

where $z^{-1} = e^{-j2\pi k f_s T_D}$.