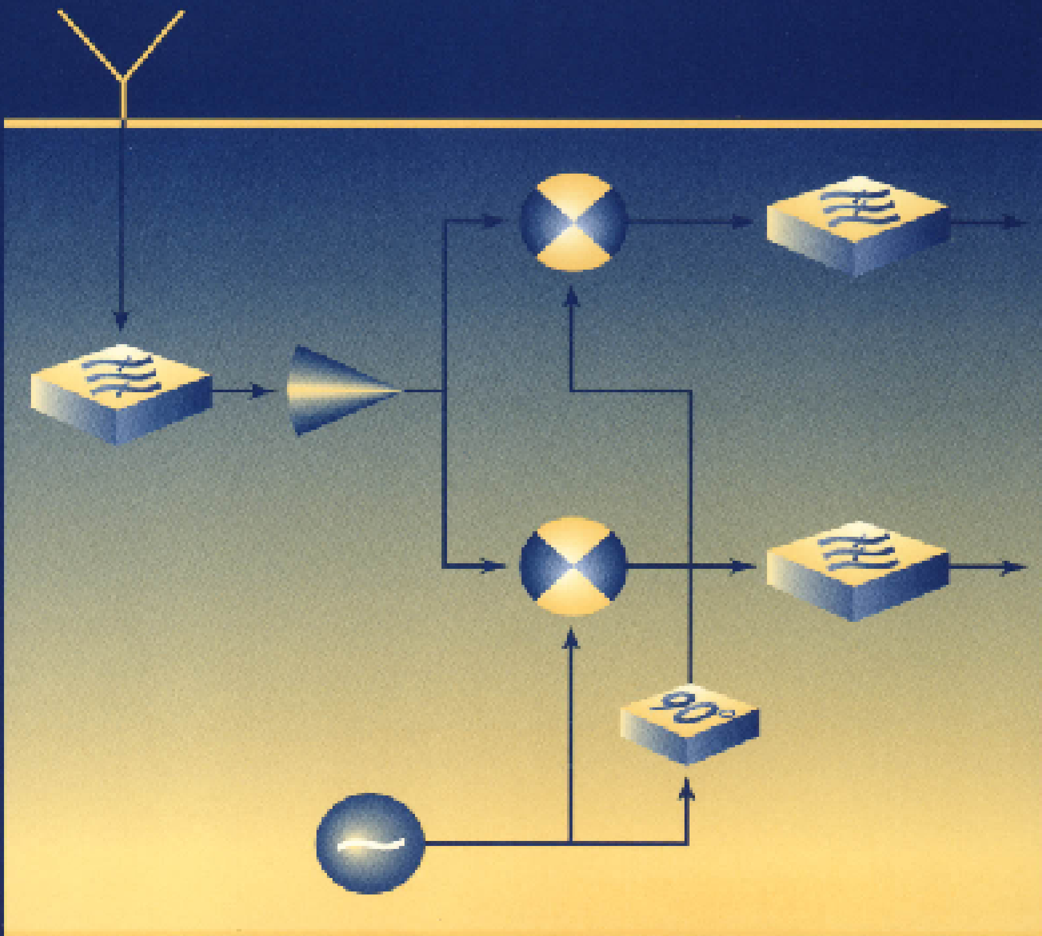
SOLUTIONS MANUAL

MICROWAVE AND RF DESIGN OF WIRELESS SYSTEMS



DAVID M. POZAR

Solutions Manual

for

Microwave and RF Design of Wireless Systems

This manual contains solutions for the end-of-chapter problems in **Microwave and RF Design of Wireless Systems**. Hopefully there is a good selection of theory versus design-type of problems, but the instructor should be able to use these as starting points to generate additional problems for homework assignments and exams. Many of the tuning, filter, amplifier, and oscillator problems will be facilitated if the reader has access to a commercial microwave CAD package, such as HP-MDS, Ansoft Serenade, or similar, but this is not essential.

Many of the solutions given here have been verified with known results, with independent solutions by others, or by computer simulation. Answers to these problems are indicated with a small check mark. Nevertheless, there likely are errors that have slipped through, and the author will be grateful if these are brought to his attention.

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Chapter 1

$\boxed{1.1}$ US cellular telephone statistics are given in the table below, from the CTIA annual survey.

Yearly subscriber growth can be estimated as:

97-98 subscriber growth = (69.2 - 55.3)/55.3 = 25%

98-99 subscriber growth = (86.0 - 69.2)/69.2 = 24%

Then we estimate year 2000 subscribers at

86.0M (1.24) = 106.6 M

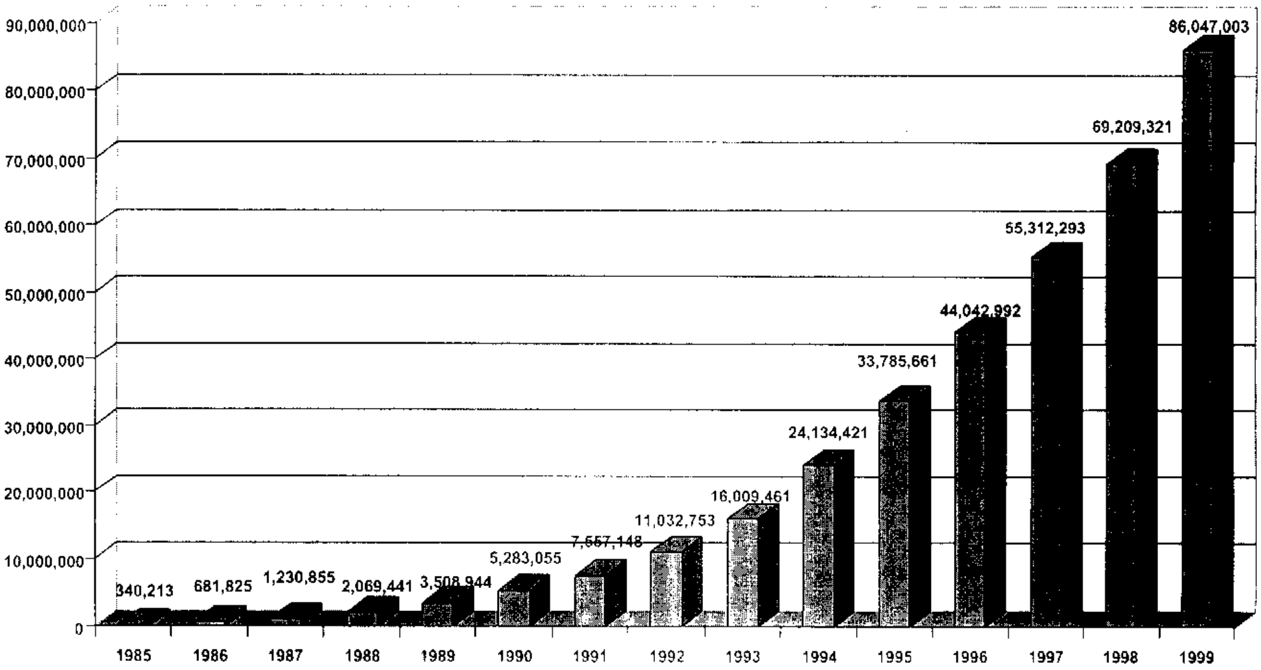
Graphs showing subscriber growth and average monthly bills are shown on the following page.

THE CELLULAR TELECOMMUNICATIONS INDUSTRY ASSOCIATION'S ANNUALIZED WIRELESS INDUSTRY SURVEY RESULTS
December 1985 to December 1999

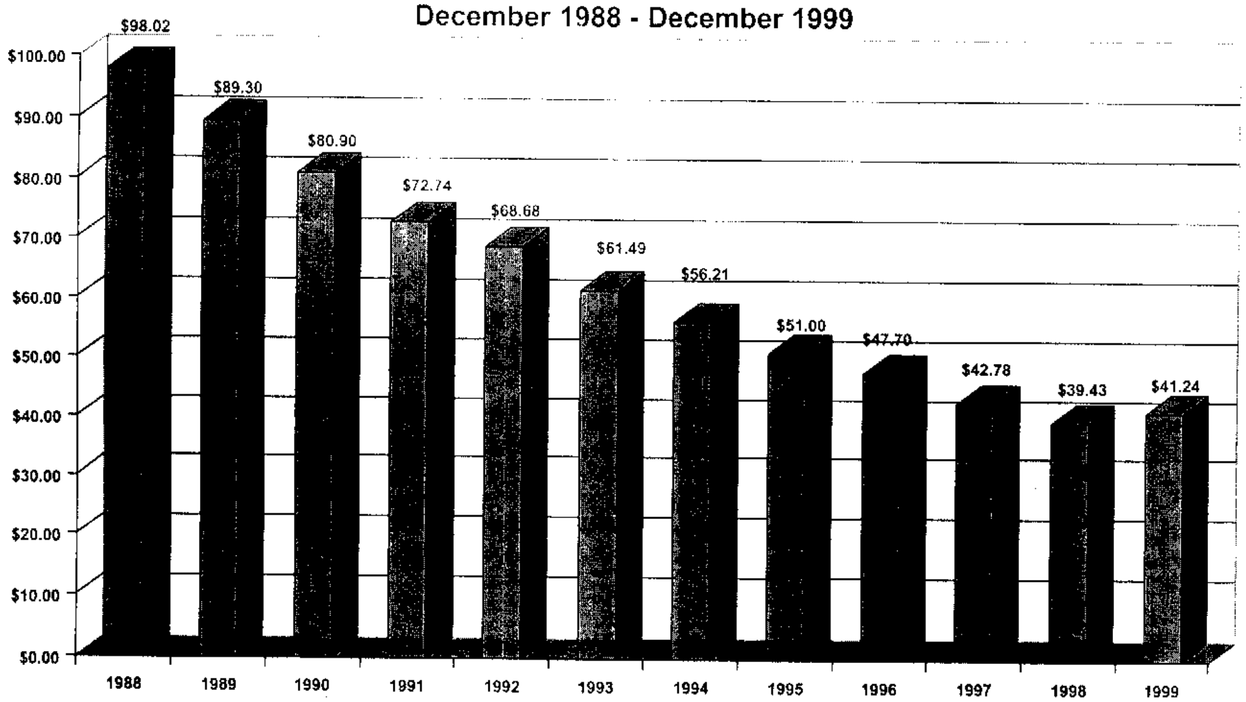
Reflecting Domestic U.S. Commercially-Operational Cellular, ESMR and PCS Providers

Date	Estimated Total Subscribers	Annualized Total Service Revenues (in 000s)	Annualized Roamer Revenues (in 000s)	Cell Sites	Direct Service Provider Employees	Cumulative Capital Investment (in 000s)	Average Local Monthly Bill	Average Local Call Length
1985	340,213	482,428	N/a	913	2,727	911,167	N/a	N/a
1986	681,825	823,052	N/a	1,531	4,334	1,436,753	N/a	N/a
1987	1,230,855	1,151,519	N/a	2,305	7,147	2,234,635	$96.83	2.33
1988	2,069,441	1,959,548	N/a	3,209	11,400	3,274,105	$98.02	2.26
1989	3,508,944	3,340,595	294,567	4,169	15,927	4,480,142	$89.30	2.48
1990	5,283,055	4,548,820	456,010	5,616	21,382	6,281,596	$80.90	2.20
1991	7,557,148	5,708,522	703,651	7,847	26,327	8,671,544	$72.74	2.38
1992	11,032,753	7,822,726	973,871	10,307	34,348	11,262,070	$68.68	2.58
1993	16,009,461	10,892,175	1,361,613	12,824	39,810	13,956,366	$61.49	2.41
1994	24,134,421	14,229,922	1,830,782	17,920	53,902	18,938,678	$56.21	2.24
1995	33,785,661	19,081,239	2,542,570	22,663	68,165	24,080,467	$51.00	2.15
1996	44,042,992	23,634,971	2,780,935	30,045	84,161	32,573,522	$47.70	2.32
1997	55,312,293	27,485,633	2,974,205	51,600	109,387	46,057,910	$42.78	2.31
1998	69,209,321	33,133,175	3,500,469	65,887	134,754	60,542,774	$39.43	2.39
1999	86,047,003	40,018,489	4,085,417	81,698	155,817	71,264,865	$41.24	2.38

Wireless Subscribership: December 1985 - December 1999

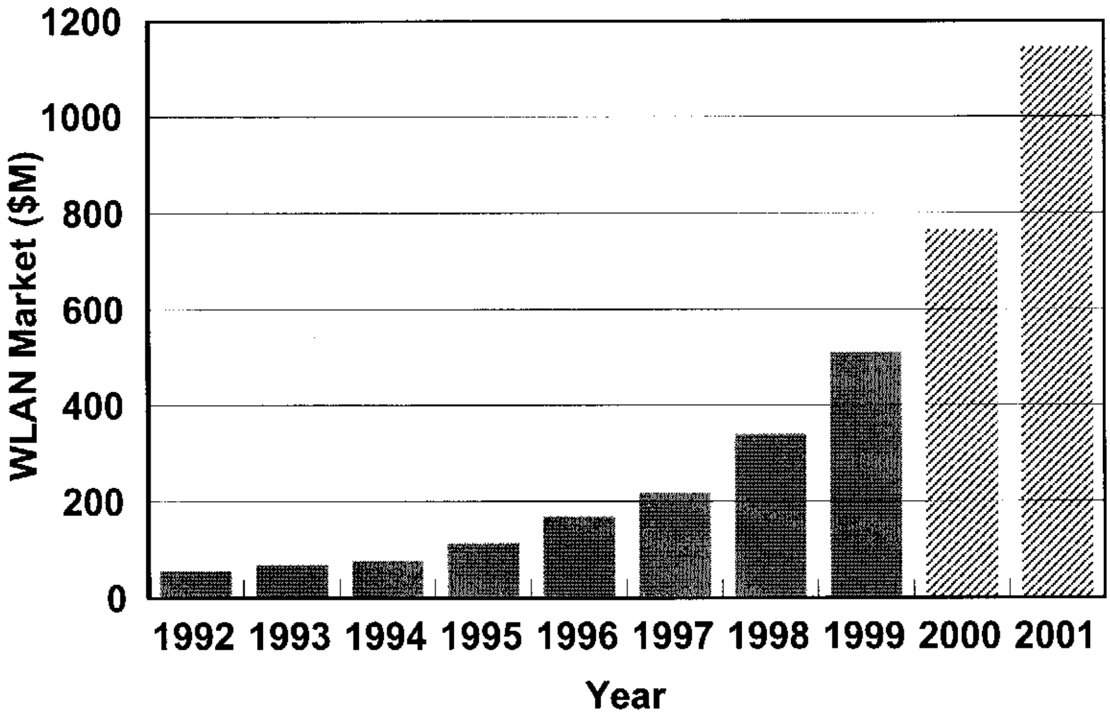


Average Local Monthly Bill: December 1988 - December 1999



1.2

WIRELESS LAN MARKET DATA WAS COLLECTED FROM SEVERAL SOURCES, INCLUDING THE YANKEE GROUP, THE GARTNER GROUP, AND OTHERS. VALUES WERE AVERAGED, AND PLOTTED BELOW. THE HASHED BARS ARE ESTIMATES FOR THE YEARS 2000 AND 2001.



1.3 THE MAJOR SPECTRUM ALLOCATIONS FOR 50 MHz to 2 GHz ARE LISTED BELOW:

BROADCAST RADIO AND TV:

FM:	88 – 108 MHz	→	20 MHz
TV:	54 – 72 MHz	→	18 MHz
	76 – 88 MHz	→	12 MHz
	174 – 216 MHz	→	42 MHz
	470 – 890 MHz	→	420 MHz
			512 MHz

WIRELESS SYSTEMS:

AMPS:	869 – 894 MHz	→	25 MHz
	824 – 849 MHz	→	25 MHz
PCS:	1710 – 1785 MHz	→	75 MHz
	1805 – 1880 MHz	→	75 MHz
PAGING:	931 – 932 MHz	→	1 MHz
ISM:	902 – 928 MHz	→	26 MHz
			227 MHz

We see that wireless systems occupy approximately half the bandwidth freely allocated to commercial radio and TV broadcasting.

Essays could take differing viewpoints. Useful information can be found in the following references:

[1] R. J. Orsulak, et al, U.S. Department of Commerce, NTIA TM-94-160, National Land Mobile Spectrum Requirements, January 1994.

[2] R. J. Mayher, et al, The SUM Data Base: A New Measure of Spectrum Use, U.S. Department of Commerce, NTIA TR-88-236, August 1988.

[3] U.S. National Spectrum Requirements: Projects and Trends NTIA Special Publication 94-31, 1995.

$\boxed{1.4}$ We consider two different cell phones:

CASE A: NOKIA 232 ANALOG (AMPS) PHONE
$P_A = 0.6\,W$ (TALK POWER CONSUMPTION), 6V BATTERY

CASE B: NOKIA 2170 DIGITAL (CDMA) PHONE
$P_B = 0.2\,W$ (TALK POWER CONSUMPTION), 6V BATTERY

ENERGY PER MINUTE OF TALK-TIME:

$$E_A = P_A T = (0.6w)(60\,sec) = \underline{36\ W\text{-}sec}\quad (/\text{MIN T.T.})$$

$$E_B = P_B T = (0.2w)(60\,sec) = \underline{12\ W\text{-}sec}\quad (/\text{MIN T.T.})$$

TYPICAL SOLAR PANELS:

1.5v, 500 mA PANEL (6.2cm x 12.0cm) COST ~ \$800!
FOUR OF THESE WILL PROVIDE 6.0V AT 500 mA
IN FULL SUNLIGHT. SIZE WILL BE ABOUT 24 cm x 12 cm,
OR 9" x 4.7" (~ 6" x 6" AREA)

500 mA WILL PROVIDE A "SLOW CHARGE" TO THE BATTERY.
WITH A TYPICAL CHARGING EFFICIENCY OF 70%, THE ENERGY
SUPPLIED TO THE BATTERY IN <u>ONE HOUR</u> WILL BE,

$$E_C = e \cdot P \cdot T = (0.7)(0.5)(6)(60)^2 = 7560\ W\text{-}sec.$$

RESULTING TALK TIME:

$$T_A = E_C/E_A = \frac{7560}{36} = 210\ sec = \underline{3.5\ min}$$

$$T_B = E_C/E_B = \frac{7560}{12} = 630\ sec = \underline{10.5\ min}$$

ALL DATA WAS OBTAINED FROM MANUFACTURER'S WEB SITES.
THERE ARE SEVERAL VARIABLES THAT CAN AFFECT THIS RESULT,
SUCH AS BATTERY TYPE, PHONE TYPE, SOLAR EFFICIENCY, CHARGING
EFFICIENCY, AND SUNLIGHT VARIATION WITH TIME AND LOCATION.
MUCH MORE WORK COULD BE DONE ON THIS PROBLEM.

1.5

The Iridium satellite telephone system consisted of 66 satellites in low Earth orbit, and was advertised as providing worldwide coverage with a single handset. The system cost was about \$5B. The satellites had an expected lifetime of about 5 years, after which the entire constellation would have to be replaced. The handsets were large and bulky, with a typical price tag of about \$1000. Service charges ranged from about \$1.40 to \$3.00 per minute.

While the Iridium system was technologically sophisticated, it was doomed to failure for several reasons. First, the rapid growth of land-based cellular systems provided service to large percentage of the population at rates that typically were a tenth that of Iridium (Typically about \$0.35 per minute during peak times, often with free talk time during off-peak hours. Handsets are usually free, or with a small nominal charge). Iridium claimed that their system was the only one to offer seamless coverage to people in lesser-developed countries, remote desert or mountainous regions, or even on the oceans. This was true, but they seemed to miss an important point – there are not many paying customers in those regions. Another serious problem with the Iridium system (and one that was never mentioned in their advertisements) is that Iridium handsets required a line-of-sight path to the satellite, meaning that it was rarely possible to use an Iridium phone in a building or vehicle. Land-based cellular systems, working at lower frequencies with better link margins and propagation properties, work quite well in buildings and vehicles. Iridium declared bankruptcy in August 1999, and the present plan is that the satellites will be de-orbited into the oceans. A sad outcome to well-engineered system, but one that was not unexpected.

The Globalstar satellite system consists of 48 LEO satellites, and is also designed to provide worldwide telephone coverage. Globalstar handsets typically cost about \$750, and service charges are about \$1 per minute. Satellite lifetime is expected to be about 7.5 years. Service began in late 1999, and at the present time (Spring 2000) the Globalstar system is struggling to meet its market projections. It, too, has trouble providing service to users in buildings or vehicles, and so suffers from the same type of problems as did Iridium. We expect it to suffer the same fate, but probably at a slower pace.

The lesson here is that large constellations of LEO satellites simply cannot compete with land-based systems that provide essentially the same service. Land-based facilities are much cheaper to build, install, and operate than satellites, and they can be much more easily modified, upgraded, and repaired. In addition, the quality of service (including factors such as coverage in buildings and vehicles, handset size, weight, and battery life) of land-based telephone systems is significantly better than that provided by satellite systems. This is ultimately due to the difference in link loss between satellite systems and land-based cellular systems – a fact of nature that no amount of marketing can change. Users will not pay substantially more for inferior service, even if the system can work worldwide. The same conclusion applies to data-oriented LEO systems, such as the proposed Teledesic system.

Chapter 2

$\boxed{2.1}$ $L = 0.3\,\mu H/m$, $C = 450\ pF/m$, $R = 5\,\Omega/m$, $G = 0.01\ S/m$
$f = 880\ MHz$.

From (2.5), $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(5+j1659)(.01+j2.49)}$

$$= \sqrt{(1659\angle 89.83^\circ)(2.49\angle 89.77^\circ)} = 64.3\angle 89.80^\circ$$

$$= 0.224 + j\,64.3 = \alpha + j\beta.$$

From (2.7), $$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{1659\angle 89.83^\circ}{2.49\angle 89.77}} = 25.8\angle 0.03^\circ$$

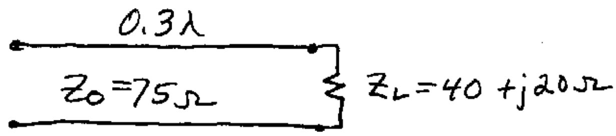
$$= 25.8 + j0.014\ \Omega$$

If $R=G=0$, then $\alpha = 0$ and $\beta = \omega\sqrt{LC} = 64.2$ rad/m.

$$Z_0 = \sqrt{\frac{L}{C}} = 25.8\,\Omega$$

Note that β and Z_0 for the lossless case are very close to the corresponding values for the lossy (but low loss) case.

$\boxed{2.2}$



$z_L = \frac{Z_L}{Z_0} = 0.53 + j\,0.266$

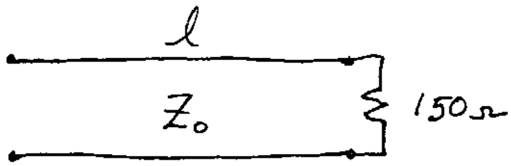
From a Smith chart, $z_{in} = 0.93 - j0.7$

So, $Z_{in} = Z_0 z_{in} = 69.8 - j\,52.5\,\Omega$ ✓

$SWR = 2.05$ ✓

$\Gamma = 0.34\angle 140^\circ$ ✓

$\boxed{2.3}$



SWR = 1.6

From (2.23), $\quad SWR = \dfrac{1+|\Gamma|}{1-|\Gamma|}$ so $|\Gamma| = \dfrac{SWR-1}{SWR+1} = \dfrac{0.6}{2.6} = 0.231$

From (2.17), $\quad \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$, so $|\Gamma| = \left|\dfrac{Z_L - Z_0}{Z_L+Z_0}\right|$

So either,

$$|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_0 = Z_L \frac{1-|\Gamma|}{1+|\Gamma|} = 150\left(\frac{1-.231}{1+.231}\right) = \underline{93.7\,\Omega} \checkmark$$

OR,

$$-|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_0 = Z_L \frac{1+|\Gamma|}{1-|\Gamma|} = 150\left(\frac{1+.231}{1-.231}\right) = \underline{240.\,\Omega} \checkmark$$

(verified by Smith chart)

$\boxed{2.4}$ $\quad Z_L = 80 + j40\ \Omega, \quad Z_0 = 50\,\Omega, \quad P_{in} = 30\,W$

From (2.17), $\quad \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{30+j40}{130+j40} = \dfrac{50\angle 53^\circ}{136\angle 17^\circ} = 0.367\angle 36^\circ \checkmark$

$$P_{LOAD} = P_{INC}\left(1 - |\Gamma|^2\right) = 30\left[1 - (0.367)^2\right] = \underline{\underline{25.9\ W}} \checkmark$$

$\boxed{2.5}$ a)

Γ	SWR	RL(dB)
0.01	1.02	40.
0.1	1.22	20.
0.25	1.67	12.
0.5	3.00	6.0
0.75	7.00	2.5

b)

Γ	SWR	RL(dB)
0.89	17.2	1.0
0.71	5.9	3.0
0.316	1.92	10.
0.1	1.22	20.
0.0316	1.07	30

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$RL = -20 \log |\Gamma| \text{ dB}$$

$$|\Gamma| = 10^{-RL/20}$$

$\boxed{2.6}$ $V_g = 10\,\text{V RMS}$, $Z_g = 50\,\Omega$, $Z_0 = 50\,\Omega$, $Z_L = 60 - j40\,\Omega$, $\ell = 0.6\lambda$

a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{10 - j40}{110 - j40} = \frac{41.2\angle -76.0}{117.0\angle -20.0} = 0.352\angle -56^\circ = 0.197 - j0.292 \quad \checkmark$$

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}\left(1 - |\Gamma|^2\right) = \underline{0.438\,W} \quad \checkmark$$

This method is based on $P_L = P_{inc}(1 - |\Gamma|^2)$. It is the simplest method, but is only valid for lossless lines.

b) $$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 50\,\frac{(60 - j40) + j36.3}{50 + j(43.6 - j29.1)} = 50\,\frac{60 - j3.7}{79.1 + j43.6}$$

$$= 50\,\frac{60.1\angle -3.5^\circ}{90.3\angle 28.9^\circ} = 33.3\angle -32.4^\circ = 28.1 - j17.8\,\Omega \quad \checkmark$$

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 Re(Z_{in}) = \left|\frac{10}{78.1 - j17.8}\right|^2 (28.1) = \underline{0.438\,W} \quad \checkmark$$

This method is based on $P_L = |I_{in}|^2 R_{in}$, and also applies only to lossy lines.

c) $$V(z) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$V_L = V(0) = V^+(1 + \Gamma) = \frac{V_g}{2}(1 + \Gamma) = 5(1.197 - j0.292) = 6.161\angle -13.7^\circ$$

$$|Z_L| = 72.1$$

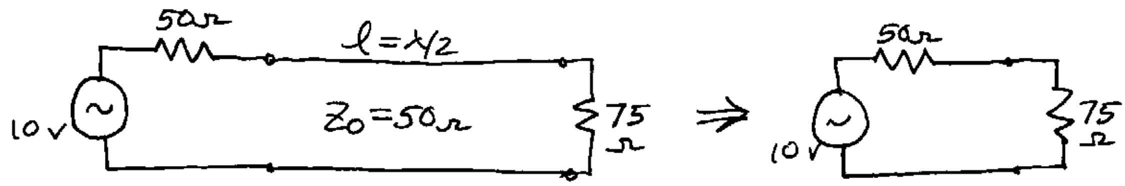
$$P_L = \left|\frac{V_L}{Z_L}\right|^2 Re(Z_L) = \left(\frac{6.161}{72.1}\right)^2 (60) = \underline{0.438\,W} \quad \checkmark$$

This method computes $P_L = |I_L|^2 R_L$, and applies to lossy or lossless lines. Note that $V^+ = V_g/2$ only applies here because $Z_g = Z_0$!

2.7 $\quad Z_L = jX \quad , \quad \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{jX - Z_0}{jX + Z_0}$

Then, $\quad |\Gamma|^2 = \Gamma \cdot \Gamma^* = \left(\dfrac{jX - Z_0}{jX + Z_0}\right)\left(\dfrac{-jX - Z_0}{-jX + Z_0}\right) = \dfrac{X^2 \cancel{-jXZ_0} + \cancel{jXZ_0} + Z_0^2}{X^2 + Z_0^2} = 1$ ✓

2.8



(ASSUMING 10V PEAK-TO-PEAK)

POWER DELIVERED BY SOURCE $= P_{SOURCE} = \frac{1}{2}\dfrac{(10)^2}{50+75} = \underline{0.400\text{ W}}$ ✓

POWER DISSIPATED IN 50Ω LOAD $= P_{DISS} = \frac{1}{2}(50)\left(\dfrac{10}{50+75}\right)^2 = \underline{0.160\text{ W}}$ ✓

INCIDENT POWER $= P_{INC} = \frac{1}{2}(50)\left(\dfrac{10}{50+50}\right)^2 = \underline{0.250\text{ W}}$ ✓

POWER TRANSMITTED DOWN LINE $= P_{TRANS} = \frac{1}{2}(75)\left(\dfrac{10}{50+75}\right)^2 = \underline{0.240\text{ W}}$ ✓

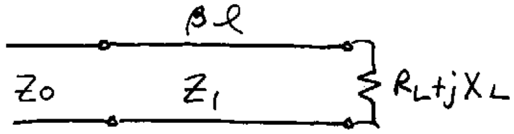
REFLECTED POWER $= P_{REF} = P_{INC}\,|\Gamma|^2 = 0.25\left|\dfrac{75-50}{75+50}\right|^2 = \underline{0.010\text{ W}}$ ✓

CHECK:

$P_{INC} - P_{REF} = 0.250 - 0.010 = 0.240 = P_{TRANS}$ ✓

$P_{DISS} + P_{TRANS} = 0.160 + 0.240 = 0.400 = P_{SOURCE}$ ✓

$\boxed{2.9}$



let $t = \tan \beta\ell$

The (real) characteristic impedance and the length $\beta\ell$ must satisfy

$$Z_{in} = Z_0 = Z_1 \frac{(R_L + jX_L) + jZ_1 t}{Z_1 + j(R_L + jX_L)t}$$

$$Z_0 Z_1 - Z_0 X_L t + jZ_0 R_L t = Z_1 R_L + jZ_1 X_L + jZ_1^2 t$$

Separating real and imaginary parts:

$$\text{Re:} \quad Z_0 Z_1 - Z_0 X_L t = Z_1 R_L \quad \Rightarrow \quad t = \frac{Z_1(Z_0 - R_L)}{Z_0 X_L}$$

$$\text{Im:} \quad Z_0 R_L t = Z_1 X_L + Z_1^2 t$$

Eliminate t:
$$(Z_0 R_L - Z_1^2)\frac{Z_1(Z_0 - R_L)}{Z_0 X_L} = Z_1 X_L$$

$$Z_0^2 R_L - Z_0 R_L^2 - Z_0 Z_1^2 + Z_1^2 R_L = Z_0 X_L^2$$

$$(R_L - Z_0) Z_1^2 = Z_0 (X_L^2 + R_L^2 - Z_0 R_L)$$

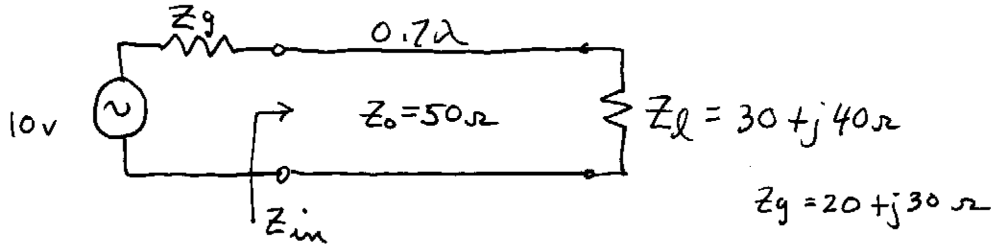
$$Z_1 = \sqrt{\frac{Z_0 (X_L^2 + R_L^2 - Z_0 R_L)}{R_L - Z_0}}$$

For $R_L = 60$, $X_L = 30$, $Z_0 = 50$:

$$Z_1 = 86.6\Omega, \quad t = -0.5773, \quad \beta\ell = 330^\circ, \quad \ell = 0.417\lambda \checkmark$$

(verified by Smith chart)

2.10



By Smith chart, $Z_{in} = 56.6 - j61.1\Omega$. Then (assuming peak voltages), since the line is lossless,

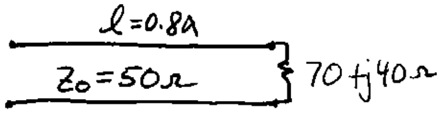
$$P_L = P_{in} = \frac{1}{2}|I_{in}|^2 \, Re(Z_{in}) = \frac{1}{2}\left|\frac{V_g}{Z_g + Z_{in}}\right|^2 Re(Z_{in}) = \frac{1}{2}\left|\frac{10}{76.6 - 31.1}\right|^2 (56.6)$$

$$= \underline{0.414\,W}$$

Maximum power will be delivered to the load when $Z_{in} = Z_g^* = 20 - j30$. By Smith chart, this requires a load impedance of $Z_\ell = 93.8 + j80.8\Omega$. The power delivered to the load is then,

$$P_L = P_{in} = \frac{1}{2}\left|\frac{V_g}{Z_g + Z_{in}}\right|^2 Re(Z_{in}) = \frac{1}{2}\left|\frac{10}{40}\right|^2 (20) = \underline{0.625\,W}$$

2.11



From a Smith chart,

$SWR = 2.094$ ✓

$\Gamma = 0.354\angle 45^\circ$ ✓

$RL = -20\log|\Gamma| = 9.02\,dB$

$Y_L = (0.5385 - j0.3077)/50$

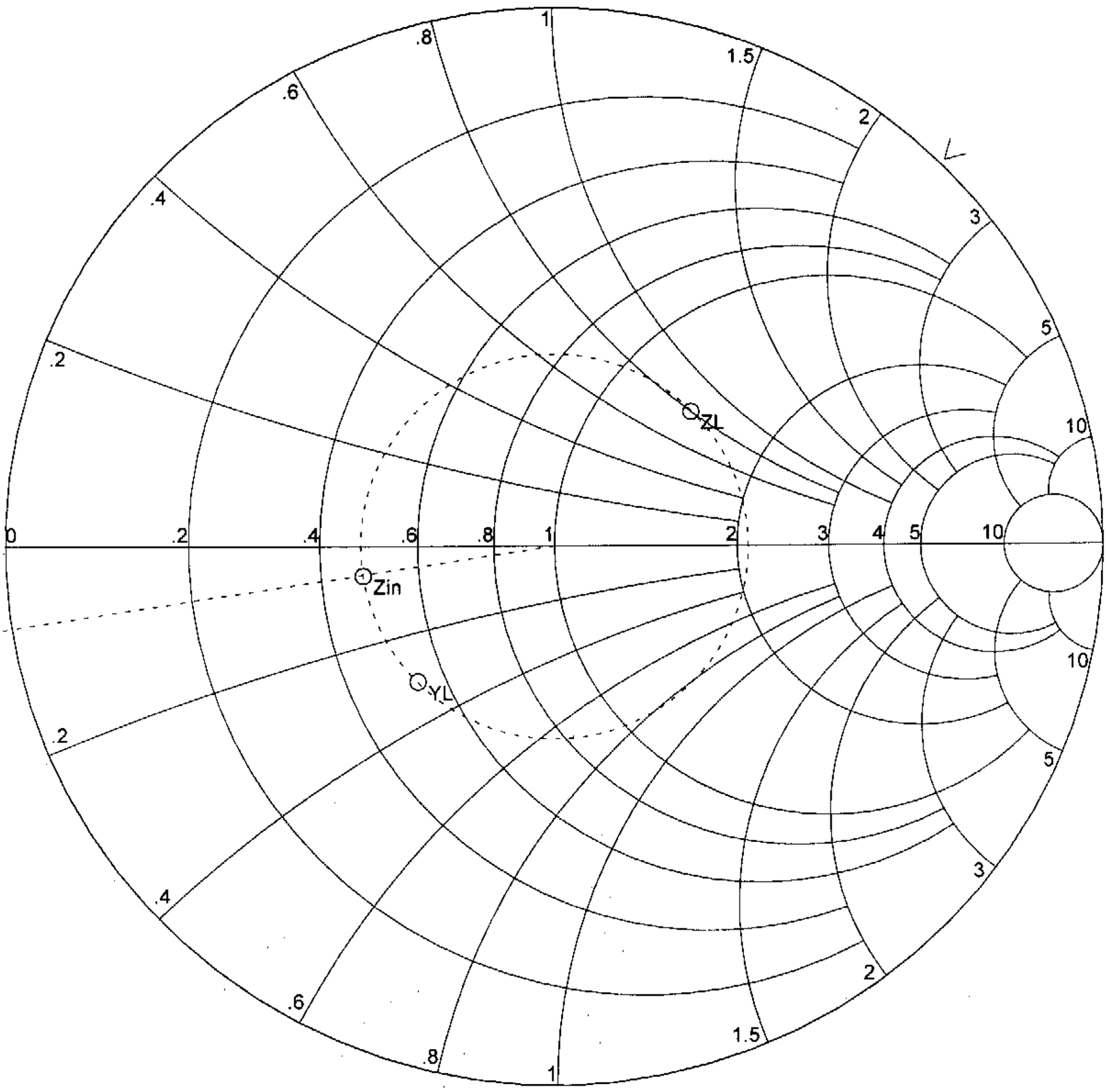
$= 0.0108 - j0.00615$ mS

$Z_{in} = 24.0 - j3.0\Omega$

$\theta - 2\beta\ell = -\pi \Rightarrow \ell = .3125\lambda$ ✓ $\quad \ell_{MIN} = 0.3125\lambda$

$\theta = 2\beta\ell \Rightarrow \ell = .0625\lambda$ ✓ $\quad \ell_{MAX} = 0.0625\lambda$

(See attached Smith chart)



Smith chart for Problem 2.11

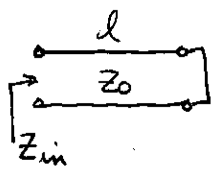
2.12

a) $\ell = 0\lambda$ or $\ell = 0.5\lambda$ ✓

b) $\ell = 0.25\lambda$ ✓

c) $\ell = 0.125\lambda$ ✓

d) $\ell = 0.375\lambda$ ✓



$Z_L = 0$
$Z_0 = 50\Omega$

(verified with $Z_{in} = jZ_0 \tan\beta\ell$)

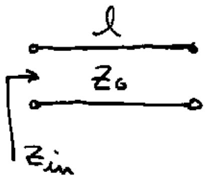
2.13

a) $\ell = 0.25\lambda$ ✓

b) $\ell = 0\lambda$ or $\ell = 0.5\lambda$ ✓

c) $\ell = 0.375\lambda$ ✓

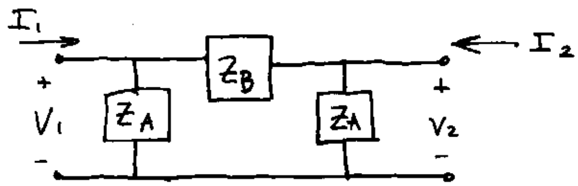
d) $\ell = 0.125\lambda$ ✓



$Z_L = \infty$
$Z_0 = 50\Omega$

(verified with $Z_{in} = -jZ_0 \cot\beta\ell$)

2.14 a)



From (2.55),

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{V_1\left(\frac{2Z_A+Z_B}{Z_A(Z_A+Z_B)}\right)} = \frac{Z_A(Z_A+Z_B)}{2Z_A+Z_B} ✓ = Z_{22} \quad \text{(by symmetry)}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_1 Z_{11}\left(\frac{Z_A}{Z_A+Z_B}\right)}{I_1} = \frac{Z_A^2}{2Z_A+Z_B} ✓ = Z_{12} \quad \text{(by reciprocity)}$$

From (2.56),

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{I_1}{I_1\left(\frac{Z_A Z_B}{Z_A+Z_B}\right)} = \frac{Z_A+Z_B}{Z_A Z_B} ✓ = Y_{22} \quad \text{(by symmetry)}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-V_1/Z_B}{V_1} = \frac{-1}{Z_B} = Y_{12} \quad \text{(by reciprocity)}$$

check that $[Z][Y] = [U]$:

$$Z_{11}Y_{11} + Z_{12}Y_{21} = \frac{(Z_A+Z_B)^2}{Z_B(2Z_A+Z_B)} - \frac{Z_A^2}{Z_B(2Z_A+Z_B)} = \frac{2Z_AZ_B+Z_B^2}{Z_B(2Z_A+Z_B)} = 1 \checkmark$$

$$Z_{11}Y_{12} + Z_{12}Y_{22} = \frac{-Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} + \frac{Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} = 0 \checkmark$$

b) SIMILAR ANALYSIS FOR THE T-NETWORK. THE RESULTS ARE:

$$Z_{11} = Z_{22} = \frac{Y_A+Y_B}{Y_A Y_B} \checkmark \qquad Z_{12} = Z_{21} = \frac{-1}{Y_B} \checkmark$$

$$Y_{11} = Y_{22} = \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} \checkmark \qquad Y_{12} = Y_{21} = \frac{Y_A^2}{2Y_A+Y_B} \checkmark$$

$\boxed{2.15}$ From (2.62), $V_n = V_n^+ + V_n^-$; $Z_0 I_n = V_n^+ - V_n^-$

Solving for V_n^+, V_n^- :

$$V_n^+ = (V_n + Z_0 I_n)/2$$
$$V_n^- = (V_n - Z_0 I_n)/2$$

at port 1, $V_1^+ = \frac{1}{2}\left[5\angle 45^\circ + 5\angle 45^\circ\right] = 5\angle 45^\circ$

$V_1^- = \frac{1}{2}\left[5\angle 45^\circ - 5\angle 45^\circ\right] = 0$

at port 2, $V_2^+ = \frac{1}{2}\left[2.12 - j2.12 + 10j\right] = 4.08\angle 75^\circ$

$V_2^- = \frac{1}{2}\left[2.12 - j2.12 - 10j\right] = 6.15\angle -80^\circ$

$\boxed{2.16}$ From Problem 2.15, $V_n^+ = (V_n + Z_0 I_n)/2$

$V_n^- = (V_n - Z_0 I_n)/2$

Then,

$$V_1^+ = \tfrac{1}{2}\left(1.314\angle 12.4^\circ + 0.77\angle -21.5^\circ\right) = 1.000\angle 0^\circ$$

$$V_1^- = \tfrac{1}{2}\left(1.314\angle 12.4^\circ - 0.77\angle -21.5^\circ\right) = 0.40\angle 45^\circ$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \underline{0.40\angle 45^\circ} \checkmark$$

Since $V_2^+ = 0$, $V_2^- = V_2 = 0.8\angle 90^\circ$. Then,

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{0.8\angle 90^\circ}{1\angle 0} = \underline{0.8\angle 90^\circ} \checkmark$$

$\boxed{2.17}$

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} 0.1\angle 90 & 0.4\angle 180 & 0.4\angle 180 \\ 0.4\angle 180 & 0.2\angle 0 & 0.6\angle 45 \\ 0.4\angle 180 & 0.6\angle 45 & 0.2\angle 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

a) For $V_1^+ = 1$, $V_2^+ = V_3^+ = 0$, $\Gamma_1 = 0.1\angle 90^\circ \Rightarrow \underline{RL = 20\,dB}$ at PORT 1 ✓

For $V_2^+ = 1$, $V_1^+ = V_3^+ = 0$, $\Gamma_2 = 0.2\angle 0^\circ \Rightarrow \underline{RL = 14\,dB}$ at PORT 2 ✓

For $V_3^+ = 1$, $V_1^+ = V_2^+ = 0$, $\Gamma_3 = 0.2\angle 0^\circ \Rightarrow \underline{RL = 14\,dB}$ at PORT 3 ✓

b) $T_{23} = 0.6\angle 45^\circ$; $IL = -20\log|T_{23}| = \underline{4.4\,dB}$ ✓

c) If $\Gamma_2 = \Gamma_3 = -1$, then $V_2^- = -V_2^+$; $V_3^- = -V_3^+$.

$$V_1^- = 0.1j\,V_1^+ + 0.4V_2^- + 0.4V_3^- \quad (1)$$
$$V_2^- = -0.4\,V_1^+ - 0.2V_2^- - 0.6\angle 45^\circ V_3^- \quad (2)$$
$$V_3^- = -0.4\,V_1^+ - 0.6\angle 45^\circ V_2^- - 0.2\,V_3^- \quad (3)$$

(2)-(3):

$$V_2^- - V_3^- = -.2(V_2^- - V_3^-) + .6\angle 45^\circ (V_2^- - V_3^-)$$

so $V_2^- = V_3^-$.

From (2), $(1.2 + 0.6\angle 45^\circ)V_2^- = -0.4V_1^+$

$$V_2^- = -0.238\angle -14.6^\circ\ V_1^+$$

Using this in (1):

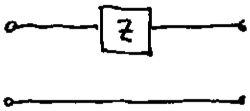
$$V_1^- = (0.1j - 0.190\angle -14.6^\circ)V_1^+$$

$$\Gamma_1 = \frac{V_1^-}{V_1^+} = 0.236\angle 141^\circ \ ✓$$

$$RL = 12.5\,dB \ ✓$$

(VERIFIED WITH ANALYSIS BY SERENADE)

$\boxed{2.18}$

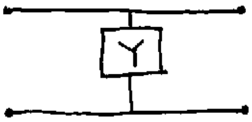


$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 \quad ✓$$

$$B = \frac{V_1}{I_2}\Big|_{V_2=0} = \frac{V_1}{V_1/Z} = Z \quad ✓$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = 0 \quad ✓$$

$$D = \frac{I_1}{I_2}\Big|_{V_2=0} = 1 \quad ✓$$

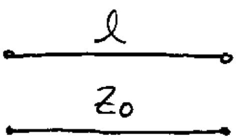


$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 \quad ✓$$

$$B = \frac{V_1}{I_2}\Big|_{V_2=0} = 0 \quad ✓$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = Y \quad ✓$$

$$D = \frac{I_1}{I_2}\Big|_{V_2=0} = 1 \quad ✓$$



For $I_2=0$, $(\Gamma=1)$

$$V_1 = V_1^+(e^{j\beta\ell} + e^{-j\beta\ell}) = 2V^+\cos\beta\ell$$

$$V_2 = 2V^+ = V_1/\cos\beta\ell$$

So,

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = \cos\beta\ell \quad ✓$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{V_1}{Z_{in}V_2} = \frac{\cos\beta\ell}{-jZ_0\cot\beta\ell} = jY_0\sin\beta\ell \quad ✓$$

For $V_2=0$, $(\Gamma=-1)$

$$V_1 = V^+(e^{j\beta\ell} - e^{-j\beta\ell}) = 2jV^+\sin\beta\ell$$

$$I_2 = 2V^+/Z_0$$

So,

$$B = \frac{V_1}{I_2}\Big|_{V_2=0} = jZ_0\sin\beta\ell \quad ✓$$

$$D = \frac{I_1}{I_2}\Big|_{V_2=0} = \frac{V_1}{Z_{in}I_2} = \frac{B}{Z_{in}} = \frac{jZ_0\sin\beta\ell}{jZ_0\tan\beta\ell} = \cos\beta\ell \quad ✓$$

$\boxed{2.19}$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

NOTE DIFFERENCE IN SIGN OF I_2 FOR [Z] AND ABCDs.

For $I_2 = 0$:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{V_2}\frac{V_2}{I_1}\right|_{I_2=0} = \frac{A}{C} \checkmark$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{1}{C} \checkmark$$

For $I_1 = 0$:

$$V_2 = DI_2/C$$
$$V_1 = \left(\frac{AD}{C} - B\right)I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{AD-BC}{C} \checkmark$$

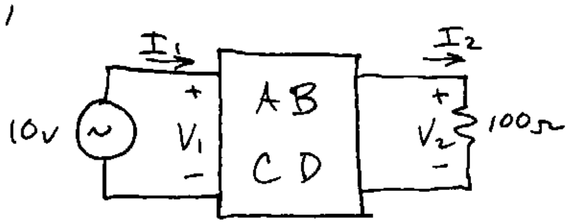
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C} \checkmark$$

$\boxed{2.20}$ From Table 2.1 the ABCD matrix of the cascade of the five components is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}}_{\substack{\text{SERIES} \\ R}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix}}_{\substack{\text{SHUNT} \\ R}} \underbrace{\begin{bmatrix} 1 & j30 \\ 0 & 1 \end{bmatrix}}_{\substack{\text{SERIES} \\ L}} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}}_{\substack{\text{TRANS-} \\ \text{FORMER}}} \underbrace{\begin{bmatrix} 0 & j50 \\ j0.02 & 0 \end{bmatrix}}_{\substack{\text{TRANS.} \\ \text{LINE}}} = \begin{bmatrix} -0.3 & j100 \\ -0.012+j0.01 & 4j \end{bmatrix}$$

CHECK: $AD - BC = (-.3)(4j) - (j100)(-.012 + j.01) = -\cancel{1.2j} + \cancel{1.2j} + 1 = 1$ ✓

Then we have,



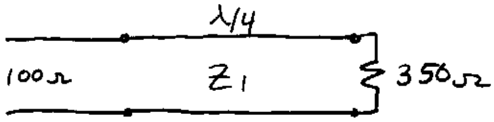
$I_2 = V_2/100$

$$V_1 = AV_2 + BI_2 = (A + B/100)\,V_2$$

$$I_1 = CV_2 + DI_2 = (C + D/100)\,V_2$$

$$V_L = V_2 = \frac{V_1}{A + B/100} = \frac{10}{-0.3 + j} = \underline{\underline{9.58\angle -107^\circ}}$$

$\boxed{2.21}$

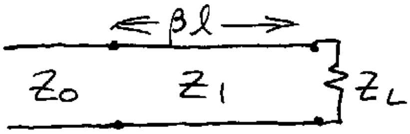


From (2.72), $\quad Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(100)(350)} = 187\,\Omega$.

For SWR = 2, $\quad \Gamma_m = \dfrac{SWR-1}{SWR+1} = 0.333$

From (2.80), $\quad \dfrac{\Delta f}{f_0} = 2 - \dfrac{4}{\pi}\cos^{-1}\left[\dfrac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\dfrac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 0.71 = 71\%$.

$\boxed{2.22}$



From the solution of Problem 2.9,

$$Z_1 = \sqrt{\frac{Z_0(X_L^2 + R_L^2 - Z_0 R_L)}{R_L - Z_0}}, \quad \tan\beta l = \frac{Z_1(Z_0 - R_L)}{Z_0 X_L}$$

For $R_L = 100$, $X_L = 200$, $Z_0 = 50$,

$Z_1 = 212\,\Omega$ ✓ $\quad l = 0.370\lambda$ ✓ (VERIFIED VIA SMITHCHART)

This method can be used for any R_L, X_L, Z_0 as long as Z_1 is real. Thus,

$$\frac{Z_0(X_L^2 + R_L^2 - Z_0 R_L)}{R_L - Z_0} > 0.$$

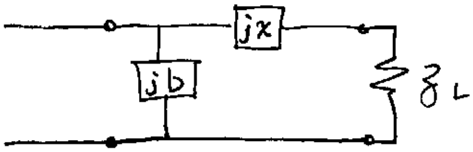
OR,

$$X_L^2 + R_L^2 - Z_0 R_L > 0 \quad \text{if } R_L > Z_0$$

$$X_L^2 + R_L^2 - Z_0 R_L < 0 \quad \text{if } R_L < Z_0.$$

$\boxed{2.23}$

a) $z_L = 0.5 - j0.8$ (z_L outside $1+jx$ circle)



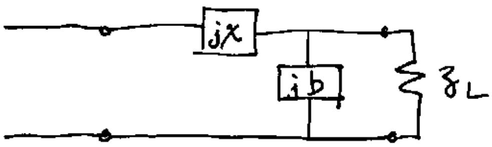
SOLUTION #1: $x = 1.30$ ✓

$b = 1.00$ ✓

SOLUTION #2: $x = 0.30$ ✓

$b = -1.00$ ✓

b) $z_L = 1.6 + j0.8$ (z_L inside $1+jx$ circle)



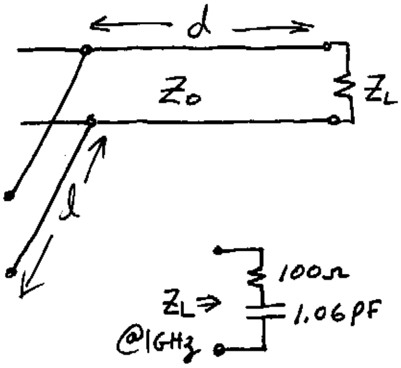
SOLUTION #1: $x = 1.00$ ✓

$b = 0.75$ ✓

SOLUTION #2: $x = -1.00$ ✓

$b = -0.25$ ✓

$\boxed{2.24}$ $Z_L = 100 - j150\,\Omega$, $Z_0 = 50\,\Omega$



SOLUTION #1: $d = 0.155\lambda$ ✓

$b_S = -2.236$

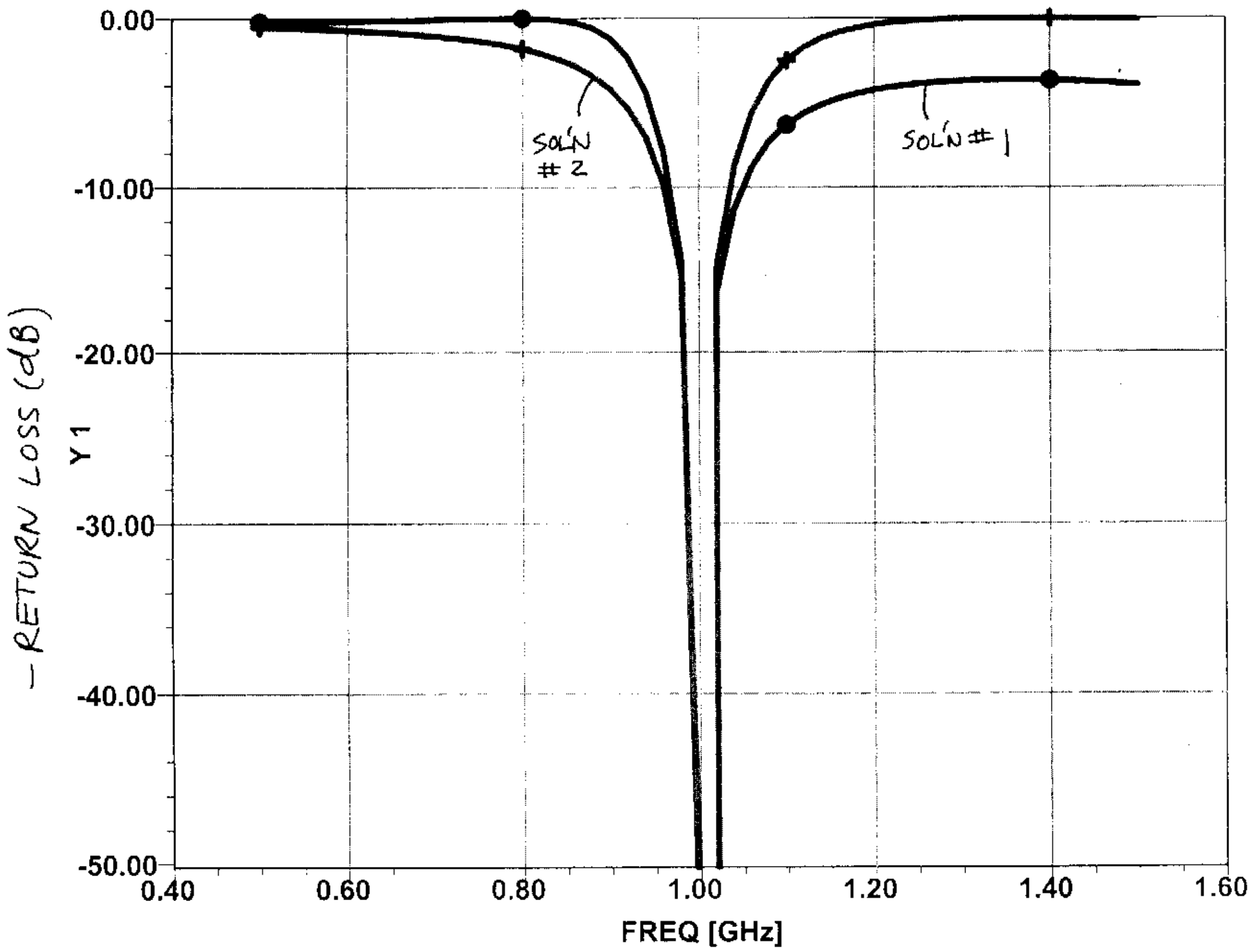
$l = 0.317\lambda$ ✓

SOLUTION #2: $d = 0.271\lambda$ ✓

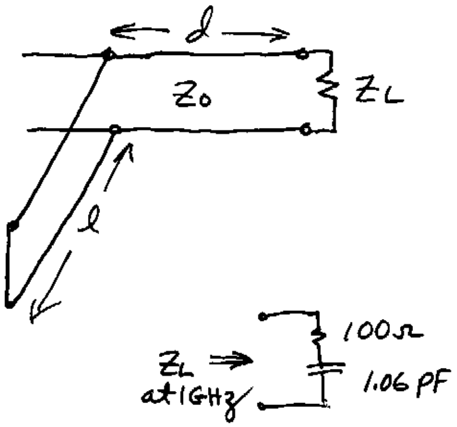
$b_S = 2.236$

$l = 0.183\lambda$ ✓

(VERIFIED WITH ANALYSIS WITH SERENADE WITH $C = \frac{1}{2\pi f X_L} = 1.06\,pF$ at 1GHz)



$\boxed{2.25}$ $Z_L = 100 - j150\,\Omega$, $Z_0 = 50\,\Omega$



SOLUTION #1: $d = 0.155\lambda$ ✓

$b_s = -2.236$

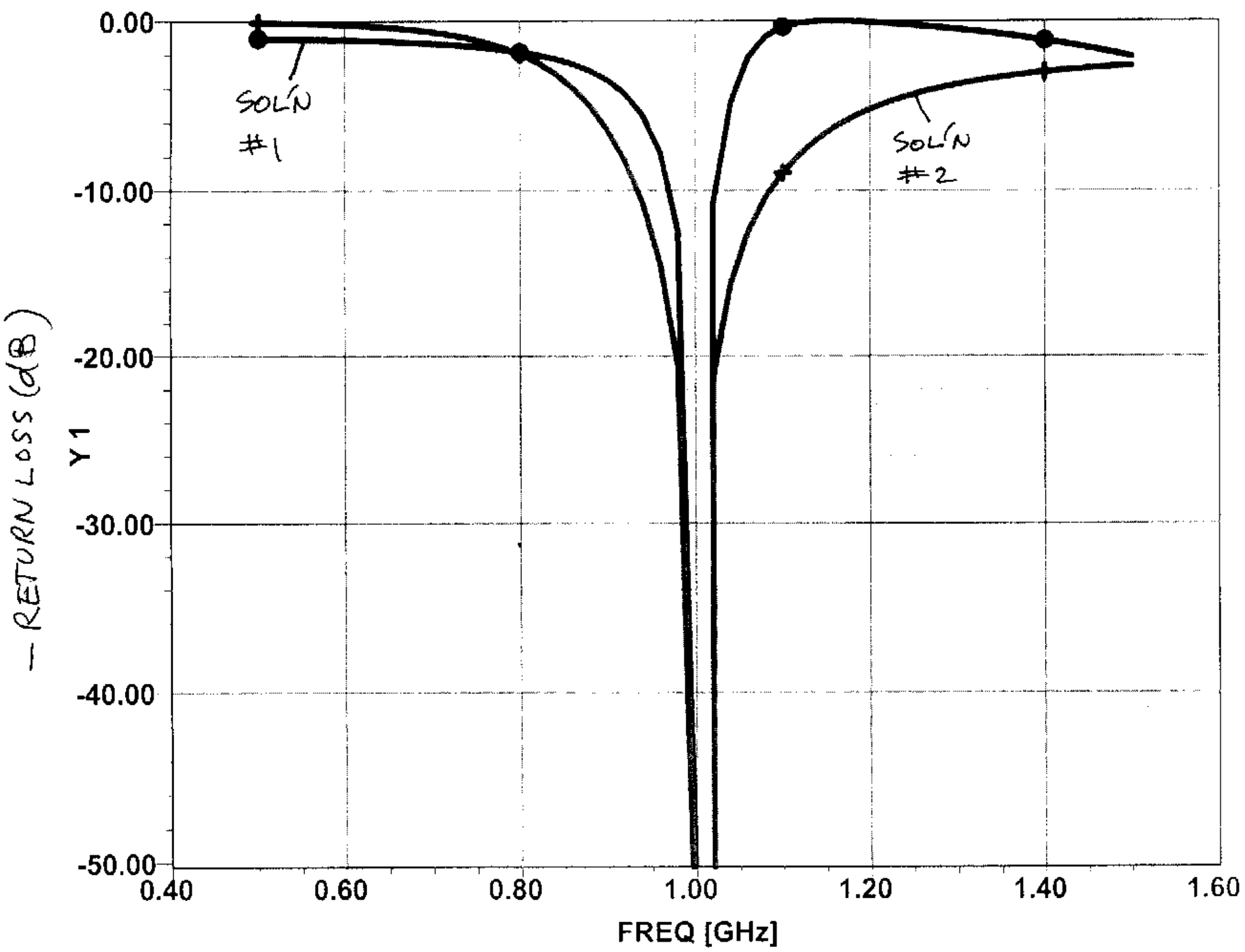
$l = 0.067\lambda$ ✓

SOLUTION #2: $d = 0.271\lambda$ ✓

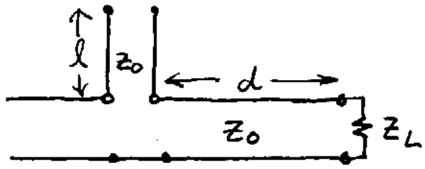
$b_s = 2.236$

$l = 0.433\lambda$ ✓

(VERIFIED WITH ANALYSIS WITH SERENADE USING C=1.06pF at 1GHz IN Z)



$\boxed{2.26}$ $Z_L = 15 + j50\ \Omega$, $Z_0 = 100\ \Omega$



SOLUTION #1: $d = 0.122\lambda$ ✓

$X_S = -2.55$

$\ell = 0.059\lambda$ ✓

SOLUTION #2: $d = 0.228\lambda$ ✓

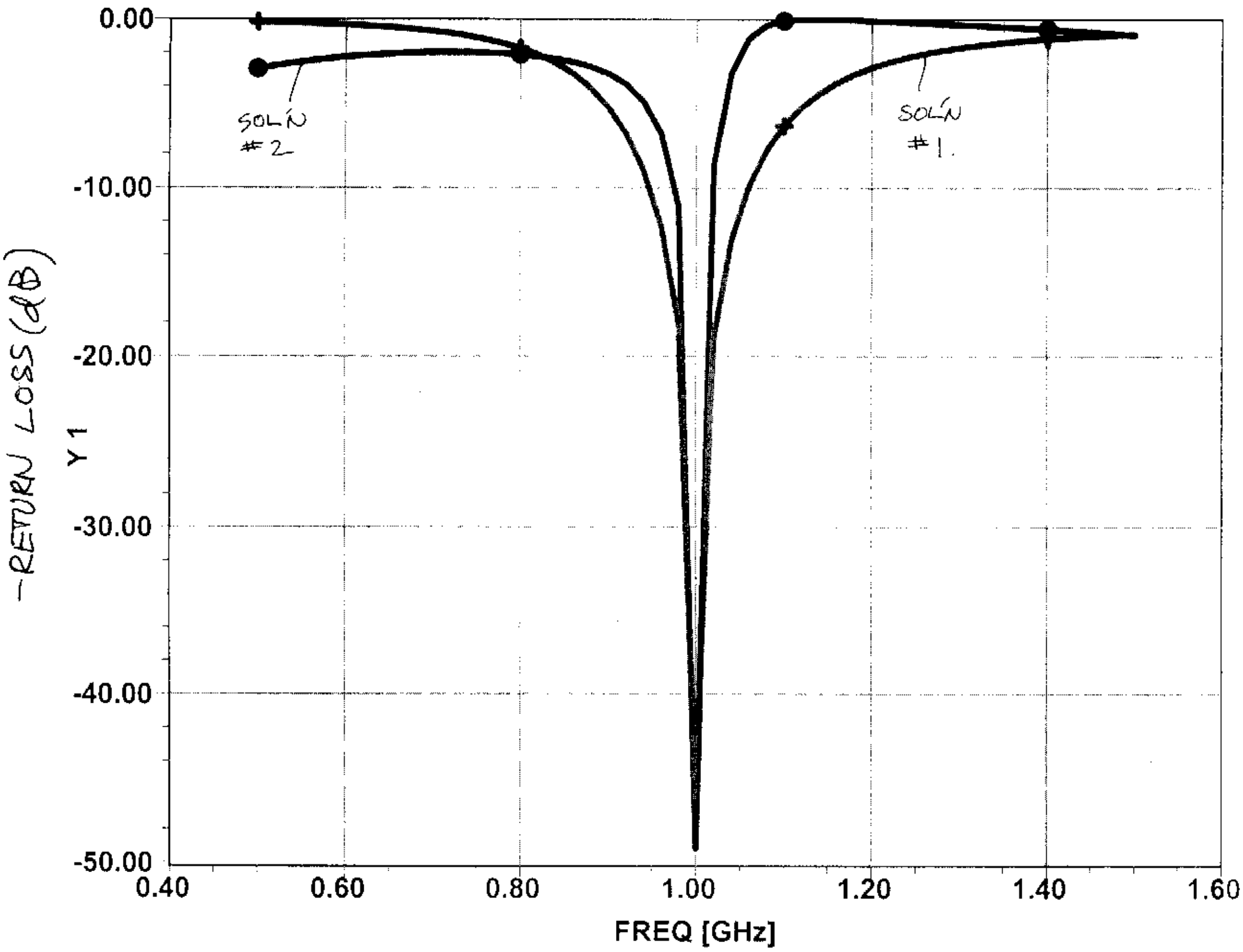
$X_S = 2.55$

$\ell = 0.441\lambda$ ✓

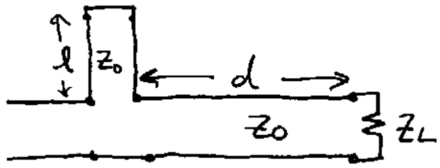
Z_L at 1GHz

$L = \frac{X_L}{2\pi f} = 7.96\ nH$ at 1GHz.

(VERIFIED BY ANALYSIS USING SERENADE WITH L = 7.96 nH at 1GHz IN Z_L)



$\boxed{2.27}$ $Z_L = 15 + j50\,\Omega$, $Z_0 = 100\,\Omega$



SOLUTION #1: $d = 0.122\lambda$ ✓

$X_s = -2.55$

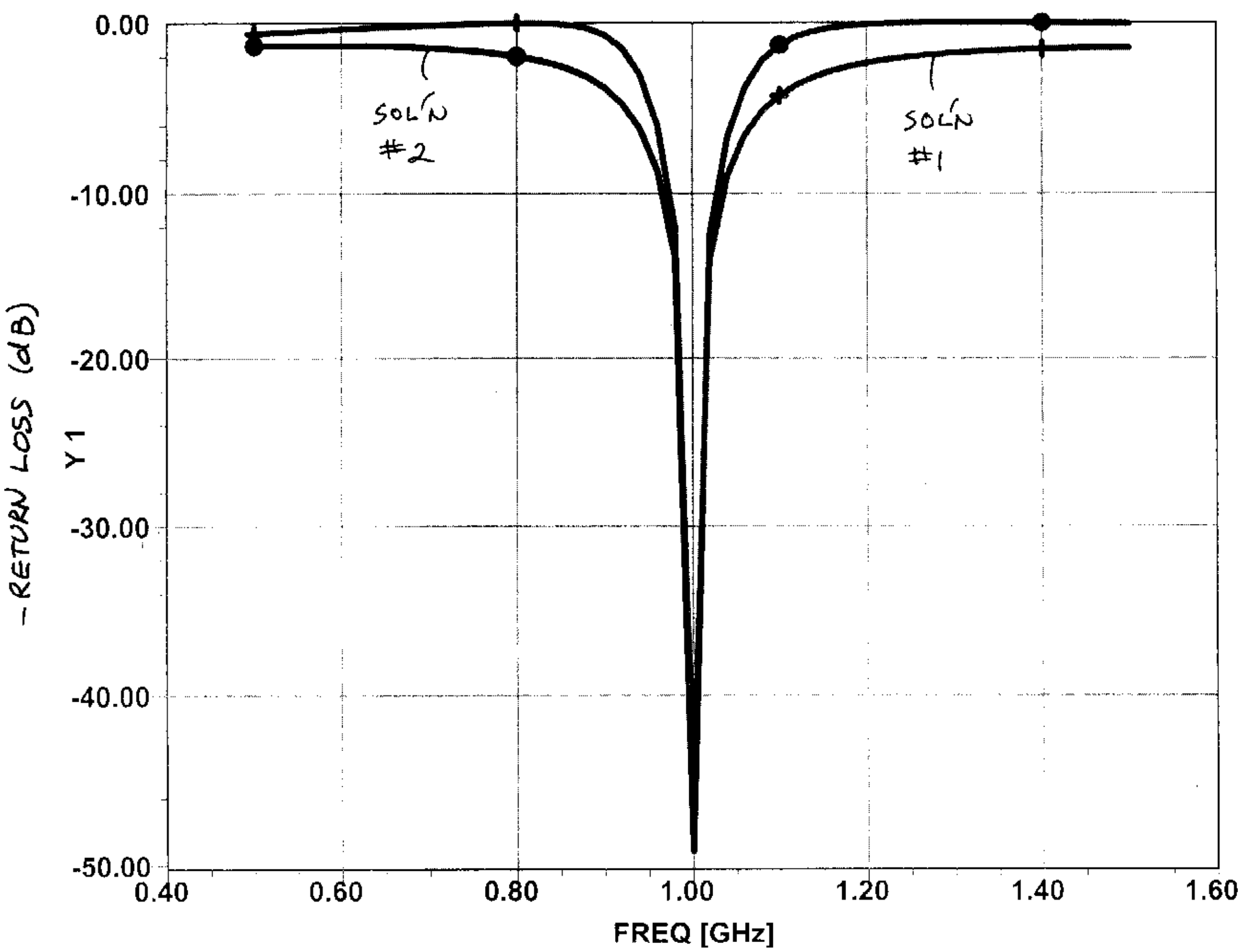
$l = 0.309\lambda$ ✓

SOLUTION #2: $d = 0.228\lambda$ ✓

$X_s = 2.55$

$l = 0.191\lambda$ ✓

(VERIFIED BY ANALYSIS USING SERENADE WITH L=7.96nH at 1GHz IN Z_L)



Chapter 3

3.1

1) $F_X(x) \geq 0$
By definition (3.1), $F_X(x) = P\{X \leq x\} \geq 0$ since $0 \leq P\{\cdot\} \leq 1$.

2) $F_X(\infty) = 1$
By definition (3.1), $F_X(\infty) = P\{X \leq \infty\} = 1$ since $X \leq \infty$ for all real x.

3) $F_X(-\infty) = 0$
By definition (3.1), $F_X(-\infty) = P\{X \leq -\infty\} = 0$ since $X > -\infty$ for all real x.

4) $F_X(x_1) \leq F_X(x_2)$ if $x_1 \leq x_2$
Since $P\{X \leq x_1\} \leq P\{X \leq x_2\}$ if $x_1 \leq x_2$, defn (3.1) gives $F_X(x_1) \leq F_X(x_2)$ if $x_1 \leq x_2$.

3.2

For continuous random variables,

$$E\{x\} = \int_{-\infty}^{\infty} x\, f_x(x)\,dx$$

a) $E\{cX\} = \int_{-\infty}^{\infty} c\,x\, f_X(x)\,dx = c\int_{-\infty}^{\infty} x\, f_X(x)\,dx = c\,E\{X\}$ ✓

b)
$$E\{X+Y\} = \iint_{-\infty}^{\infty} (x+y)\, f_{X,Y}(x,y)\,dx\,dy$$
$$= \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy\,dx + \int_{-\infty}^{\infty} y \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy$$
$$= \int_{-\infty}^{\infty} x\, f_X(x)\,dx + \int_{-\infty}^{\infty} y\, f_Y(y)\,dy = E\{X\} + E\{Y\}$$ ✓

where $f_{X,Y}(x,y)$ is the joint PDF of X, Y.
We also used the fact that $\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = f_X(x)$.

$\boxed{3.3}$

UNIFORM: $\quad f_x = \frac{1}{b-a} \quad$ for $a \le x \le b$.

$$\bar{x} = E\{x\} = \int_a^b \frac{x}{b-a}\,dx = \frac{1}{b-a}\,\frac{x^2}{2}\Big|_a^b = \frac{1}{2}\,\frac{b^2-a^2}{b-a} = \frac{b+a}{2} \checkmark$$

$$\sigma^2 = E\{(x-\bar{x})^2\} = \int_a^b \frac{(x-\bar{x})^2}{b-a}\,dx = \frac{1}{b-a}\int_a^b (x^2 - 2\bar{x}x + \bar{x}^2)\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3} - \bar{x}x^2 + \bar{x}^2 x\right]\Big|_a^b = \frac{1}{b-a}\left[\frac{b^3-a^3}{3} - \bar{x}(b^2-a^2) + \bar{x}^2(b-a)\right]$$

$$= \left[\frac{b^2+ab+a^2}{3} - \frac{(b+a)^2}{2} + \frac{(b+a)^2}{4}\right] = \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12} \checkmark$$

GAUSSIAN: $\quad f_x = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-m)^2/2\sigma^2}$

$$\bar{x} = E\{x\} = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi\sigma^2}}\, e^{-(x-m)^2/2\sigma^2}\,dx = \frac{\sqrt{2}\,\sigma}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} (\underbrace{\sqrt{2}\,\sigma y}_{0 \text{ since odd}} + m)\, e^{-y^2}\,dy$$

$$= \frac{m}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-y^2}\,dy = m \checkmark \qquad \left(\text{let } y = (x-m)/\sqrt{2}\,\sigma\right)$$

$$E\{(x-\bar{x})^2\} = \int_{-\infty}^{\infty} \frac{(x-m)^2}{\sqrt{2\pi\sigma^2}}\, e^{-(x-m)^2/2\sigma^2}\,dx = \frac{\sqrt{2}\,\sigma}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} 2\sigma^2 y^2 e^{-y^2}\,dy$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = \sigma^2 \checkmark$$

3.3 CONT.

RAYLEIGH: $$f_r = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \qquad 0 \le r < \infty$$

$$\bar{r} = E\{r\} = \int_0^\infty \frac{r^2}{\sigma^2} e^{-r^2/2\sigma^2} dr = \frac{1}{\sigma^2}\frac{\sqrt{\pi}}{4}(2\sigma^2)^{3/2} = \sqrt{\frac{\pi}{2}}\,\sigma \checkmark$$

$$E\{(r-\bar{r})^2\} = \int_0^\infty \frac{(r-\bar{r})^2 r}{\sigma^2} e^{-r^2/2\sigma^2} dr$$

$$= \frac{1}{\sigma^2}\int_0^\infty (r^3 - 2\bar{r}r^2 + \bar{r}^2 r) e^{-r^2/2\sigma^2} dr$$

$$= \frac{1}{\sigma^2}\left[\frac{1}{2}(2\sigma^2)^2 - 2\bar{r}\frac{\sqrt{\pi}}{4}(2\sigma^2)^{3/2} + \bar{r}^2\frac{1}{2}(2\sigma^2)\right]$$

$$= \frac{1}{\sigma^2}\left[2\sigma^4 - \frac{\pi\sigma}{2\sqrt{2}}2\sqrt{2}\sigma^3 + \frac{\pi}{2}\sigma^2\cdot\sigma^2\right] = (2-\frac{\pi}{2})\sigma^2 \checkmark$$

$\boxed{3.4}$ From (3.16) the n-th moment is,

$$\overline{x^n} = E\{x^n\} = \int_{-\infty}^{\infty} x^n f_x(x) dx \qquad f_x = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} x^n e^{-x^2/2\sigma^2} dx$$

If n is even, $\overline{x^n} = \frac{2}{\sqrt{2\pi\sigma^2}}\int_0^\infty x^n e^{-x^2/2\sigma^2} dx = \frac{1}{\sqrt{2\pi\sigma^2}}\frac{\Gamma(\frac{n+1}{2})}{\left(\frac{1}{2\sigma^2}\right)^{\frac{n+1}{2}}}$

CHECK FOR $n=2$:

$$\overline{x^2} = \frac{\Gamma(\frac{3}{2})}{\sqrt{2\pi\sigma^2}}(2\sigma^2)^{3/2} = \frac{\sqrt{\pi}}{2\sqrt{2\pi\sigma^2}} 2\sqrt{2}\sigma^3 = \sigma^2 \checkmark$$

If n is odd, $\overline{x^n} = 0$ by anti-symmetry of integrand.

$\boxed{3.5}$ $z = x + y$, $f_x = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-x^2/2\sigma_x^2}$, $f_y = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-y^2/2\sigma_y^2}$

Since x, y, and z are zero-mean,

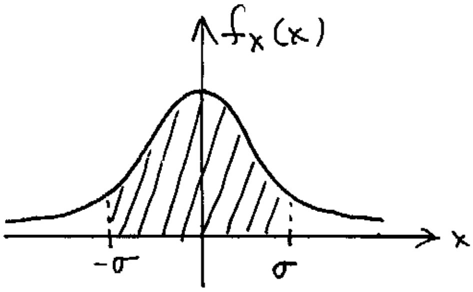
$$\sigma_z^2 = E\{z^2\} = E\{(x+y)^2\} = E\{x^2 + 2xy + y^2\}$$

$$= \sigma_x^2 + 2\cancel{m_x}^{0}\cancel{m_y}^{0} + \sigma_y^2 = \sigma_x^2 + \sigma_y^2 \checkmark$$

$\boxed{3.6}$ $f_X(x) = \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$

$$P\{-\sigma < x < \sigma\} = \int_{-\sigma}^{\sigma} f_X(x)\,dx = \int_{-\infty}^{\infty} - 2\int_{\sigma}^{\infty} = 1 - 2\int_{\sigma}^{\infty} \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dx$$

$$= 1 - \mathrm{erfc}\left(\tfrac{1}{\sqrt{2}}\right) = 1 - 0.3176 = 0.68 \checkmark$$



NOTE:

$P\{-\sigma < x < \sigma\} = 1 - P\{x < -\sigma\} - P\{x > \sigma\}$,

BUT

$P\{-\sigma < x < \sigma\} \neq P\{x < \sigma\}\, P\{-\sigma < x\}$.

$\boxed{3.7}$

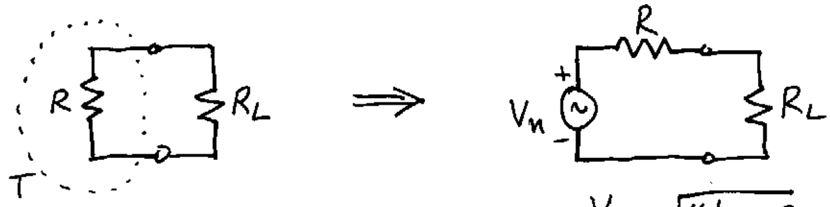
$R(\tau) = \eta_0\,\delta(\tau)/2$. From (3.22a) the power spectral density is,

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-j\omega\tau}\,d\tau = \frac{\eta_0}{2} \qquad \text{(TWO-SIDED PSD)}$$

Then the output noise power is, from (3.35),

$$N_o = (2\Delta f)(S(f)) = \underline{\Delta f\, \eta_0} \checkmark$$

$\boxed{3.8}$



$$V_n = \sqrt{4kTBR}$$

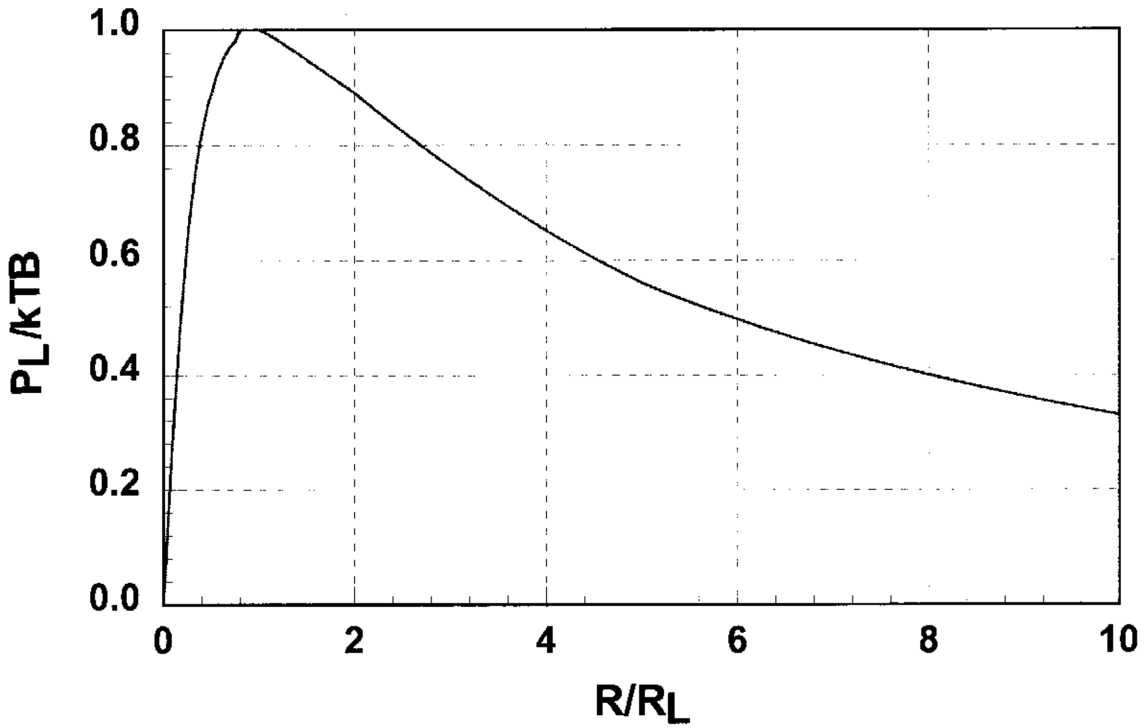
$$P_L = \left[\frac{V_n R_L}{R+R_L}\right]^2 \frac{1}{R_L} = \frac{4kTBRR_L}{(R+R_L)^2}$$

Let $A = \frac{P_L}{kTB} = \frac{4RR_L}{(R+R_L)^2} = \frac{4(R/R_L)}{(1+R/R_L)^2}$ (SEE PLOT BELOW)

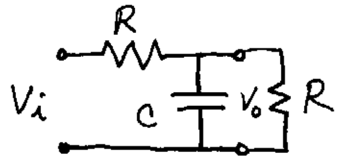
$$\frac{dA}{dR_L} = \frac{4R}{(R+R_L)^2} - \frac{8RR_L}{(R+R_L)^3} = 0$$

$$4R(R+R_L) - 8RR_L = 0$$

$$4R^2 + 4RR_L - 8RR_L = 0 \Rightarrow \underline{\underline{R = R_L}} ✓$$



$\boxed{3.9}$



$$H(f) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + jf/f_c} \quad \text{where } f_c = \frac{1}{2\pi RC}$$

$$|H(f)|^2 = \frac{1}{1 + (f/f_c)^2} \checkmark \qquad S_i(f) = n_o/2$$

$$S_o = S_i\,|H(f)|^2 = \frac{n_o}{2}\,\frac{1}{1 + (f/f_c)^2}$$

$$N_o = \int_{-\infty}^{\infty} S_o(f)\,df = \frac{n_o}{2}\int_{-\infty}^{\infty} \frac{df}{1 + (f/f_c)^2} = \frac{n_o f_c}{2}\underbrace{\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}}_{\pi} \qquad \begin{array}{l} \text{LET } x = f/f_c \\ dx = df/f_c \end{array}$$

$$= \frac{\pi n_o f_c}{2} = \frac{n_o}{4RC} \checkmark$$

$\boxed{3.10}$

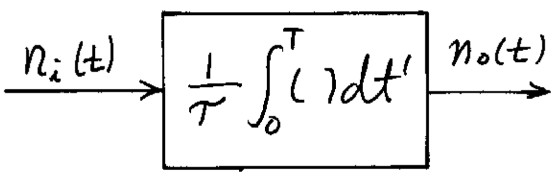
$$N_o = \sigma^2 = E\{n_o^2(T)\} = E\left\{\frac{1}{T^2}\int_0^T\!\!\int_0^T n_i(t)\,n_i(s)\,dt\,ds\right\}$$

$$= \frac{1}{T^2}\int_0^T\!\!\int_0^T E\{n_i(t)\,n_i(s)\}\,dt\,ds$$

$$E\{n_i(t)\,n_i(s)\} \xrightarrow{\text{LET } s = t + \tau} = E\{n_i(t)\,n_i(t+\tau)\} = R(\tau) \xrightarrow{\tau = s - t} = R(s-t) = \frac{n_o}{2}\,\delta(s-t)$$

Then,

$$N_o = \frac{n_o}{2T^2}\int_0^T\!\!\int_0^T \delta(s-t)\,dt\,ds = \frac{n_o}{2T^2}\int_0^T dt = \frac{n_o T}{2T^2} \checkmark$$



$\boxed{3.11}$

$$R_y(\tau) = E\{y(t+\tau)y(t)\}$$
$$= E\{x(t+\tau)\,x(t)\cos(\omega_0 t+\omega_0\tau+\theta)\cos(\omega_0 t+\theta)\}$$
$$= E\{x(t+\tau)x(t)\}\,E\{\cos(\omega_0 t+\omega_0\tau+\theta)\cos(\omega_0 t+\theta)\} \quad \text{(SINCE IND.)}$$
$$= R_x(\tau)\,\frac{1}{2\pi}\int_0^{2\pi}\cos(\omega_0 t+\omega_0\tau+\theta)\cos(\omega_0 t+\theta)\,d\theta$$
$$= \frac{1}{4\pi}R_x(\tau)\int_0^{2\pi}\left[\overset{0}{\cancel{\cos(2\omega_0 t+\omega_0\tau+2\theta)}} + \cos\omega_0\tau\right]d\theta$$
$$= \frac{1}{2}R_x(\tau)\cos\omega_0\tau \;\checkmark$$

$$S_y(\omega) = \int_{-\infty}^{\infty} R_y(\tau)e^{-j\omega\tau}\,d\tau = \frac{1}{4}\int_{-\infty}^{\infty} R_x(\tau)\left[e^{-j(\omega-\omega_0)\tau} + e^{-j(\omega+\omega_0)\tau}\right]d\tau$$
$$= \frac{1}{4}\left[S_x(\omega-\omega_0) + S_x(\omega+\omega_0)\right] \;\checkmark$$

$\boxed{3.12}$

$$P_e^{(0)} = P\{r(t) = n(t) > v_0/2\} = \int_{v_0/2}^{\infty}\frac{e^{-r^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dr$$

let $x = r/\sqrt{2\sigma^2}$

$$P_e^{(0)} = \frac{1}{\sqrt{2\pi\sigma^2}}\sqrt{2\sigma^2}\int_{\frac{v_0}{2\sqrt{2\sigma^2}}}^{\infty} e^{-x^2}\,dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{v_0}{2\sqrt{2\sigma^2}}\right) = P_e^{(1)} \;\checkmark$$

$\boxed{3.13}$

$$P_e = \frac{1}{2}\,\mathrm{erfc}(x)$$

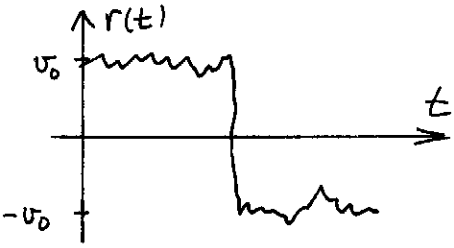
$$x = \frac{v_0}{2\sqrt{2\sigma^2}}$$

$$\frac{v_0}{\sigma} = 2\sqrt{2}\,x$$

P_e	x	v_0/σ	v_0/σ (dB)
10^{-2}	1.645	4.65	13.3
10^{-5}	3.015	8.53	18.6
10^{-8}	3.968	11.22	21.0

(agrees with Figure 3.14)

3.14



$r(t) = s(t) + n(t)$

$E\{n(t)\} = 0$

$E\{n^2(t)\} = \sigma^2$

THRESHOLD AT $v = 0$.

$$P_e^{(1)} = P\{v_0 + n(t) < 0\} = P\{n(t) < -v_0\} = \int_{-\infty}^{-v_0} \frac{e^{-n^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn \qquad \begin{array}{l}\text{LET } x = -n/\sqrt{2}\sigma \\ dx = -dn/\sqrt{2}\sigma\end{array}$$

$$= \int_{v_0/\sqrt{2}\sigma}^{\infty} \frac{e^{-x^2}}{\sqrt{2\pi\sigma^2}}(\sqrt{2}\sigma)\,dx = \frac{1}{\sqrt{\pi}}\int_{v_0/\sqrt{2}\sigma}^{\infty} e^{-x^2}\,dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{v_0}{\sqrt{2}\,\sigma}\right) = P_e^{(0)} \checkmark$$

The result of Section 3.4 was,

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{v_0}{2\sqrt{2}\,\sigma}\right),$$

which means that, for the same P_e, the SNR of the present case is 6 dB lower than for the previous case. Of course, the maximum voltage swing here is twice ($-v_0$ to v_0) that of the case in Section 3.4 (0 to v_0), so the results are really the same.

3.15

From (3.62), $$T_e = \frac{T_1 - Y T_2}{Y - 1} = \frac{320 - (1.150)(77)}{1.150 - 1} = 1543 \text{ K}$$

From (3.64),

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{1543}{290} = 6.32 = \underline{8.0 \text{ dB}}$$

$\boxed{3.16}$

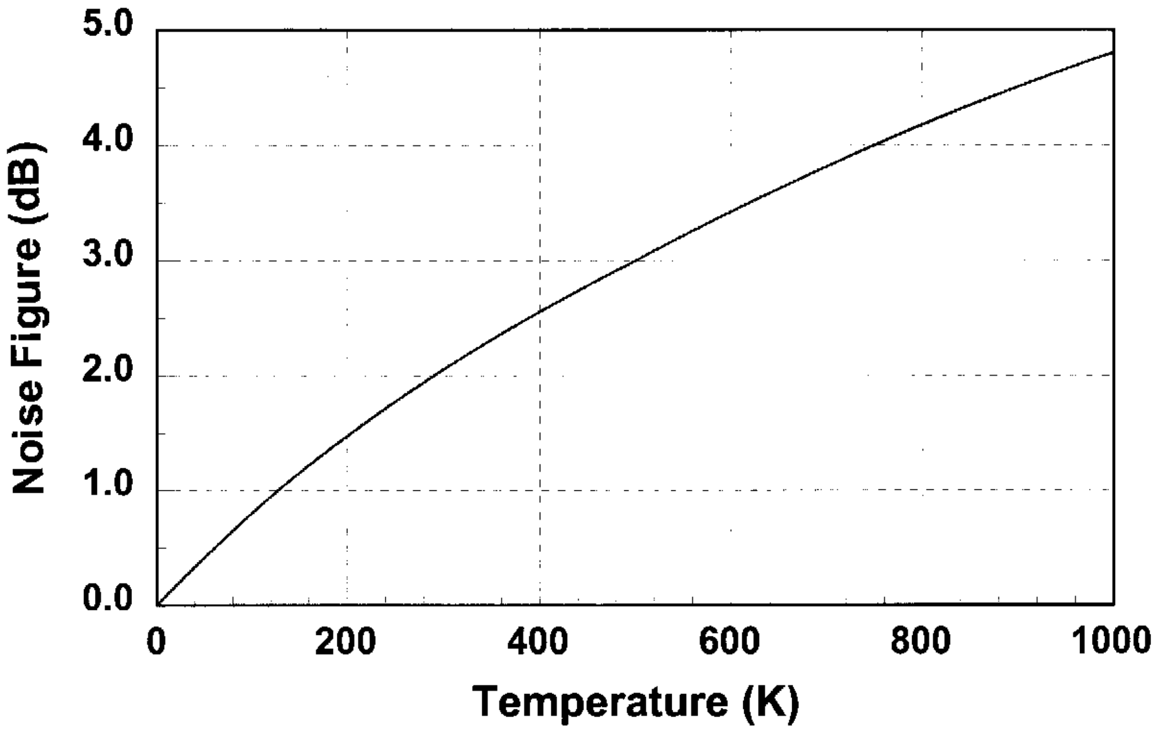
From (3.70), $F = 1 + (L-1)\frac{T}{T_0}$ for a lossy line.

Thus, for $T_1 = T_0$, $F = L = 1.58$.

For different temperature, T, $F = 1 + 0.58\frac{T}{T_0}$, $T_0 = 290K$.

T(K)	F(dB)
0	0
100	0.8
290	2.0
500	3.0
1000	4.8

DATA IS PLOTTED BELOW.



$\boxed{3.17}$

$G_1 = 15\,dB = 31.6$ $\quad T_{e_2} = 800K$

$B = 200\,MHz$

$F_1 = 3\,dB = 2.0$

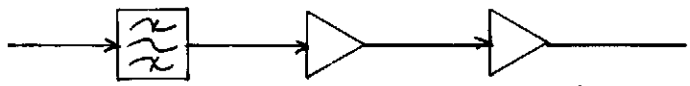
From (3.64), $F_2 = 1 + \frac{T_{e2}}{T_o} = 1 + \frac{800}{290} = 3.76$

From (3.65), $T_{e_1} = (F_1 - 1)T_o = (2-1)(290) = 290K$

From (3.74), $T_e = T_{e_1} + \frac{T_{e2}}{G_1} = 290 + \frac{800}{31.6} = 315K$

From (3.75), $F = F_1 + \frac{F_2 - 1}{G_1} = 2 + \frac{3.76-1}{31.6} = 2.09 = \underline{3.2\,dB}$

$\boxed{3.18}$



$L = 1.5\,dB$ $\quad G = 10\,dB$, $F = 2\,dB$ $\quad G = 15\,dB$, $F = 2\,dB$ $\quad B = 150\,MHz$

a) $F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + \frac{(1.58-1)}{1/1.41} + \frac{(1.58-1)}{10/1.41} = 2.31 = \underline{3.6\,dB}$ ✓

b) $S_i = -85\,dBm$

$T_e = (F-1)T_o = (2.31-1)(290) = 380K$ ✓

$N_i = kT_eB = (1.38\times10^{-23})(380)(150\times10^6) = 7.87\times10^{-13}\,W$

$= -91.0\,dBm.$

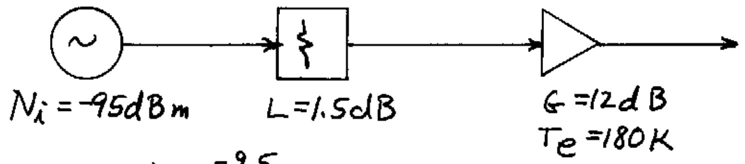
$SNR = \frac{S_i G}{N_i G} = \frac{S_i}{N_i} = S_i(dBm) - N_i(dBm) = -85\,dBm + 91.0\,dBm$

$= \underline{6\,dB}$

$\underline{G = 23.5\,dB}$

Placing the amplifier with F=2dB first will improve the noise figure, but the bandpass filter may be required before the amplifiers to avoid saturation from strong out-of-band signals.

$\boxed{3.19}$



1.5dB = 1.413
12dB = 15.8

a) $$T_e = \frac{P}{kB} = \frac{(0.001)\times 10^{-9.5}}{(1.38\times10^{-23})(75\times10^{6})} = \underline{305.5K}$$ ✓

b) $$F_\ell = 1 + (L-1)\frac{T}{T_0} = 1 + (1.413 - 1)\frac{300}{290} = 1.43$$

$$F_a = 1 + \frac{T_e}{T_0} = 1 + \frac{180}{290} = 1.62$$

FOR CASCADE:

$$F_c = F_\ell + \frac{F_a - 1}{G_\ell} = 1.43 + \frac{1.62-1}{1/1.413} = 2.30 = \underline{3.6dB}$$ ✓

$$T_c = (F_c - 1)T_0 = (2.30-1)(290) = \underline{378K}$$ ✓

c) $$N_o = k(T_c + T_i)BG = (1.38\times10^{-23})(378 + 305.5)(75\times10^{6})\left(\frac{15.8}{1.413}\right)$$
$$= 7.9\times10^{-12}\,W = 7.9\times10^{-9}\,mW = \underline{-81.\ dBm}$$ ✓

$\boxed{3.20}$

From (3.87), $$T_e = \frac{(L-1)(L+|\Gamma_s|^2)}{L(1-|\Gamma_s|^2)}T$$

let $x^2 = |\Gamma_s|^2$; $c = (L-1)T/L$. Then $T_e = c\,\frac{L+x^2}{1-x^2}$

$$\frac{dT_e}{dx} = c\,\frac{(1-x^2)(2x)+(2x)(L+x^2)}{(1-x^2)^2} = \frac{2x(1+L)}{(1-x^2)^2} = 0$$

Thus $x = 0$, so $|\Gamma_s| = 0$ minimizes T_e ✓

$\boxed{3.21}$ Solution using available gain:

$$\Gamma_{out} = S_{11} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{22}\Gamma_S} = S_{11} = 0$$

$$G_{21} = \frac{|S_{21}|^2}{1 - |\Gamma_{out}|^2} = |S_{21}|^2 = \frac{1}{4}$$

$$T_e = \frac{1 - G_{21}}{G_{21}} T = 3T$$

$$F = 1 + \frac{T_e}{T_0} = \underline{\underline{1 + \frac{3T}{T_0}}} \checkmark$$

Solution using the thermodynamic argument:

$$P_2^- = \frac{N_{INPUT}}{4} + \frac{N_{ADDED}}{4} = \frac{kTB}{4} + \frac{N_{ADDED}}{4} = kTB$$

← REF. AT INPUT

$$\therefore \ N_{ADDED} = 3kTB$$

Then

$$T_e = \frac{N_{ADDED}}{kB} = 3T \checkmark$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0} \checkmark$$

CHECK: IF $T = T_0$, $F = 4 = 6\,dB$ ✓ (VERIFIED WITH HP-MDS)

3.22 *Solution using available gain:*

$\Gamma_S = 0$, $\Gamma_{out} = S_{22} = 1/2$.

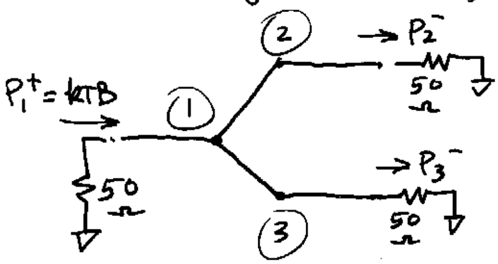
$$G_{21} = \frac{|S_{21}|^2}{1-|\Gamma_{out}|^2} = \frac{1/2}{1-1/4} = \frac{1/2}{3/4} = \frac{2}{3} \checkmark$$ (VERIFIED WITH SEREMADE)

$$T_e = \frac{1-G_{21}}{G_{21}} T = \frac{1/3}{2/3} T = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

CHECK: IF $T = T_0$, $F = \frac{3}{2} = 1.76\,dB$ – AGREES WITH HP-MDS.

Solution by thermodynamic argument:



$P_1^+ = T$ $\quad P_1^- = T/2 + T/2$

$P_2^+ = T$ $\quad P_2^- = T/2 + T/4 + T/4$

$P_3^+ = T$ $\quad P_3^- = T/2 + T/4 + T/4$

$\sum P_i^+ = 3T$ $\quad \sum P_i^- = 3T \checkmark$

(EQUILIBRIUM)

$$P_2^- = \frac{kTB}{2} + \frac{N_{ADDED}}{2} = \frac{3}{4} kTB$$ *(not including reflected power for P_2, since available)*

$$N_{ADDED} = \frac{kTB}{2}$$

$$T_e = \frac{N_{ADDED}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0} \checkmark$$

$\boxed{3.23}$ $$[S] = \frac{-1}{\sqrt{2L}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Solution using available gain:

$$\Gamma_S = \Gamma_{out} = 0$$

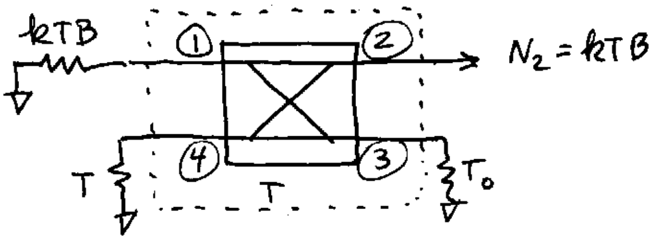
$$G_{21} = |S_{21}|^2 = \frac{1}{2L}$$

$$T_e = \frac{1-G_{21}}{G_{21}} T = (2L-1)T \checkmark$$

$$F = 1 + \frac{T_e}{T_0} = 1 + (2L-1)\frac{T}{T_0} \checkmark$$

CHECK: IF $T=T_0$, $F = 2L$.
IF $L=1$, $F = 2 = 3dB \checkmark$ (VERIFIED WITH SERENADE)

Solution by thermodynamic argument:



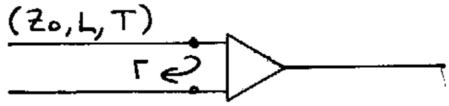
$$N_2 = \frac{kTB}{2L} + \frac{N_{ADDED}}{2L} = kTB$$

$$N_{ADDED} = kTB(2L-1)$$

$$T_e = \frac{N_{ADDED}}{kB} = (2L-1)T$$

$$F = 1 + (2L-1)\frac{T}{T_0} \checkmark$$

$\boxed{3.24}$



Solution using noise temperature:

Let $N_i = kT_oB$

Then $N_o = \underbrace{\frac{kT_oBG}{L}(1-|\Gamma|^2)}_{\text{INPUT NOISE}} + \underbrace{\frac{(L-1)}{L}kTB(1-|\Gamma|^2)G}_{\text{NOISE ADDED BY LINE}} + \underbrace{kT_o(F-1)GB}_{\text{NOISE ADDED BY AMP}}$

also,

$$S_o = \frac{G(1-|\Gamma|^2)}{L}S_i$$

So,

$$F_{CAS} = \frac{S_iN_o}{S_oN_i} = \frac{L}{\cancel{G}(1-|\Gamma|^2)} \cdot \frac{\frac{\cancel{kT_oBG}}{L}(1-|\Gamma|^2)+\left(\frac{L-1}{L}\right)\cancel{G}\cancel{k}T\cancel{B}(1-|\Gamma|^2)+\cancel{k}T_o(F-1)\cancel{GB}}{\cancel{kT_oB}}$$

$$= 1 + (L-1)\frac{T}{T_o} + \frac{L(F-1)}{1-|\Gamma|^2} \checkmark$$

Solution using cascade formula:

$$T_{e(\text{LINE})} = (L-1)T$$

$$F_{(\text{LINE})} = 1 + (L-1)T/T_o$$

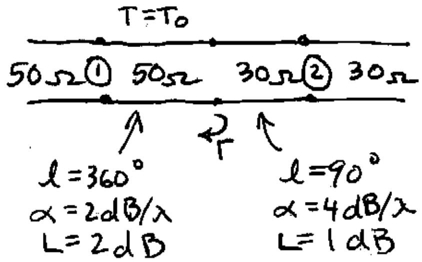
$$G_{(\text{LINE})} = \frac{1}{L}(1-|\Gamma|^2)$$

$$F_{CAS} = F_{(\text{LINE})} + \frac{F_{AMP}-1}{G_{(\text{LINE})}} = 1 + (L-1)\frac{T}{T_o} + \frac{L}{1-|\Gamma|^2}(F-1) \checkmark$$

CHECK: IF $\Gamma = 0$, $F_{CAS} = 1 + (L-1)\frac{T}{T_o} + L(F-1)$ ✓

IF $\Gamma = 0$ AND $T = T_o$, $F_{CAS} = 1 + (L-1) + L(F-1) = LF$ ✓

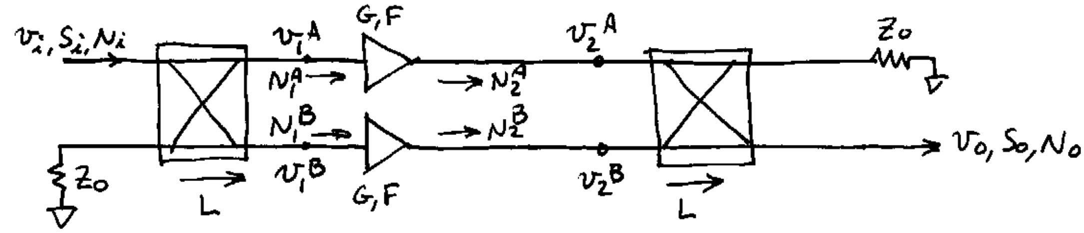
NUMERICAL CHECK:



$$\Gamma = \frac{30-50}{30+50} = \frac{-1}{4}$$

$F_{CAS} = 3.06\,dB$ – AGREES WITH SERENADE.

3.25



$$S_i = v_i^2/2$$

$$v_1^A = \frac{v_i}{\sqrt{2L}} \qquad v_1^B = -j\frac{v_i}{\sqrt{2L}}$$

$$v_2^A = \frac{v_i\sqrt{G}}{\sqrt{2L}} \qquad v_2^B = -j\frac{v_i\sqrt{G}}{\sqrt{2L}}$$

$$v_0 = -j\frac{v_2^A}{\sqrt{2L}} + \frac{v_2^B}{\sqrt{2L}} = -j\frac{v_i\sqrt{G}}{2L} - j\frac{v_i\sqrt{G}}{2L} = -j\frac{v_i\sqrt{G}}{L}$$

$$S_0 = \frac{v_0^2}{2} = \frac{v_i^2 G}{2L^2} = \frac{GS_i}{L^2} \checkmark$$

$$N_1^A = N_1^B = kT_0B$$

$$N_2^A = N_2^B = kT_0BG + kT_eBG = kT_0BG(1+F-1) = kT_0BGF$$

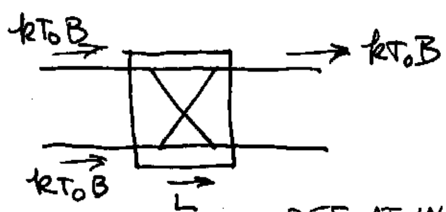
$$N_0 = \frac{N_2^A}{2L} + \frac{N_2^B}{2L} + \frac{N_{ADDED}}{2L} = \frac{kT_0BG}{L}F + \underbrace{\frac{kT_0B}{2L}(2L-2)}_{\text{SEE BELOW}}$$

$$= \frac{kT_0BG}{L}F + kT_0B\left(1-\frac{1}{L}\right)$$

$$F_{TOT} = \frac{S_iN_0}{S_0N_i} = \frac{L^2}{G}\left[\frac{GF}{L} + \left(1-\frac{1}{L}\right)\right] = LF + \frac{L}{G}(L-1) \checkmark$$

CHECK: IF $L=1$, $F_{TOT} = F$ ✓

N_{ADDED} FOR HYBRID:



$$N_0 = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{ADDED}}{2L} = kT_0B$$

REF. AT INPUT

$\therefore N_{ADDED} = 2kT_0B(L-1)$ (REF. AT INPUT)

$\boxed{3.26}$

$N_i = -105\,dBm = 3.16\times10^{-14}\,W$

$F = 6dB = 4$
$G = 30dB = 10^3$
$B = 20MHz$

$T_e = (F-1)T_o = 3(290) = 870K$ ✓

$N_o = G(N_i + kT_eB)$

$= 10^3\left[3.16\times10^{-14} + (1.38\times10^{-23})(870)(20\times10^6)\right]$

$= 2.72\times10^{-10}\,W = -65.7\,dBm$ ✓

$DR_l = \frac{P_1}{N_o} = 21dBm + 65.7\,dBm = \underline{86.7dB}$ ✓

$DR_f = \frac{2}{3}(P_3 - N_o - SNR) = \frac{2}{3}(33 + 65.7 - 8) = \underline{60.5\,dB}$ ✓

$\boxed{3.27}$

Moving the reference for P_3' to the output of the mixer gives,

$P_3' = 13dBm - 6dB = 7dB$ (ref. at output)

Numerical values:

$P_3'' = 22dBm = 158\,mW$ (AMP)

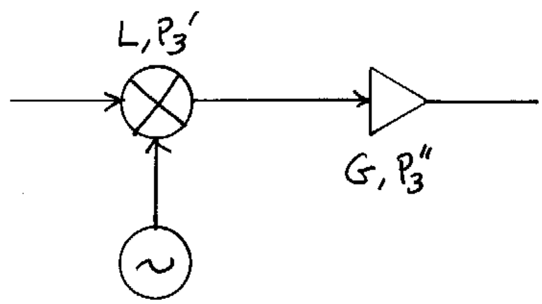
$P_3' = 7\,dBm = 5\,mW$ (MIXER)

$G_2 = 20dB = 100$ (AMP)

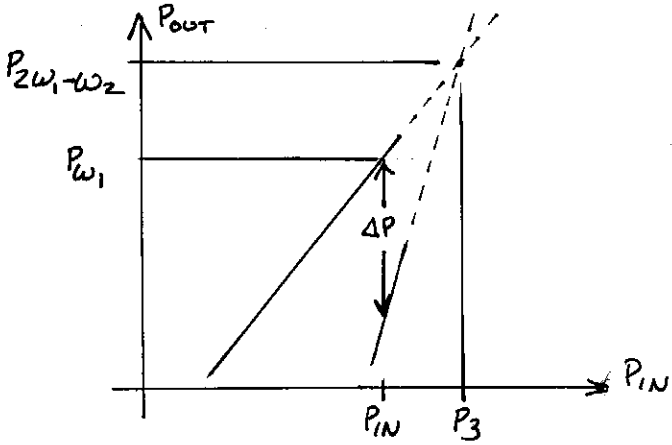
Then from (3.114),

$$P_3 = \left(\frac{1}{G_3 P_3'} + \frac{1}{P_3''}\right)^{-1} = \left[\frac{1}{(100)(5)} + \frac{1}{158}\right]^{-1} = 120\,mW$$

$= \underline{20.8\,dBm}$



$\boxed{3.28}$



$P_{\omega_1} = P_{in} + b_1$ (EQ. OF LINE, SLOPE = 1)

$P_{2\omega_1-\omega_2} = 3P_{in} + b_2$ (EQ. OF LINE, SLOPE = 3)

SUBTRACT:

$$\Delta P = P_{\omega_1} - P_{2\omega_1-\omega_2} = -2P_{in} + b_1 - b_2$$

Now, $P_3 = P_{in}$ when $\Delta P = 0$, so

$$\Delta P = -2P_{in} + b_1 - b_2$$
$$0 = -2P_3 + b_1 - b_2$$

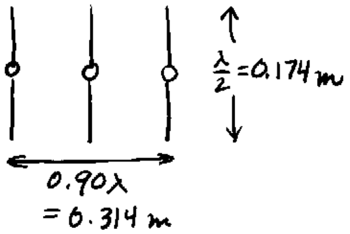
so, $\Delta P = -2P_{in} + 2P_3$

or, $$P_3 = \underbrace{P_{in} + \Delta P/2}_{\text{RELATIVE TO INPUT}} = \underbrace{P_{\omega_1} + \Delta P/2}_{\text{RELATIVE TO OUTPUT}} \checkmark$$

Chapter 4

$\boxed{4.1}$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{860\times10^6} = 0.349\ m$$



From (4.5), $\quad R_{ff} = \dfrac{2D^2}{\lambda} = \dfrac{2(0.314)^2}{0.349} = 0.57\ m$

$\boxed{4.2}$

$$F_\theta(\theta,\phi) = A \sin\theta \sin\phi$$

MAIN BEAM OCCURS AT $\theta = 90°$; $\phi = 90°, 270°$ ✓

3 dB POINTS IN $\theta = 90°$ PLANE :

$$\sin\phi = 0.707 \Rightarrow \phi = 45°, 135°$$

$$HPBW_\theta = 135-45 = \underline{90°} \checkmark$$

3 dB POINTS IN $\phi = 90°$ PLANE :

$$\sin\theta = 0.707 \Rightarrow \theta = 45°, 135°$$

$$HPBW_\phi = 135-45 = \underline{90°} \checkmark$$

$$D = \frac{4\pi F_{MAX}^2}{\iint F^2(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{4\pi}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin^3\theta \sin^2\phi\, d\theta\, d\phi}$$

$$= \frac{4\pi}{\frac{4}{3}\pi} = 3 = \underline{\underline{4.8\,dB}}$$

$\boxed{4.3}$ $F_\phi(\theta,\phi) = \sin\theta$

main beam in $\theta = 90°$ plane; omni in ϕ-plane

$HPBW_\theta = 90°$ (same as short dipole)

$D = 1.5$ (same as short dipole)

$\boxed{4.4}$ $F_\theta(\theta,\phi) = A\sin\theta$ for $0<\theta<\pi/2$
$= 0$ otherwise

$$D = \frac{4\pi F_{MAX}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F_\theta^2(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{4\pi}{2\pi\int_{\theta=0}^{\pi/2}\sin^3\theta\, d\theta} = \frac{4\pi}{2\pi\left(\frac{2}{3}\right)}$$

$= 3 = \underline{4.8 \text{ dB}}$ ✓ (VERIFIED WITH PCAAD 4.0)

$\boxed{4.5}$ $f = 12.4$ GHz, $Diam = 18'' = 0.457$ m, $e_{ap} = 65\%$

$\lambda = \frac{c}{f} = 0.0242$ m ✓

$A = \pi R^2 = \pi\left(\frac{Diam}{2}\right)^2 = 0.164 \text{ m}^2$

From (4.13),

$$D = e_{ap}\frac{4\pi A}{\lambda^2} = (0.65)\frac{4\pi(0.164)}{(0.0242)^2} = 2287 = \underline{\underline{33.6 \text{ dB}}} \checkmark$$

4.6 $G = 32\,dB$, $e_{ap} = 60\%$

$$D = e_{ap} \cdot G = (0.6)(10^{32/10}) = 67,554.$$

From (4.9), $$D \simeq \frac{32,400}{\theta_1 \theta_2}$$

If $\theta_1 = \theta_2$, $$\theta_1 = \sqrt{\frac{32,400}{67,554}} = \underline{\underline{0.69^\circ}}$$

4.7 From Example 4.2, $D = 1.5$ for a short dipole.
$f = 900\,MHz \Rightarrow \lambda = 0.333\,m$, $L = 3\,cm$, $Diam = 0.5\,mm$
The physical cross-section is,

$$A_p = L \cdot Diam = (3\,cm)(0.05\,cm) = \underline{\underline{0.15\,cm^2}}$$

From (4.15) the effective aperture is,

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{(1.5)(0.333)^2}{4\pi} = 0.013\,m^2 = \underline{\underline{132\,cm^2}}$$

4.8 Solving (4.19) for G gives

$$G = \frac{4\pi S R^2}{P_t} = \frac{4\pi(7.5\times10^{-3})(300)^2}{85} = 100 = \underline{\underline{20\,dB}}$$

$\boxed{4.9}$ The directivity is found as,

$$D = \frac{4\pi F_{MAX}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F(\theta,\phi)\sin\theta d\theta d\phi} = \frac{4\pi}{2\pi\int_{\theta=0}^{12°}\sin\theta d\theta} = \frac{2}{(-\cos\theta)\Big|_0^{12°}}$$

$$= \frac{2}{0.0218} = 91.5 = \underline{19.6\,dB} \checkmark$$

Then the gain is $G = eD = (0.8)(91.5) = 73.2 = \underline{18.6\,dB}$

The radiated power is,

$$P_{rad} = \frac{1}{2\eta_0}\int_\theta\int_\phi |E(\theta,\phi)|^2 r^2 \sin\theta d\theta d\phi = \frac{(21.8)^2}{2(377)}(2\pi)(0.0218)$$

$$= 0.086\,W \checkmark$$

Then, $R_{rad} = \frac{2P_{rad}}{|I_0|^2} = \underline{108\,\Omega} \checkmark$

$\boxed{4.10}$ $f = 2.4\,GHz \Rightarrow \lambda = \frac{300}{2400} = 0.125\,m.$

$3dB = 2.$
$22dBm = 158\,mW$

From (4.20) the received power is,

$$P_r = \frac{G_t G_r \lambda^2 P_t}{(4\pi R)^2} = \frac{(158)(2)(0.125)^2}{(4\pi)^2(500)^2} = 1.25\times10^{-7}\,mW$$

$$V_r = \sqrt{R\,P_r} = \sqrt{(50)(1.25\times10^{-7})(10^{-3})} = \underline{79.1\,\mu V}$$

$\boxed{4.11}$

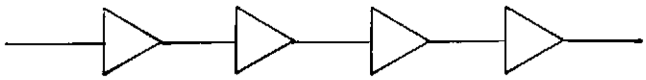
1) RADIO LINK: $f = 28\,\mathrm{GHz} \Rightarrow \lambda = 0.0107\,m$; $G_t = G_r = 25\,dB = 316$.

Let $P_r = 1\,W$. Then,

$$P_t = \frac{(4\pi R)^2}{P_r G_t G_r \lambda^2} = \frac{(4\pi)^2 (5000)^2}{(1)(316)^2 (0.0107)^2} = 3.45 \times 10^8\, W$$

$$\text{ATTENUATION} = 10 \log \frac{P_r}{P_t} = 10 \log \frac{1}{3.45 \times 10^8} = \underline{-85.4\, dB}$$

2) WIRED LINK: $\alpha = 0.05\, dB/m = 0.0057\ \text{neper}/m$; 4 30dB REPEATERS.



$$\text{ATTENUATION OF LINE} = 10 \log e^{-2\alpha R}$$
$$= 10 \log e^{-2(0.0057)(5000)}$$
$$= -250\, dB.$$

$$\text{TOTAL LOSS} = -250 + 4(30) = \underline{-130\, dB}$$

The radio link has much less link loss, and will thus require less transmit power.

$\boxed{4.12}$ $f = 28\,GHz \Rightarrow \lambda = 0.0107\ m$; $G = 32\,dB = 1585$; $P_t = 3\ W$.

From (4.19), $$R = \sqrt{\frac{G P_t}{4\pi S}}.$$

So in the main beam, with $S = 10\ mW/cm^2$,

$$R = \sqrt{\frac{(1585)(3W)}{4\pi\,(0.01\,W/cm^2)}} = 195\ cm = \underline{1.95\ m}$$

In the sidelobe region, let $G = 32-15 = 17\,dB = 50.1$

$$R = \sqrt{\frac{(50.1)(3W)}{4\pi(0.01\,W/cm^2)}} = 35\ cm = \underline{0.35\ m}$$

The directivity of the antenna is

$$D = \frac{G}{e_{ap}} = \frac{1585}{0.6} = 2642 = \frac{4\pi\left(\frac{\pi d^2}{4}\right)}{\lambda^2}$$

So the diameter is about,

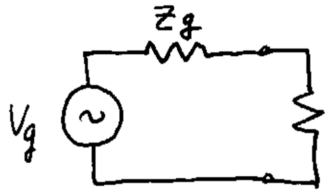
$$d = \sqrt{\frac{\lambda^2 D}{\pi^2}} = 0.175\ m$$

Then the far-field distance is,

$$R_{ff} = \frac{2d^2}{\lambda} = 5.7\ m.$$

So neither of the above distances are in the far-field of the antenna.

4.13



$Z_d = 72 + j40\,\Omega = R_d + jX_d$

$e_{RAD} = 0.8$

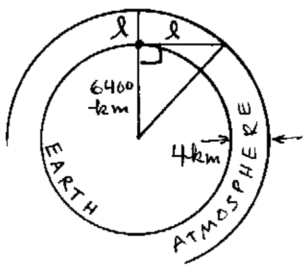
Maximum power transfer will occur when the generator is conjugately matched to the load. Thus, $Z_g = Z_d^* = 72 - j40\,\Omega$. The power <u>delivered</u> to the antenna is,

$$P_d = |I_d|^2 R_d = \left|\frac{V_g}{2}\right|^2 \frac{1}{R_d} \qquad \text{(VOLTAGE DIVIDER)}$$

The radiated power is $P_{rad} = e_{RAD} P_d$. So,

$$|V_g| = \sqrt{\frac{4 P_{RAD} R_{RAD}}{e_{RAD}}} = \sqrt{\frac{(4)(0.1)(72)}{0.8}} = \underline{6.0\ V\ (RMS)}$$

4.14



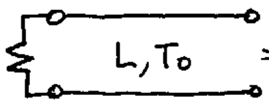
LOOKING TOWARD ZENITH, $l = 4000$ m. $= 4$ km.

LOOKING TOWARD HORIZON,

$$l = \sqrt{(6404)^2 - (6400)^2} = 226 \text{ km}.$$

$\alpha = 0.005$ dB/km

T= 4K



$$\Rightarrow T_e = \frac{4}{L} + (L-1)T_o$$

AT ZENITH: $L = (0.005 \text{ dB/km})(4 \text{ km}) = 0.02\text{dB} = 1.0046$

$\underline{T_e = 5.3K}$

AT HORIZON: $L = (0.005\text{dB/km})(226\text{km}) = 1.13\text{dB} = 1.297$

$\underline{T_e = 89K}$

$\boxed{4.15}$ $N_o = kT_b B = S_o$ FOR $S_o/N_o = 0\,dB$

$4\,dB = 2.51$
$\lambda_o = 4.48\,m$

$$kT_b B = P_r = \frac{P_t G^2 \lambda^2}{(4\pi R)^2}$$

So,
$$R = \sqrt{\frac{P_t G^2 \lambda^2}{16\pi^2 kT_b B}} = \sqrt{\frac{(1000)(2.51)^2(4.48)^2}{16\pi^2(1.38\times10^{-23})(4)(4\times10^6)}} = \underline{1.9\times10^9\,m}$$

DISTANCE TO VENUS = $4.2\times10^{10}\,m$

IF SNR = 30dB = 1000,

$$R = \sqrt{\frac{(1000)(2.51)^2(4.48)^2}{16\pi^2(1.38\times10^{-23})(4)(4\times10^6)(1000)}} = 6.0\times10^7\,m$$

Such a signal could not be received on the nearest planet, let alone beyond the solar system!

$\boxed{4.16}$ $f = 2GHz \Rightarrow \lambda = 0.15\,m$; $G = 60\,dB = 10^6$; $P_r = kT_b B$; $T_b = 4K$.
$R = 4.35$ LIGHT YRS $= (4.35)(9.46\times10^{15}\,m) = 4.1\times10^{16}\,m$ (ALPHA CENTAURI)

$$P_t = \frac{16\pi^2 R^2 P_r}{G^2\lambda^2} = \frac{16\pi^2 R^2 kT_b B}{G^2\lambda^2}$$

$$= \frac{16\pi^2(4.1\times10^{16})^2(1.38\times10^{-23})(4)(10^3)}{(10^{12})(0.15)^2}$$

$$= 6.5\times10^5\,W = \underline{650\,kW}$$

NOTE: TYPICAL SETI SEARCHES USE $B \simeq 1\,Hz$!
THEN $P_t \simeq 650\,W$.

$\boxed{4.17}$ CARRIER POWER AT RECEIVER:

$$C = S_i G_A G/L \qquad (S_i \text{ REF. TO ANTENNA W/ 0dBi})$$

AT INPUT TO AMPLIFIER:

$$T_e = T_A + (F-1)T_0 + (L-1)T_0/G$$

THE NOISE POWER AT INPUT TO RECEIVER:

$$N = kT_eG/L = C/(C/N)$$

$L = 25\text{dB} = 316.2$
$S_i = 1\times10^{-16}$
$G_A = 5\text{dB} = 3.16$
$\frac{C}{N} = 32\text{dB} = 1.58\times10^3$

So,

$$T_e = \frac{CL}{k(\frac{C}{N})G} = \frac{S_i G_A}{k(\frac{C}{N})}$$

$$F = 1 + \frac{T_e}{T_0} - \frac{T_A}{T_0} - \frac{(L-1)}{G} = 1 + \frac{S_i G_A}{kT_0(\frac{C}{N})} - \frac{T_A}{T_0} - \frac{(L-1)}{G}$$

$$= 1 + \frac{(1\times10^{-16})(3.16)}{(1.38\times10^{-23})(290)(1.58\times10^3)} - \frac{300}{290} - \frac{(316.2-1)}{10}$$

$$= 18.4 = \underline{\underline{12.6\text{ dB}}}$$

$\boxed{4.18}$ $T = eT_b + (1-e)T_p + T_R$

$T_b = 50\text{K}$
$T_p = 290\text{K}$
$F = 1.1\text{dB} = 1.29$
$L = 2.5\text{dB} = 1.78$
$G = 33.5\text{dB} = 2240$

The noise temp. of the receiver is,

$$T_R = (F-1)T_0 = (1.29-1)(290) = 84\text{ K}$$

The efficiency of the array is,

$$e = \frac{1}{L} = \frac{1}{1.78} = 0.56$$

Thus, $T = (0.56)(50) + (1-0.56)(290) + 84 = \underline{240\text{K}}$

Then, $\frac{G}{T}(\text{dB}) = 10\log\frac{2240}{240} = \underline{9.7\text{dB/K}}$

This value is well below the desired minimum of 12dB/K.

$\boxed{4.19}$ From (4.28), $T_A = e_{RAD} T_b + (1 - e_{RAD}) T_p$

$$= (T_b - T_p) e_{RAD} + T_p$$

So,

$$e_{RAD} = \frac{T_A - T_p}{T_b - T_p} = \frac{105 - 290}{5 - 290} = \underline{65\%}$$

$\boxed{4.20}$ $f = 882\ MHz \Longrightarrow \lambda = 0.34\,m$; $F = 6\,dB = 4$; $SNR = 18\,dB = 63.1$

$G = 2\,dB = 1.58$

$$T_{SYS} = T_A + T_R = T_A + (F-1)T_0$$
$$= 200 + (4-1)(290) = 1070\ K$$

$$N_0 = kT_{SYS}B = (1.38 \times 10^{-23})(1070)(3 \times 10^4)$$
$$= 4.47 \times 10^{-16}\ W \quad \text{(AT RECEIVER INPUT)}$$

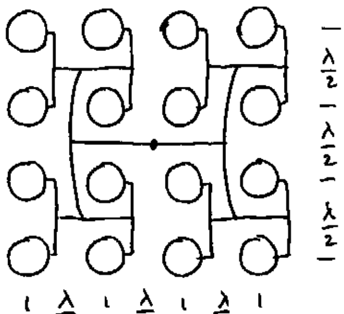
$$S_0 = \left(\frac{S_0}{N_0}\right) N_0 = (63.1)(4.47 \times 10^{-16})$$
$$= 2.82 \times 10^{-14}\ W = \underline{-98.2\ dBm}$$

$$R = \sqrt{\frac{P_t G_t G_r \lambda^2}{S_0 (4\pi)^2}} = \sqrt{\frac{(10)(10)(1.58)(0.34)^2}{(2.82 \times 10^{-14})(4\pi)^2}} = 2.2 \times 10^6\ m$$

$$= \underline{220\ km}$$

(This is unrealistically large due to assumption of a free-space propagation model.)

$\boxed{4.21}$



SIDE LENGTH $= \frac{N\lambda}{2}$ FOR $N \times N$ ARRAY

$$D = \frac{4\pi A}{\lambda^2} = \frac{4\pi N^2 \lambda^2}{4\lambda^2} = \pi N^2$$

L_f = FEED LENGTH = $N\lambda/2$

α = ATTENUATION (neper/λ)

L = LOSS = $e^{2\alpha L_f} = e^{\alpha N \lambda} \geq 1$

OPTIMIZE GAIN:

$$G = \frac{D}{L_f} = \pi N^2 e^{-\alpha N \lambda}$$

$$\frac{dG}{dN} = 2\pi N e^{-\alpha N \lambda} - \pi N^2 \alpha \lambda e^{-\alpha N \lambda} = 0$$

so, $\quad 2 = N\alpha\lambda$, or $\boxed{N_{OPT} = \frac{2}{\alpha\lambda}}$ (GAIN OPTIMIZATION)

NOISE TEMPERATURE:

$$T_A = e T_b + (1-e) T_o = \frac{T_b}{L} + \left(1 - \frac{1}{L}\right) T_o$$

So,

$$\frac{G}{T} = \frac{\pi N^2 e^{-\alpha N \lambda}}{T_b e^{-\alpha N \lambda} + (1 - e^{-\alpha N \lambda}) T_o} = \frac{\pi N^2}{T_b + (e^{\alpha N \lambda} - 1) T_o}$$

$$\frac{d(G/T)}{dN} = \frac{2\pi N}{T_b + (e^{\alpha N \lambda} - 1) T_o} - \frac{\pi N^2 T_o \alpha \lambda e^{\alpha N \lambda}}{[T_b + (e^{\alpha N \lambda} - 1) T_o]^2} = 0$$

$$2N[T_b + (e^{\alpha N \lambda} - 1) T_o] - T_o N^2 \alpha \lambda e^{\alpha N \lambda} = 0$$

LET $x = \alpha N \lambda$: $\quad 2T_o(e^x - 1) + 2T_b - T_o x e^x = 0$

THIS CAN BE SOLVED NUMERICALLY FOR x, GIVEN T_b, T_o.

LET $\alpha = 0.016$ neper/λ. Then the optimum values of N and G/T are, for various T_b, given below:

4.21 CONTINUED:

T_b (K)	x_{OPT}	N_{OPT}	$\frac{G}{T}$ (dB/K)
0	1.595	99.7	14.4
5	1.60	100.	14.4
50	1.60	100.	14.2
100	1.78	111.	14.0
290	2.0	125.	13.6

$$N_{OPT} = \frac{x_{OPT}}{0.016}$$

(G/T OPTIMIZATION)

The case where $T_b = 290\,K$ can be solved directly:

$$2e^x - 2 + 2 - xe^x = 0$$

$$2 - x = 0$$

$$x = 2 \checkmark \qquad N_{OPT} = \frac{2}{\alpha\lambda} = 125.$$

$\boxed{4.22}$ From (4.41) the radiated field is, (far-zone)

$$E_\phi = V_0 \sin\theta \, \frac{e^{-jk_0 r}}{r} \qquad V_0 = \frac{k_0^2 \eta_0 b^2 I_0}{4}$$

$$P_{RAD} = \frac{r^2}{2\eta_0} \int_\theta \int_\phi |E_\phi|^2 \sin\theta \, d\theta \, d\phi = \frac{V_0^2}{2\eta_0} (2\pi)\left(\tfrac{4}{3}\right) = \frac{4\pi V_0^2}{3\eta_0} = \tfrac{1}{2} |I_0|^2 R_{RAD}$$

Thus,
$$R_{RAD} = \frac{8\pi V_0^2}{3\eta_0 |I_0|^2} = \frac{\pi k_0^4 \eta_0 b^4}{6} = \frac{\pi (2\pi)^4 (120\pi) b^4}{6\lambda^4}$$

$$= (2\pi)^4 (20) \left(\frac{\pi b^2}{\lambda^2}\right)^2 = 31{,}171 \left(\frac{\pi b^2}{\lambda^2}\right)^2 \checkmark$$

$\boxed{4.23}$ $f = 900\,MHz \Rightarrow \lambda = \frac{300}{900} = 0.333\,m \qquad \sigma = 5.8\times10^{7}\ S/m.$

$R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}} = 7.8\times10^{-3}\,\Omega;\ R_r = 20\pi^2\left(\frac{L}{\lambda}\right)^2;\ R_\ell = \frac{R_s L}{6\pi a};\ e = \frac{R_r}{R_r + R_\ell}$

L/λ	$a = 10^{-3}\lambda = 3.3\times10^{-4}\,m$			$a = 10^{-5}\lambda = 3.3\times10^{-6}\,m$		
	R_r	R_ℓ	e	R_r	R_ℓ	e
.01	.0197	4.2E-3	0.82	.0197	.42	0.045
.02	.0790	8.4E-3	0.90	.0790	.84	0.086
.05	.493	2.1E-2	0.96	.493	2.1	0.19
.10	1.97	4.2E-2	0.98	1.97	4.2	0.32

$\boxed{4.24}$ $R_r = 31{,}200\left(\frac{\pi b^2}{\lambda^2}\right)^2 \quad ; \quad R_\ell = \frac{b}{a}R_s \quad ; \quad e = \frac{R_r}{R_r + R_\ell}$

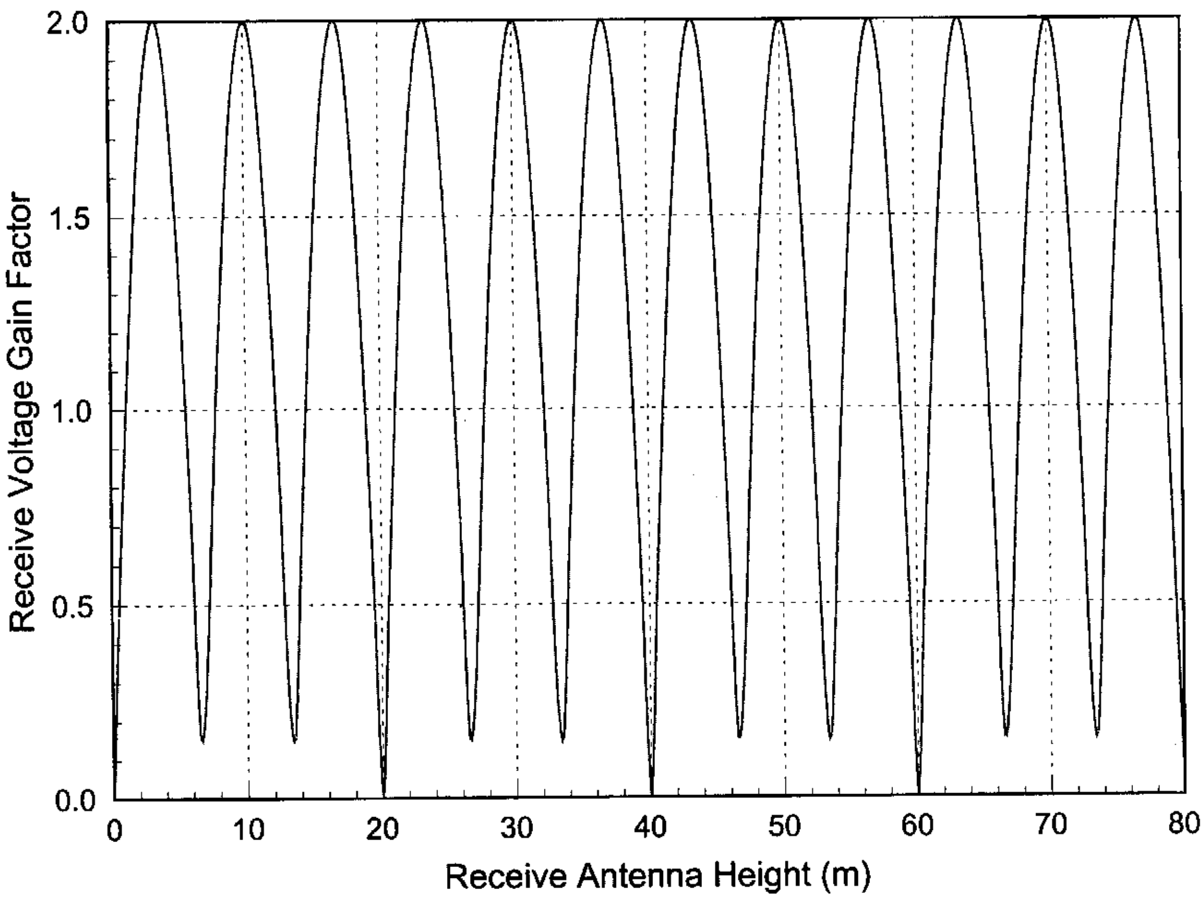
$2\pi b/\lambda$	$a = 10^{-3}\lambda$			$a = 10^{-5}\lambda$		
	R_r	R_ℓ	e	R_r	R_ℓ	e
.01	2E-6	1.2E-2	1.7E-4	2E-6	1.2	1.7E-6
.02	3E-5	2.5E-2	1.2E-3	3E-5	2.5	1.2E-5
.05	1.2E-3	6.2E-2	1.9E-2	1.2E-3	6.2	1.9E-4
.10	2E-2	1.2E-1	1.4E-1	2E-2	12.4	1.6E-3

$\boxed{4.25}$ $f = 900\,MHz$, $h_1 = 50\,m$, $d = 2000\,m$, $0 \le h_2 \le 80\,m$.

$F = 2\left|\sin(k_0 h_1 h_2 / d)\right|$ — PATH GAIN FACTOR NORMALIZED TO FREE-SPACE.

h_2 (m)	F
0	0
2	1.62
5	1.41
10	2.00

a short computer program was used to plot values at a fine resolution.



$\boxed{4.26}$ $f_r(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad r>0$

$$F_r(r_0) = \int_0^{r_0} f_r(r)\,dr = \frac{1}{\sigma^2}\int_0^{r_0} r e^{-r^2/2\sigma^2} dr = \frac{1}{\sigma^2}(-\sigma^2)\, e^{-r^2/2\sigma^2}\Big|_0^{r_0}$$

$$= 1 - e^{-r_0^2/2\sigma^2} \checkmark$$

$$\text{RMS} = \sqrt{E\{r^2\}}, \quad E\{r^2\} = \int_0^\infty r^2 f_r(r)\,dr = \frac{1}{\sigma^2}\int_0^\infty r^3 e^{-r^2/2\sigma^2} dr = \frac{1}{\sigma^2}\frac{1}{2\left(\frac{1}{2\sigma^2}\right)^2} = 2\sigma^2$$

$\underline{\text{RMS} = \sqrt{2}\,\sigma}$

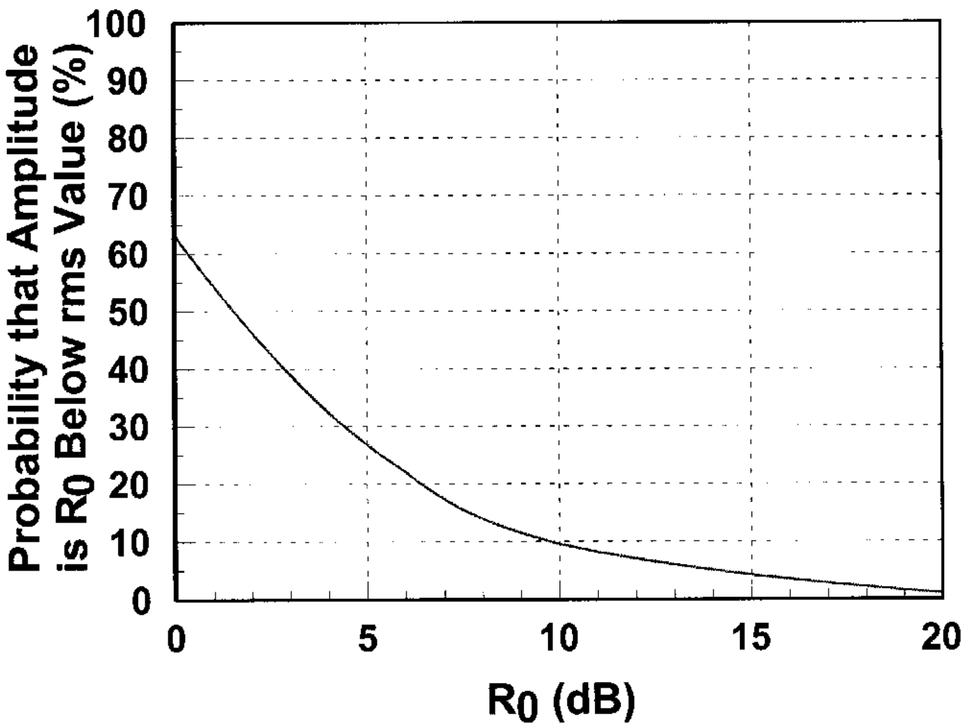
R_0 (dB)	$r_0/\sqrt{2}\sigma$	$P\{r<r_0/\sigma\}\times 100$
0	1	63%
3	.708	39
6	.501	22
10	.316	9.5
20	.100	1.

$R_0 = 20 \log r_0/\sqrt{2}\sigma$

$\frac{r_0}{\sqrt{2}\sigma} = 10^{-R_0/20}$

$P\{r<r_0\} = 1 - e^{-r_0^2/2\sigma^2}$

$= 1 - e^{-1/R_0^2}$



Chapter 5

5.1 Since $v(t)$ and $i(t)$ must be real functions, then Fourier transforms must satisfy $V(-\omega) = V^*(\omega)$ and $I(-\omega) = I^*(\omega)$. Then,

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} = \frac{V^*(-\omega)}{I^*(-\omega)} = Z^*(-\omega) \checkmark$$

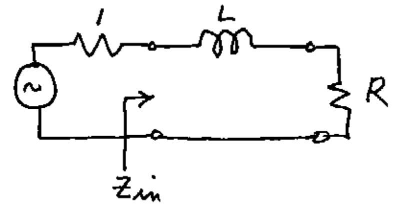
Thus, if $Z(\omega) = R(\omega) + jX(\omega)$, then $R(\omega)$ must be even in ω, and $X(\omega)$ must be odd in ω. The reflection coefficient is,

$$\Gamma(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} = \frac{Z^*(-\omega) - Z_0}{Z^*(-\omega) + Z_0} = \Gamma^*(-\omega) \checkmark$$

Then, $|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = \Gamma^*(-\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2$

Thus $|\Gamma(\omega)|^2$ is an even function of ω.

5.2



$N = 1$, $\omega_c = 1$, 1dB EQUAL RIPPLE

$Z_{in} = R + j\omega L$

From (5.6) and appendix E, $P_{LR} = 1 + k^2 T_1^2\left(\frac{\omega}{\omega_c}\right) = 1 + k^2\omega^2$

$$P_{LR} = \frac{|Z_{in}+1|^2}{2(Z_{in} + Z_{in}^*)} = \frac{|R+1+j\omega L|^2}{4R} = \frac{1}{4R}(R^2 + 2R + 1 + \omega^2 L^2) = 1 + k^2\omega^2$$

EQUATING POWERS OF ω GIVES:

ω^0: $\frac{1}{4R}(R^2 + 2R + 1) = 1 \implies \underline{R = 1 = g_0} \checkmark$

ω^2: $\frac{L^2}{4R} = k^2$

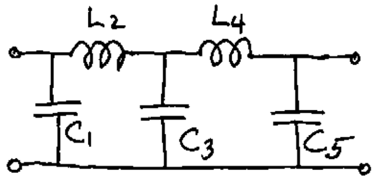
For 1dB = 1.259 ripple, $1 + k^2 = 1.259 \implies k = 0.509 \checkmark$

So, $L = 2Rk = \underline{1.018 = g_1} \checkmark$

These results agree with data from Matthai, Young, Jones.

$\boxed{5.3}$ $f_0 = 3\,GHz$, LOW-PASS, MAXIMALLY-FLAT, $Z_0 = 75\Omega$,
$\alpha = 20dB$ at $5\,GHz$.

To use Figure 5.5, $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{5}{3} - 1 = 0.667$
The figure shows that $N=5$ will give $\alpha > 20dB$.
Then from Table 5.1,



$g_1 = 0.618$
$g_2 = 1.618$
$g_3 = 2.000$
$g_4 = 1.618$
$g_5 = 0.618$

Using (5.18) for scaling gives

$$C_1 = \frac{g_1}{R_0 \omega_c} = 0.437\,pF \checkmark$$

$$L_2 = \frac{R_0 g_2}{\omega_c} = 6.44\,nH \checkmark$$

$$C_3 = \frac{g_3}{R_0 \omega_c} = 1.41\,pF \checkmark$$

$$L_4 = \frac{R_0 g_4}{\omega_c} = 6.44\,nH \checkmark$$

$$C_5 = \frac{g_5}{R_0 \omega_c} = 0.437\,pF \checkmark$$

The simulated filter response is shown below.

5.3 CONTINUED

04/27/00 Ansoft Corporation - Microwave Harmonica (tm) v7.5 11:09:22

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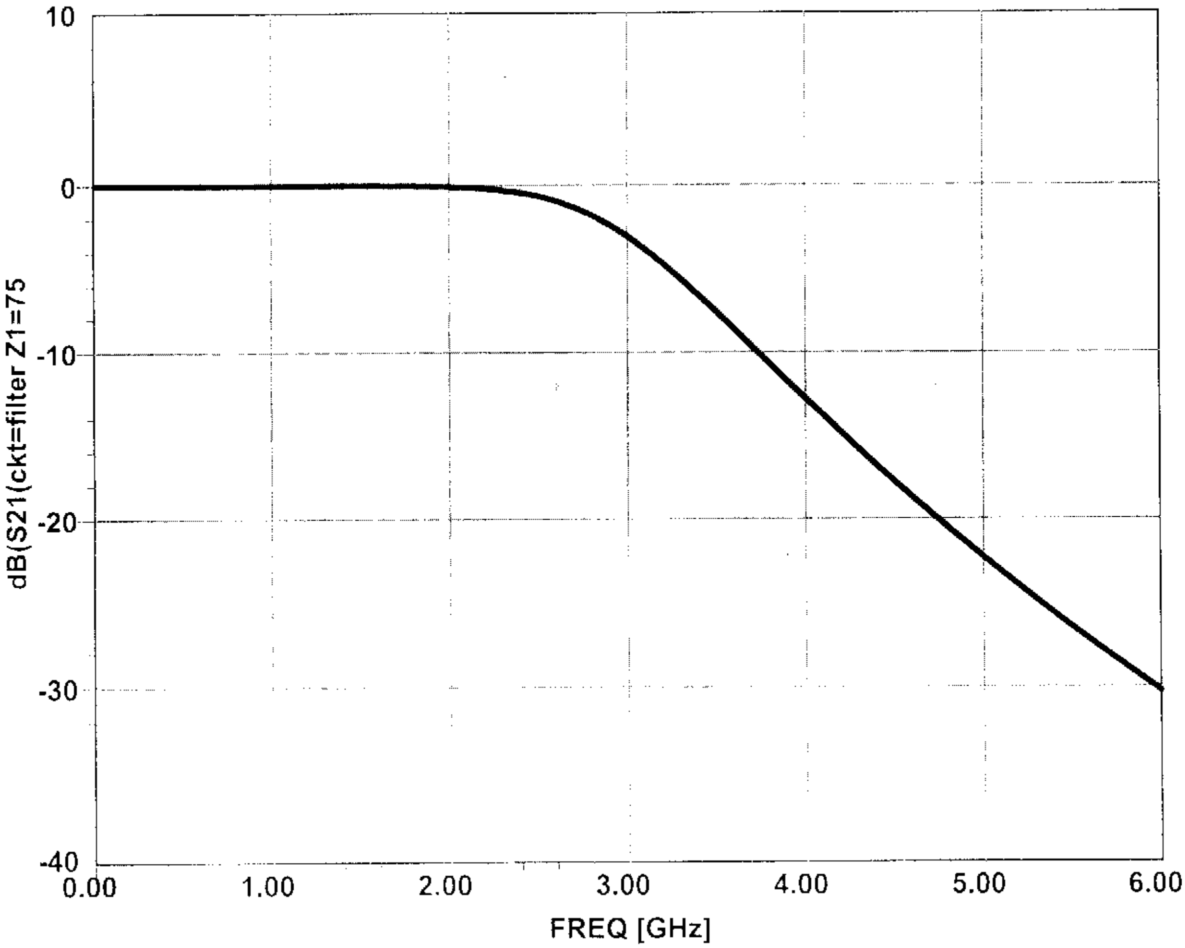
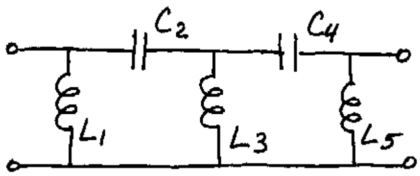


$\boxed{5.4}$ $f_o = 1\text{ GHz}$, HIGH-PASS, 3 dB EQUAL-RIPPLE, $N = 5$, $Z_o = 50\Omega$

To use Figure 5.6b, $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{1}{0.6} - 1 = 0.667$. From the figure, the attenuation at 0.6 GHz is about 41 dB. From Table 5.2, the prototype element values are,

$g_1 = 3.4817$
$g_2 = 0.7618$
$g_3 = 4.5381$
$g_4 = 0.7618$
$g_5 = 3.4817$



Scaling using (5.21) gives

$$L_1 = \frac{Z_o}{\omega_c g_1} = 2.28\text{ mH} \checkmark$$

$$C_2 = \frac{1}{Z_o \omega_c g_2} = 4.18\text{ pF} \checkmark$$

$$L_3 = \frac{Z_o}{\omega_c g_3} = 1.75\text{ mH} \checkmark$$

$$C_4 = \frac{1}{Z_o \omega_c g_4} = 4.18\text{ pF} \checkmark$$

$$L_5 = \frac{Z_o}{\omega_c g_5} = 2.28\text{ mH} \checkmark$$

The simulated filter response is shown below.

5.4 CONTINUED.

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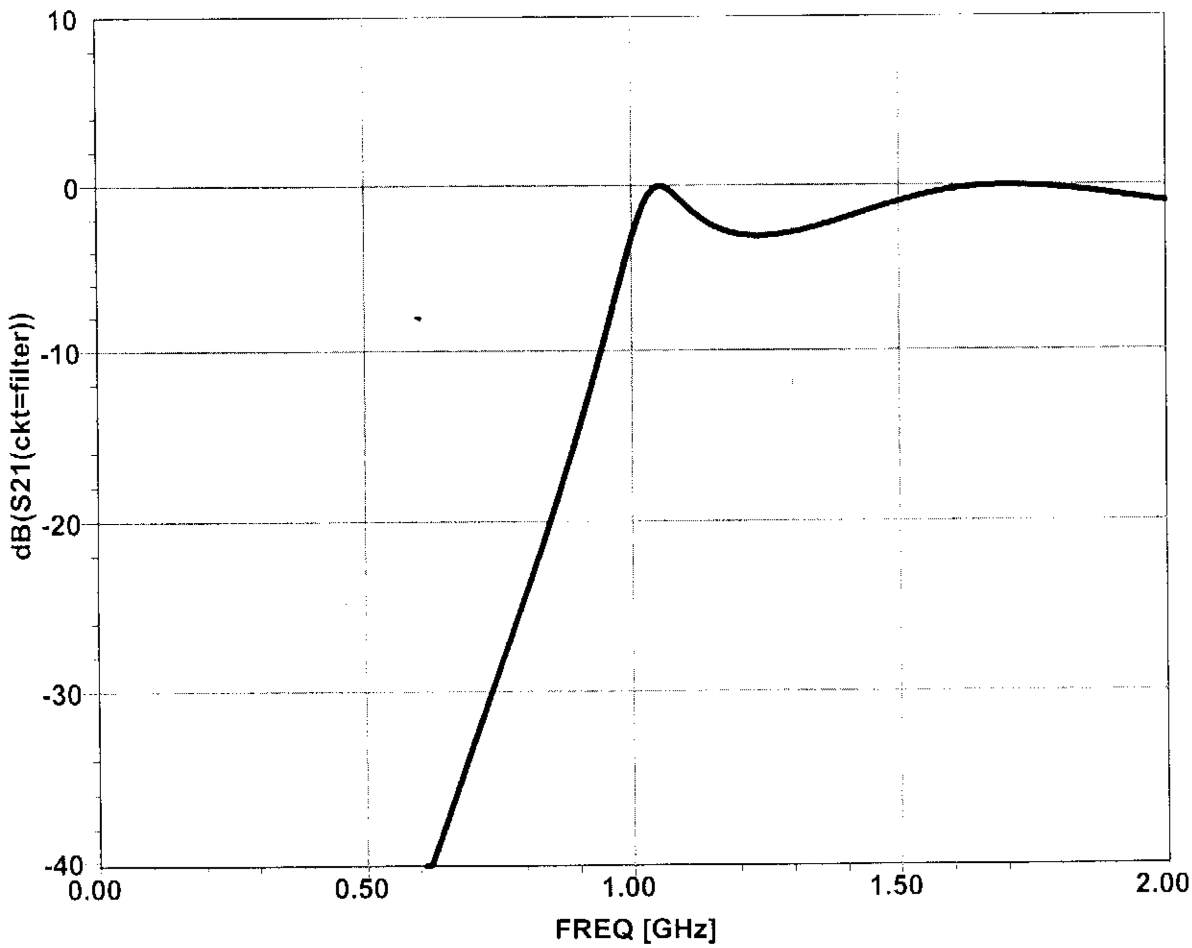


$\boxed{5.5}$ $f_0 = 2GHz$, BANDPASS, MAXIMALLY FLAT GROUP DELAY

$\Delta = 0.05$, $N=4$, $Z_0 = 50\Omega$

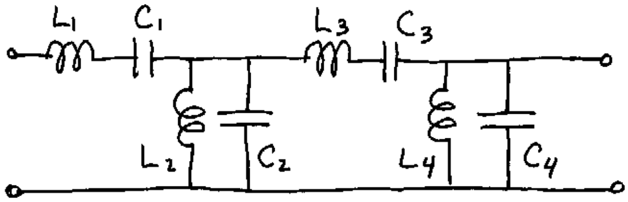
From Table 5.3 the prototype element values are

$g_1 = 1.0598$

$g_2 = 0.5116$

$g_3 = 0.3181$

$g_4 = 0.1104$



From Table 5.4 and (5.25) the scaled element values are,

$$L_1 = \frac{g_1 Z_0}{\omega_0 \Delta} = 84.3 \text{ nH} \qquad C_1 = \frac{\Delta}{\omega_0 g_1 Z_0} = 0.075 \text{ pF}$$

$$L_2 = \frac{\Delta Z_0}{\omega_0 g_2} = 0.388 \text{ nH} \qquad C_2 = \frac{g_2}{\omega_0 \Delta Z_0} = 16.3 \text{ pF}$$

$$L_3 = \frac{g_3 Z_0}{\omega_0 \Delta} = 25.3 \text{ nH} \qquad C_3 = \frac{\Delta}{\omega_0 g_3 Z_0} = 0.25 \text{ pF}$$

$$L_4 = \frac{\Delta Z_0}{\omega_0 g_4} = 1.80 \text{ nH} \qquad C_4 = \frac{g_4}{\omega_0 \Delta Z_0} = 3.51 \text{ pF}$$

The simulated filter response is shown below.

5.5 CONTINUED

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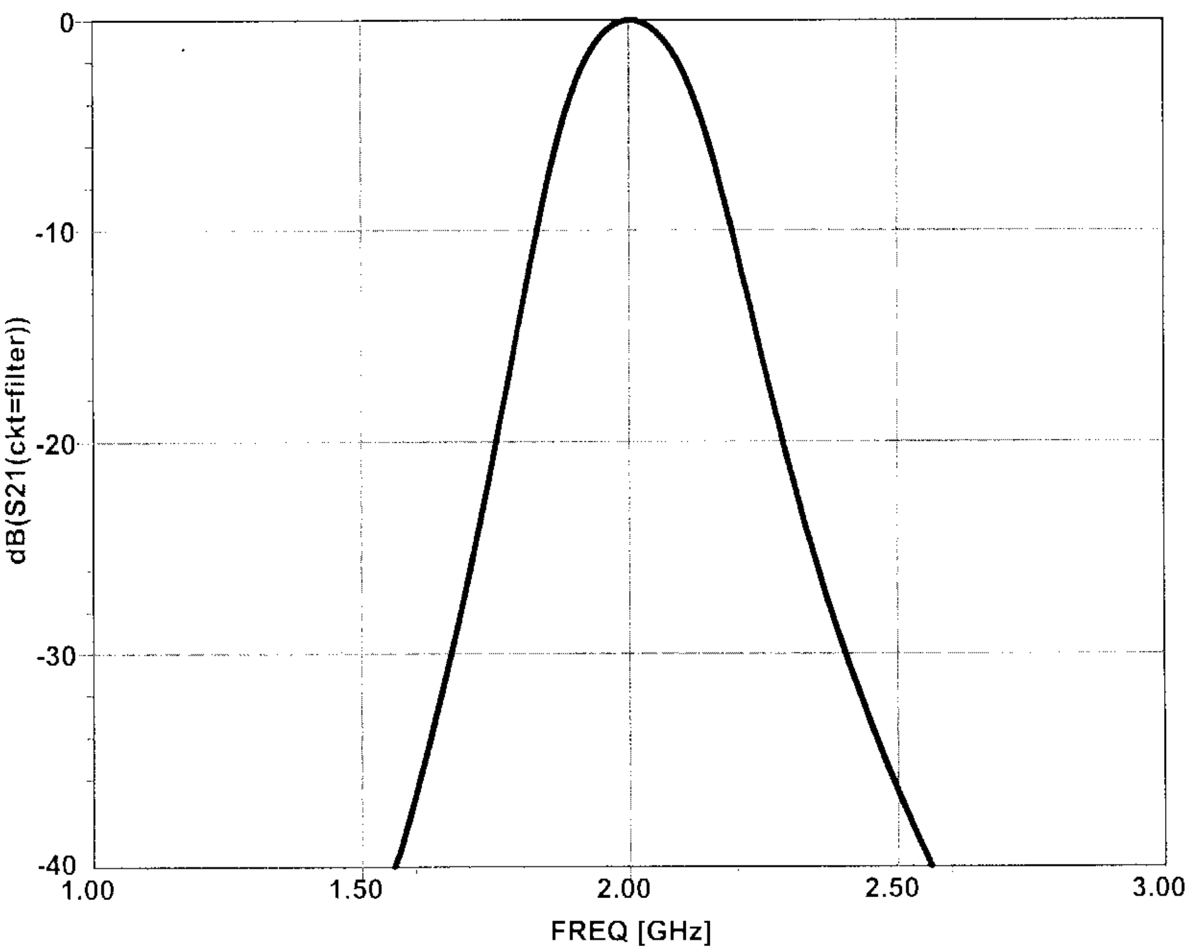


$\boxed{5.6}$ $f_o = 3\,GHz$, $Z_o = 75\,\Omega$, $N = 3$, BANDSTOP, 0.5dB EQUAL-RIPPLE

First use (5.26) to transform 3.1 GHz to a low-pass prototype response frequency:

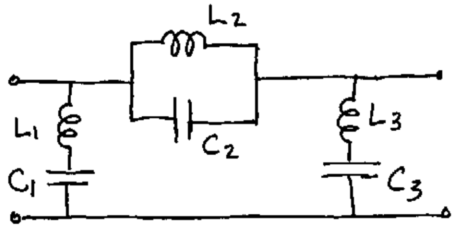
$$\omega \leftarrow \Delta\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^{-1} = 0.1\left(\frac{3.1}{3} - \frac{3}{3.1}\right)^{-1} = 1.52$$

Then $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.52$, and Figure 5.6a gives an attenuation of about 11 dB for $N=3$. From Table 5.2 the prototype element values are,

$g_1 = 1.5963$

$g_2 = 1.0967$

$g_3 = 1.5963$



Scaling with Table 5.4 and (5.27) gives

$$L_1 = \frac{Z_o}{\omega_o g_1 \Delta} = 24.9\ nH \qquad C_1 = \frac{g_1 \Delta}{\omega_o Z_o} = 0.113\ pF$$

$$L_2 = \frac{g_2 \Delta Z_o}{\omega_o} = 0.436\ nH \qquad C_2 = \frac{1}{Z_o \omega_o g_2 \Delta} = 6.45\ pF$$

$$L_3 = \frac{Z_o}{\omega_o g_3 \Delta} = 24.9\ nH \qquad C_3 = \frac{g_3 \Delta}{Z_o \omega_o} = 0.113\ pF$$

The simulated filter response is shown below.

5.6 CONTINUED.

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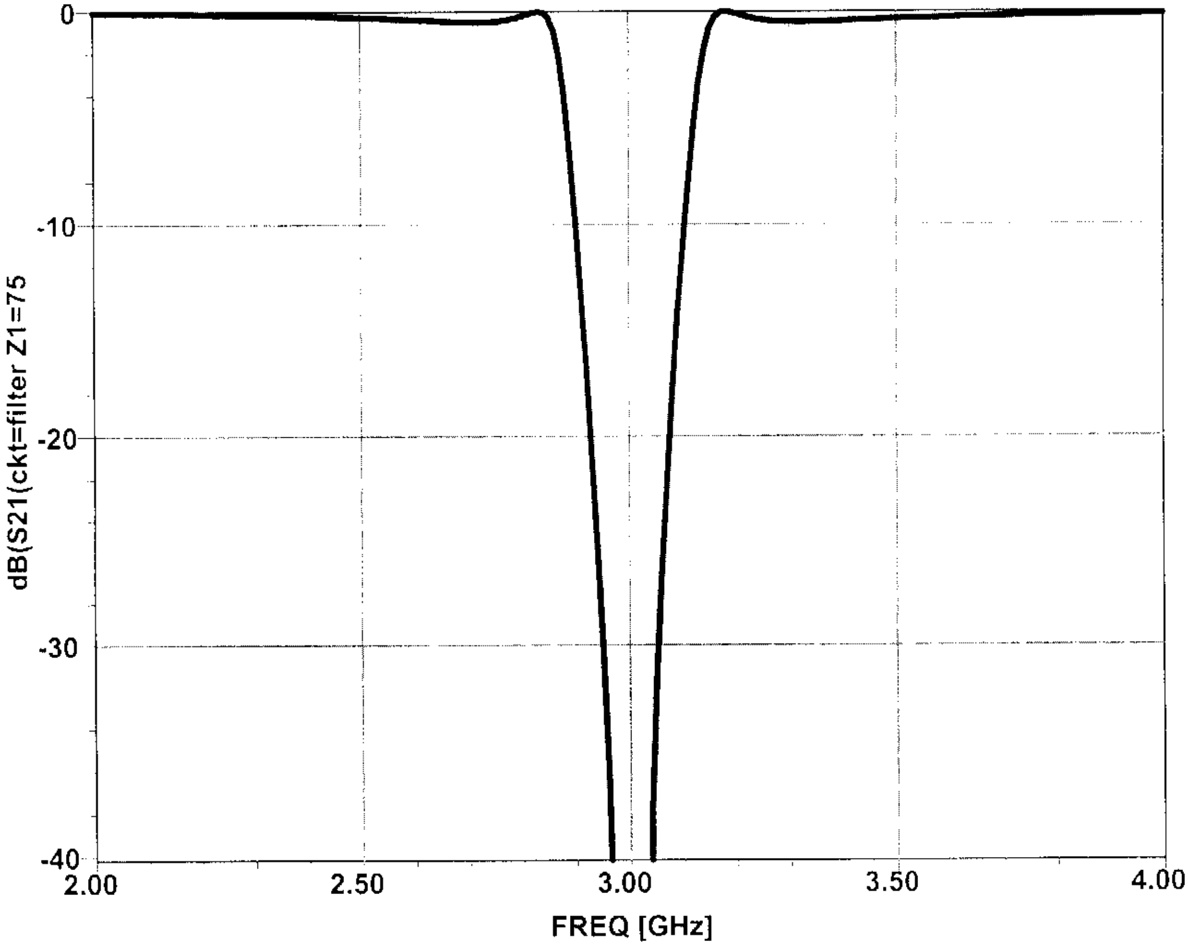
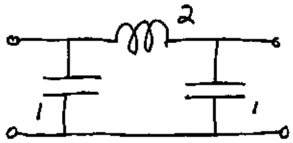


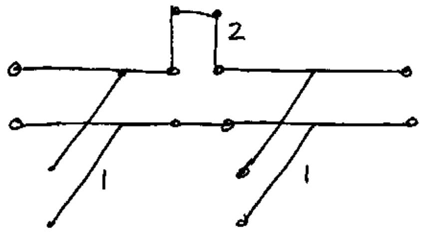
$\boxed{5.7}$ $f_0 = 2.5\,\text{GHz}$, $N=3$, MAXIMALLY-FLAT, $Z_0 = 50\,\Omega$, SERIES STUBS.

From Table 5.1 the low-pass prototype is,

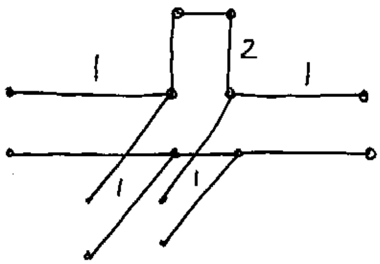


(choosing a π-circuit simplifies the design)

applying Richards' transform:

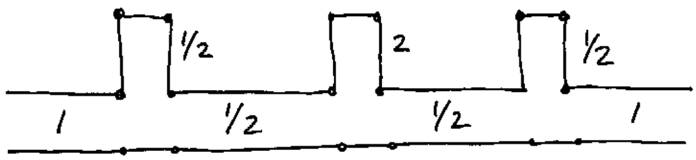


add unit elements:

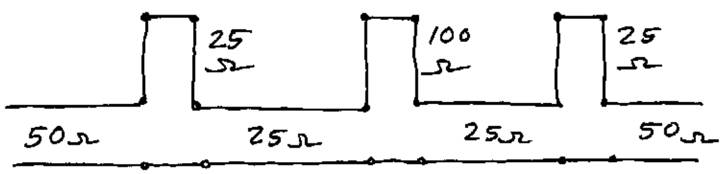


apply the first Kuroda identity (twice)

$Z_1 = 1$
$Z_2 = 1$
$n^2 = 2$



Scale to $50\,\Omega$:



all lines and stubs are $\lambda/8$ long at 2.5 GHz. The simulated filter response is shown below.

5.7 CONTINUED

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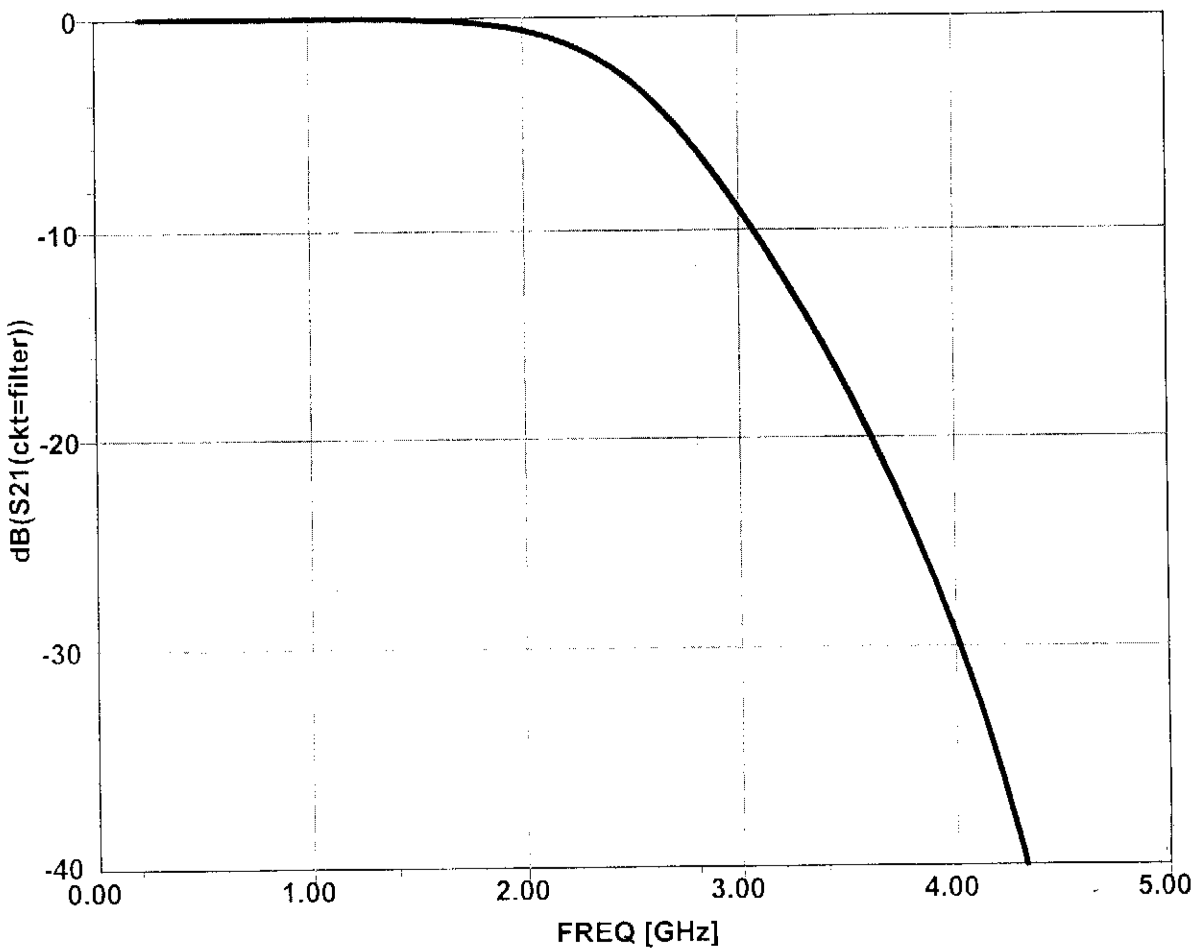
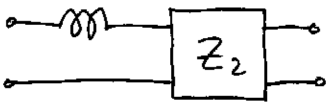
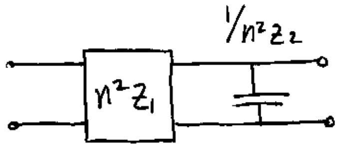
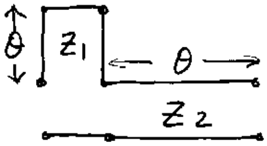


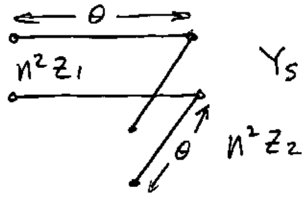
$\boxed{5.8}$







$Z_s = j Z_1 \tan\theta$



$Y_s = \dfrac{j}{n^2 Z_2} \tan\theta$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jZ_1\tan\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j(Z_1+Z_2)\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jn^2Z_2\sin\theta \\ \frac{j\sin\theta}{n^2Z_1} & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\tan\theta}{n^2Z_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & jn^2Z_1\sin\theta \\ \frac{j}{n^2}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\theta & \cos\theta \end{bmatrix}$$

These two matrices are equal if,

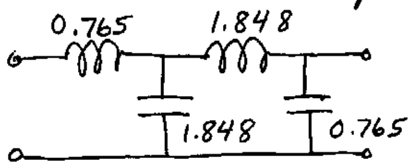
$$Z_1 + Z_2 = n^2 Z_1$$

or,

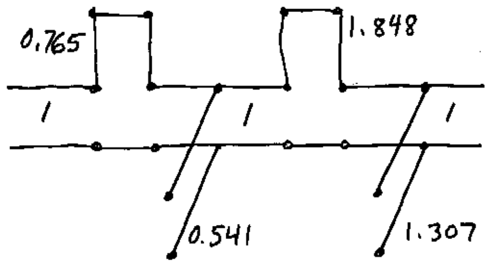
$$n^2 = 1 + Z_2/Z_1$$

$\boxed{5.9}$ $f_o = 2.5$ GHz, LOW-PASS, MAXIMALLY-FLAT, $Z_0 = 50\Omega$, SHUNT STUBS, $N=4$.

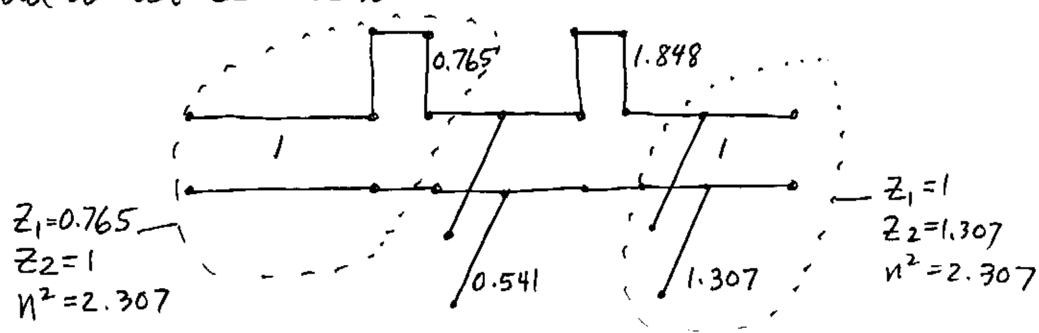
From Table 5.1 the low-pass prototype is,



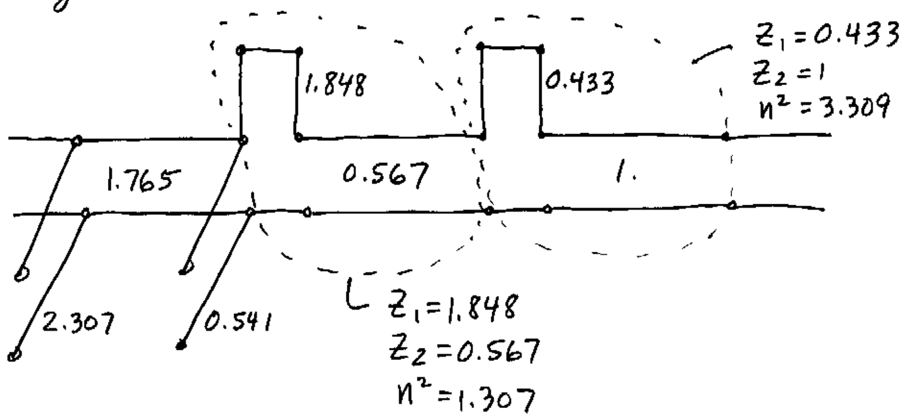
Applying Richard's transform:



Add unit elements:

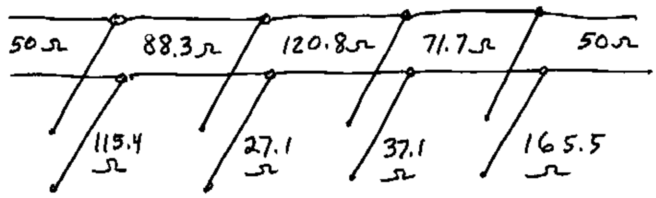


Use the second Kuroda identity on left; first Kuroda identity on right:

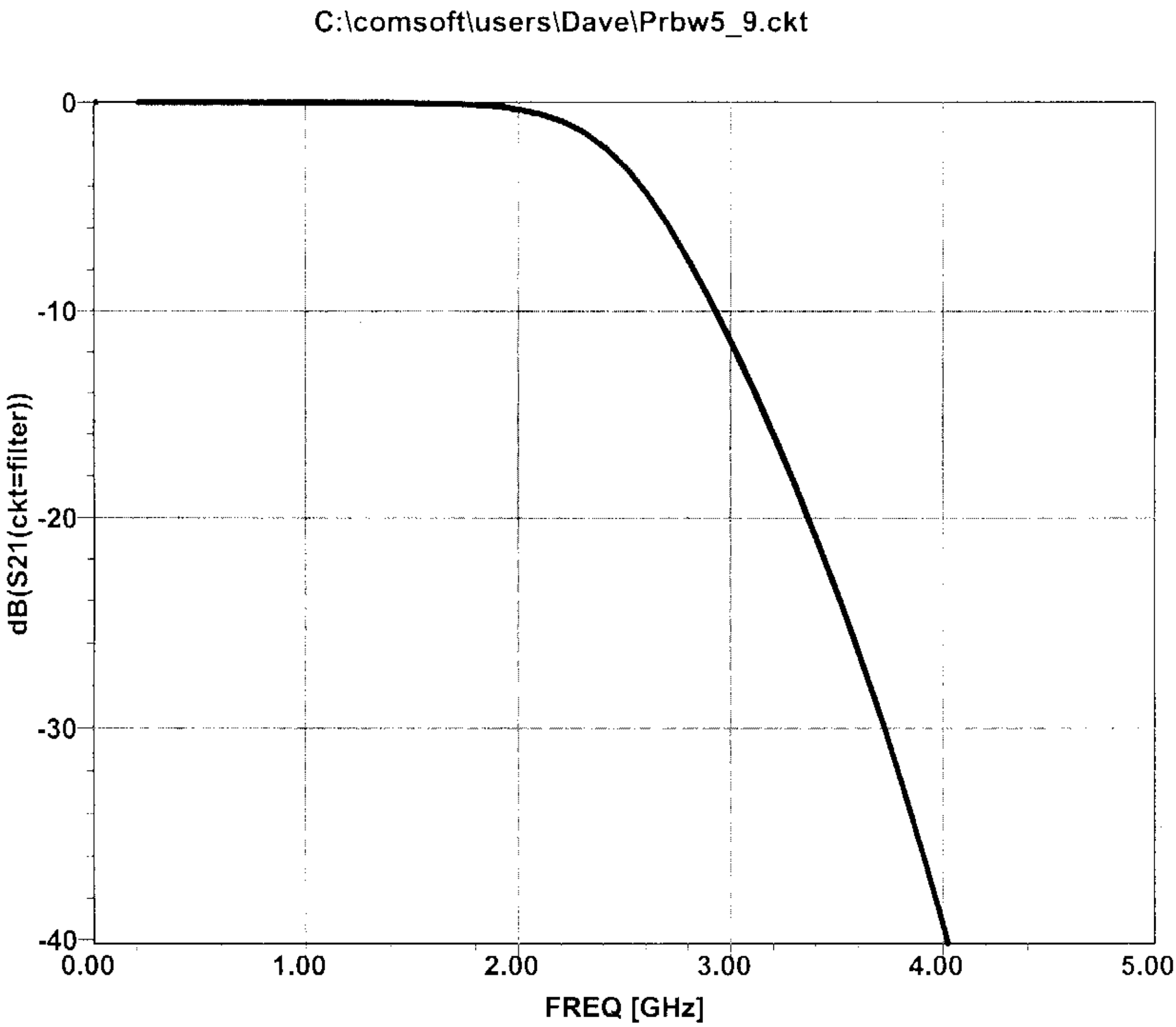


5.9 CONTINUED

Applying the second Kuroda identity twice, and scaling to 50Ω gives the final filter circuit:

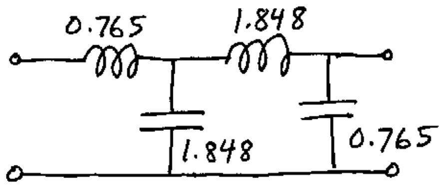


All lines and stubs are λ/8 long at 2.5 GHz. The simulated filter response is shown below:

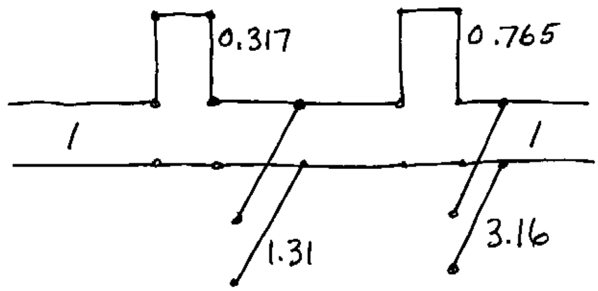


$\boxed{5.10}$ $f_0 = 2.5\,GHz$, BAND-STOP, $\Delta = 0.5$, $N=4$, MAXIMALLY-FLAT, $Z_0 = 50\Omega$, SHUNT STUBS.

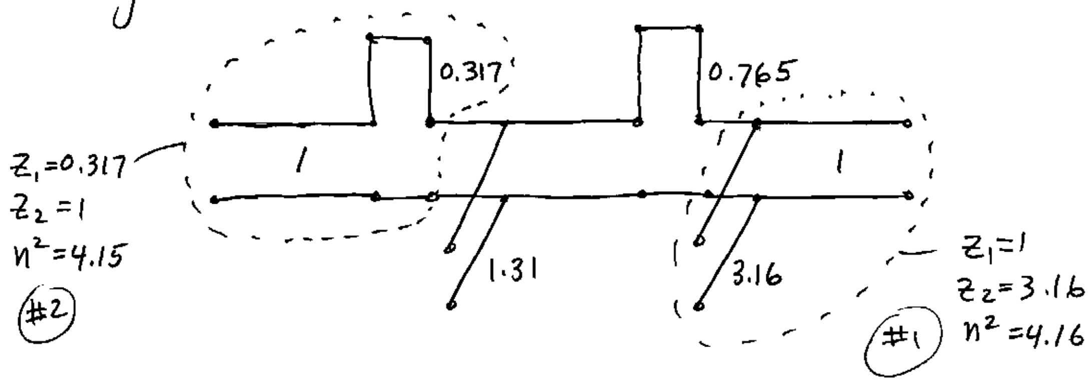
The low-pass prototype, from Table 5.1, is



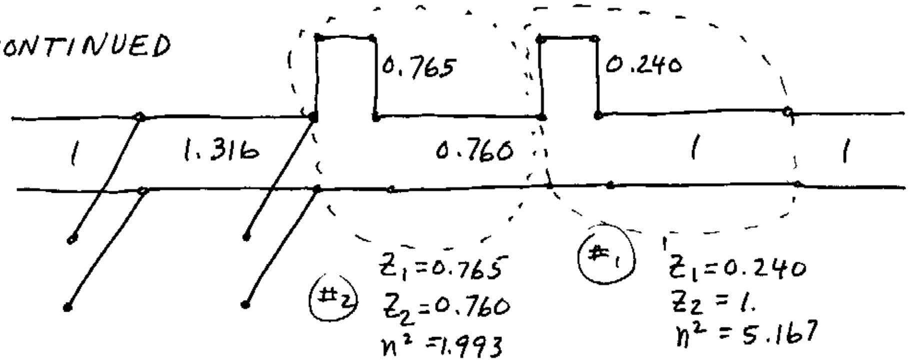
If Richard's transform is used with stub lengths of $\lambda/4$ at f_0, a bandstop response can be achieved, since the s.c. stubs (series L) will look like opens, while the o.c. stubs (shunt C) will look like shorts. Also the values of Z, Y must be scaled by $\gamma = \cot\left[\frac{\pi}{2}\left(1-\frac{\Delta}{2}\right)\right] = 0.414$ so that the stub impedances/admittances will be unity at the band edges. Thus we have,



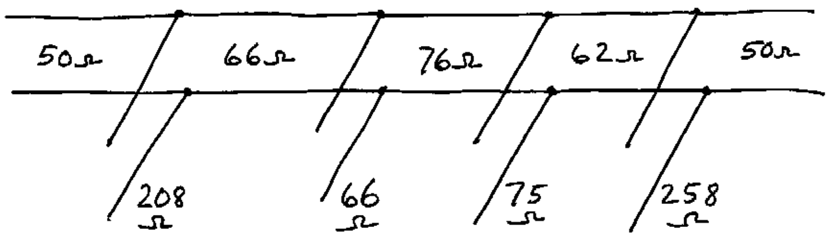
Adding unit elements:



5.10 CONTINUED



Scaling:



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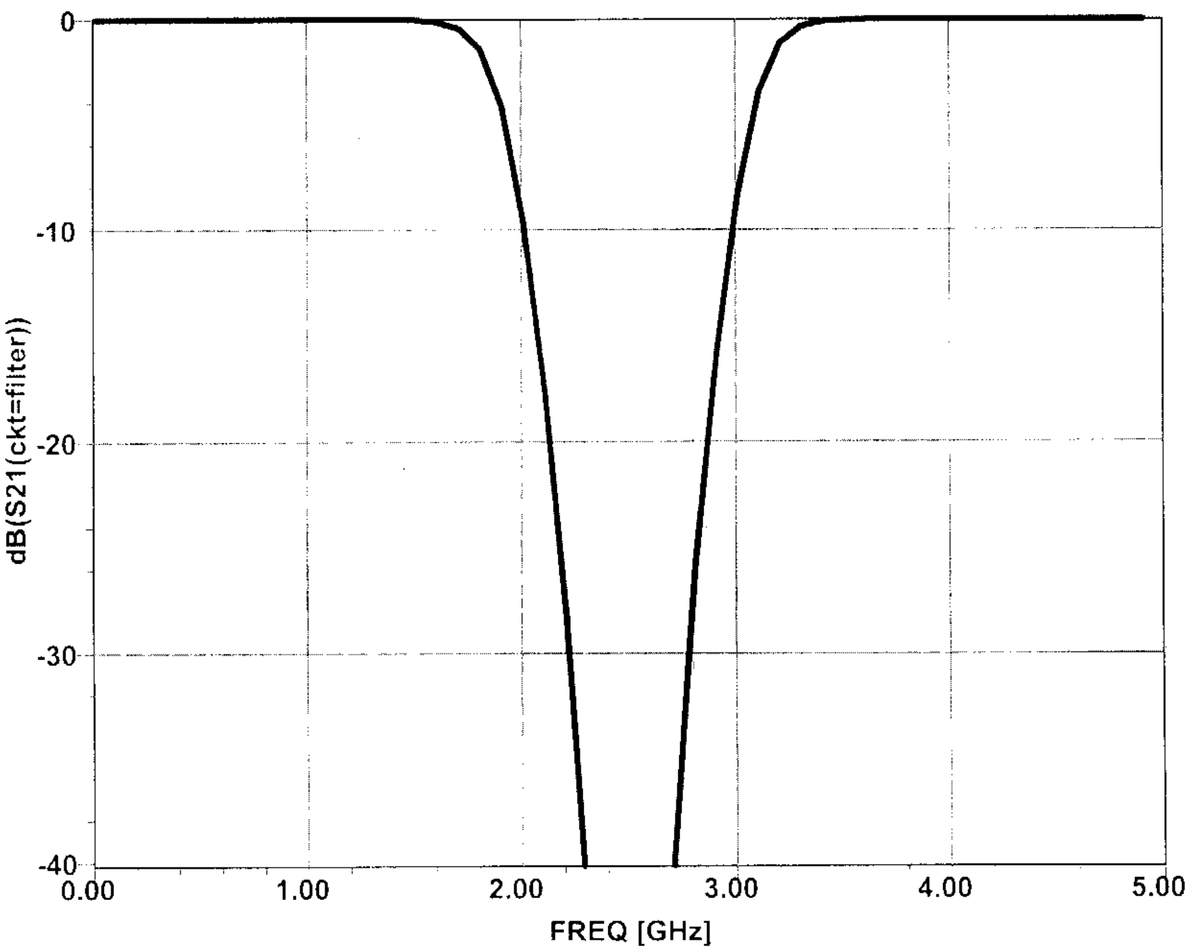
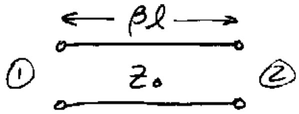


$\boxed{5.11}$



From Table 2.1, the ABCD parameters are

$$A = \cos\beta l \qquad B = jZ_0 \sin\beta l$$
$$C = jY_0 \sin\beta l \qquad D = \cos\beta l$$

Converting to Z-parameters (See Problem 2.19):

$$Z_{11} = Z_{22} = \frac{A}{C} = -jZ_0 \cot\beta l \ ✓$$

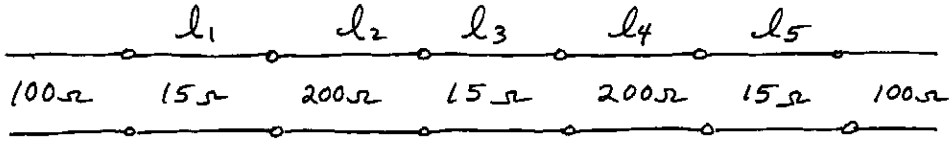
$$Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc\beta l \ ✓$$

$\boxed{5.12}$ $f_0 = 4\,GHz$, $N=5$, LOW-PASS, 0.5 dB EQUAL-RIPPLE, $Z_0 = 100\,\Omega$

From Table 5.2 and (5.41), with $Z_\ell = 15\,\Omega$ and $Z_h = 200\,\Omega$,

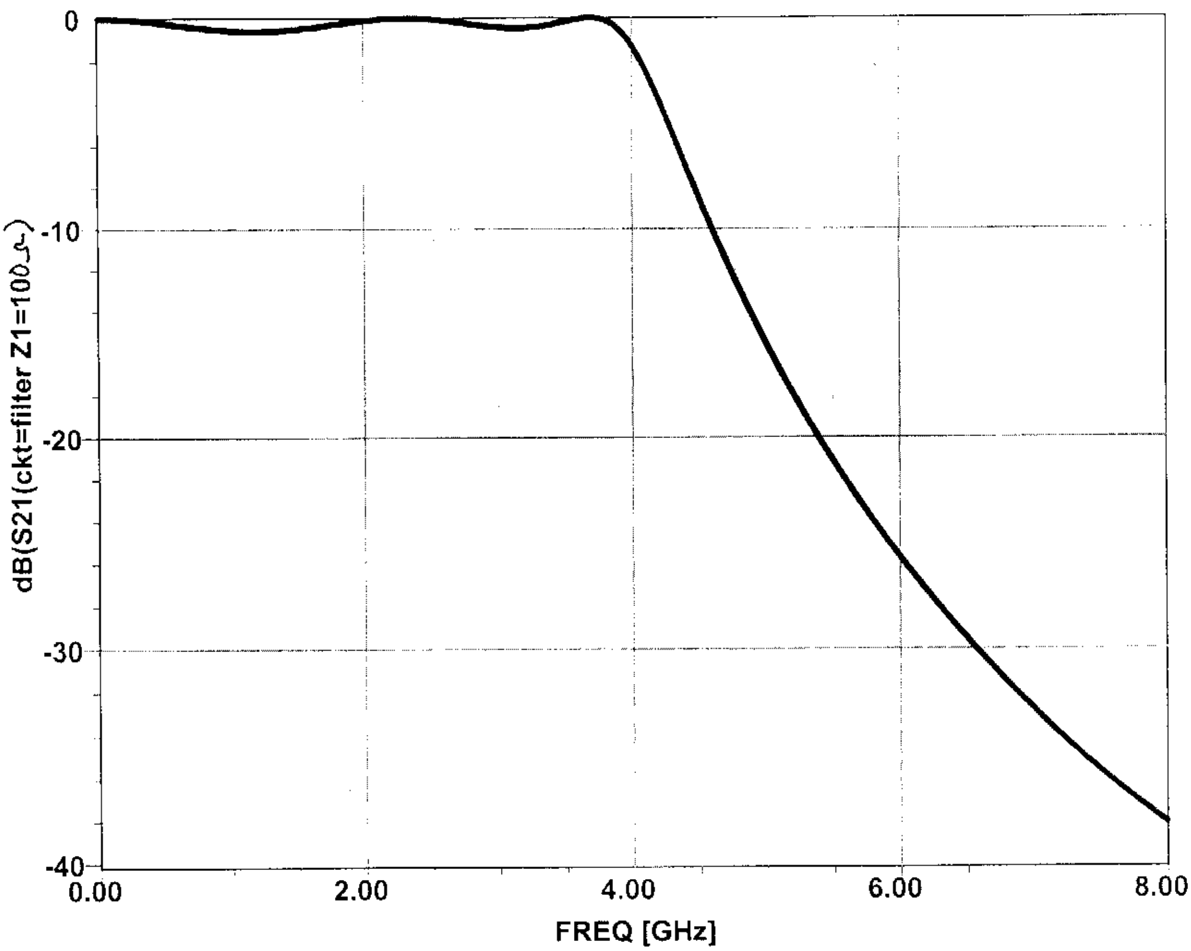
$$g_1 = 1.7058 = C_1 \quad\Rightarrow\quad \beta l_1 = g_1 Z_\ell / Z_0 = 14.7^\circ \ ✓$$
$$g_2 = 1.2296 = L_2 \quad\rightarrow\quad \beta l_2 = g_2 Z_0 / Z_h = 35.2^\circ \ ✓$$
$$g_3 = 2.5408 = C_3 \quad\Rightarrow\quad \beta l_3 = g_3 Z_\ell / Z_0 = 21.8^\circ \ ✓$$
$$g_4 = 1.2296 = L_4 \quad\Rightarrow\quad \beta l_4 = g_4 Z_0 / Z_h = 35.2^\circ \ ✓$$
$$g_5 = 1.7058 = C_5 \quad\Rightarrow\quad \beta l_5 = g_5 Z_\ell / Z_0 = 14.7^\circ \ ✓$$

Note that $\beta l < 45^\circ$ for all cases. Lengths are at 4 GHz.

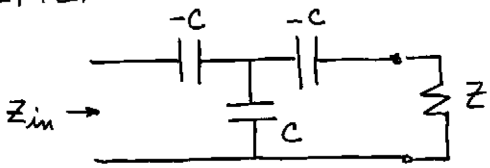


The simulated filter response is shown below.

5.12 CONTINUED



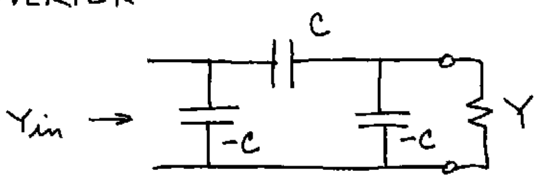
5.13 K INVERTER



$$Z_{in} = \frac{j}{\omega C} + \left[j\omega C + \frac{1}{Z + j/\omega C} \right]^{-1} = \frac{j}{\omega C} + \frac{Z + j/\omega C}{1 + j\omega C Z - 1} = \frac{j}{\omega C} + \frac{-j + 1/\omega C Z}{\omega C}$$

$$= \frac{1}{\omega^2 C^2 Z} = \frac{K^2}{Z} \checkmark$$

J INVERTER



$$Y_{in} = -j\omega C + \left[\frac{1}{j\omega C} + \frac{1}{Y - j\omega C} \right]^{-1} = -j\omega C + \frac{j\omega C Y + \omega^2 C^2}{Y \cancel{-j\omega C} \cancel{+j\omega C}} = \cancel{-j\omega C} + \cancel{j\omega C} + \frac{\omega^2 C^2}{Y}$$

$$= \frac{\omega^2 C^2}{Y} = \frac{J^2}{Y} \checkmark$$

$\boxed{5.14}$ $f_0 = 836.5$ MHz, $\Delta = 0.03$, BANDPASS, 0.5 dB, EQUAL-RIPPLE, $Z_0 = 50\,\Omega$.
$\lambda/4$ COUPLED $\lambda/4$ RESONATORS, $\alpha = 30$ dB at 869 MHz.

Use (5.22) to convert 869 MHz to normalized low-pass form:

$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{0.03}\left(\frac{869}{836.5} - \frac{836.5}{869}\right) = 2.54$$

Then $\left|\frac{\omega}{\omega_c}\right| - 1 = 1.54$, so from Figure 5.6a we see that $N=4$ will provide the required attenuation, but since we require N to be <u>odd</u>, we choose $\underline{\underline{N=5}}$.

Table 5.2 gives the required g_n's, and (5.52) can be used to find the stub impedances:

$$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}$$

n	g_n	Z_{0n} (Ω)
1	1.7058	0.69
2	1.2296	0.96
3	2.5408	0.46
4	1.2296	0.96
5	1.7058	0.69
6	1.000 ✓	

NOTE IMPRACTICALLY LOW Z_0 DUE TO SMALL Δ!

The simulated filter response is shown below. The result looks good, but the design is obviously impractical, because of the very narrow bandwidth. The capacitively coupled design of the following problem is better.

5.14 CONTINUED

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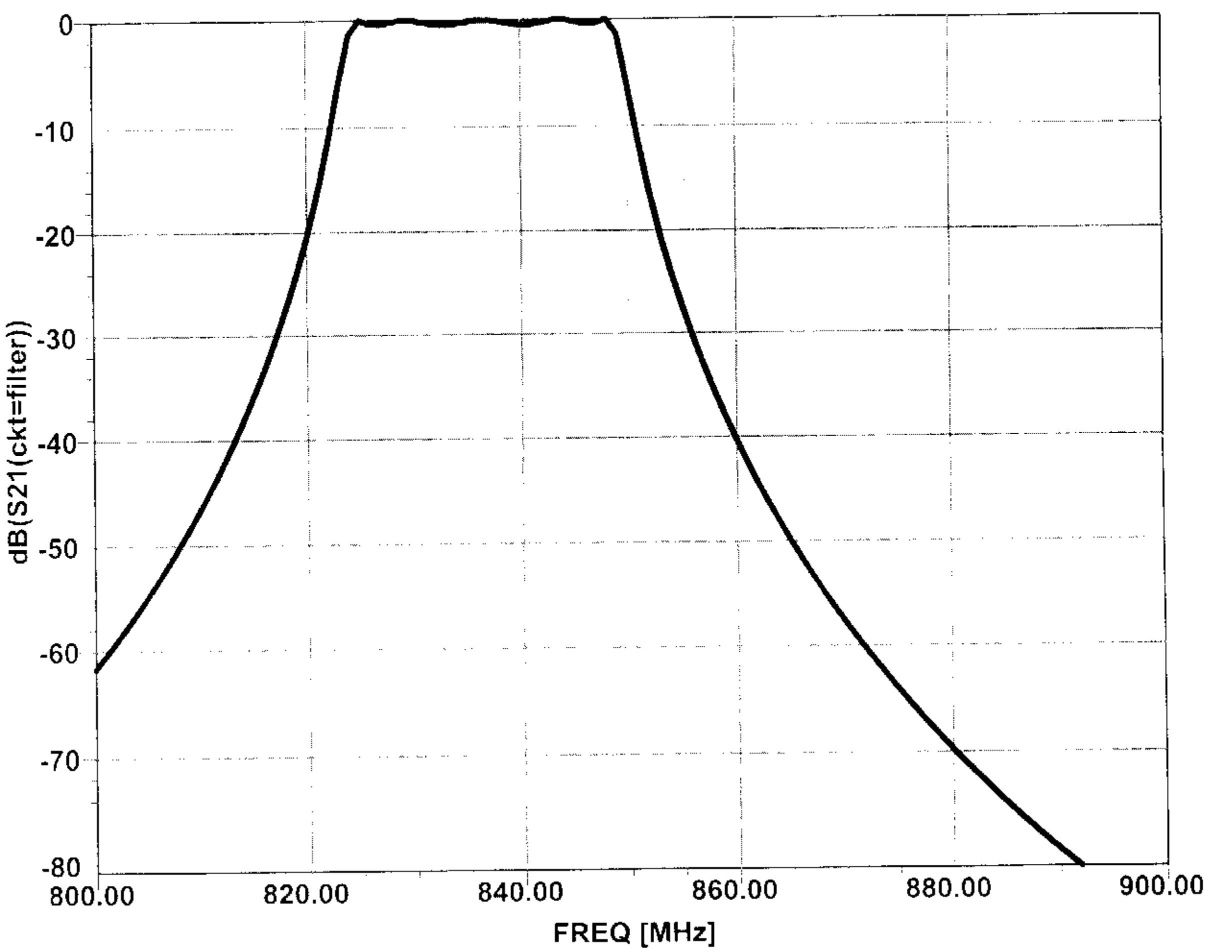


$\boxed{5.15}$ $f_0 = 836.5$ MHz, $\Delta = 0.03$, BANDPASS, 0.5dB EQUAL-RIPPLE, $Z_0 = 50\Omega$. CAPACITIVELY-COUPLED RESONATORS.

In this case we can use $N=4$. Table 5.2 gives the g_n's, and (5.53)-(5.54) can be used to find the admittance inverter constants and coupling capacitor values:

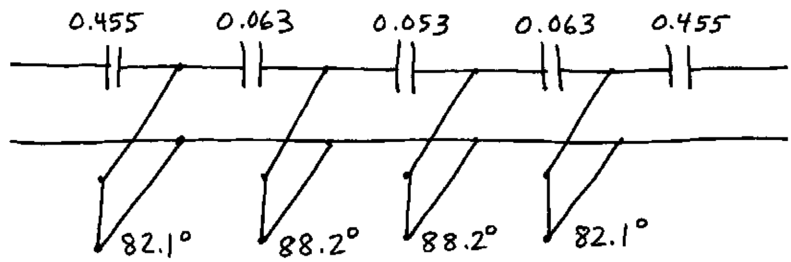
n	g_n	$Z_0 J_{n-1,n}$	$C_{n-1,n}$ (pF)
1	1.6703	0.1188	0.455
2	1.1926	0.0167	0.063
3	2.3661	0.0140	0.053
4	0.8419	0.0167	0.063
5	1.9841	0.1188	0.455

Then use (5.55) and (5.58) to find the resonator lengths:

$C_n' = C_n + \Delta C_n$

n	ΔC_n (pF)	ℓ_n (λ)	ℓ (°)
1	−0.518	0.228	82.1°
2	−0.116	0.245	88.2°
3	−0.116	0.245	88.2°
4	−0.518	0.228	82.1°

Final filter circuit:



The simulated filter response is shown below. This is a much more practical implementation than the previous case.

5.15 CONTINUED

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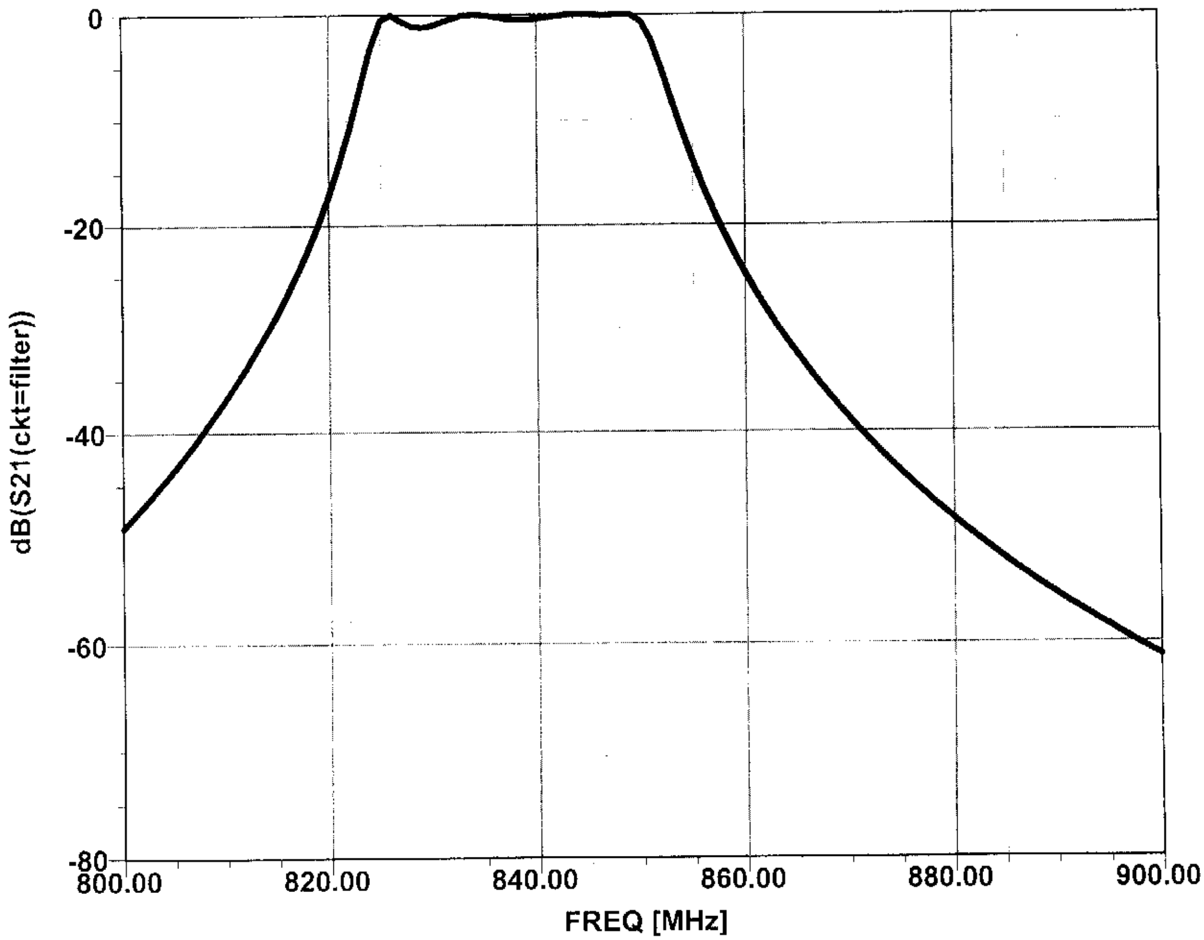
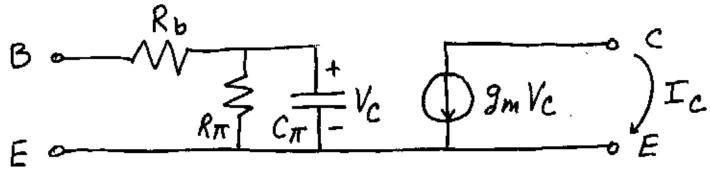


Chapter 6

6.1

UNILATERAL BIPOLAR TRANSISTOR MODEL:



From (6.1) the short-circuit current is,

$$G_i^{SC} = \left|\frac{I_c}{I_b}\right|\Bigg|_{V_{CE}=0} = \frac{g_m V_c}{I_b} = \frac{g_m I_b \left|\frac{R_\pi/j\omega C_\pi}{R_\pi + 1/j\omega C_\pi}\right|}{I_b}$$

$$= g_M \frac{R_\pi}{|1+j\omega R_\pi C_\pi|} = \frac{g_m}{\left|\frac{1}{R_\pi} + j\omega C_\pi\right|} \simeq \frac{g_m}{\omega C_\pi} \quad \text{since } R_\pi \gg \frac{1}{\omega C_\pi}$$

(e.g., $R_\pi \sim 110\Omega$, $C_\pi = 18pF$, $f = 1GHz$ then $\frac{1}{\omega C_\pi} = 9\Omega$)

6.2

$R_i = 7\Omega$, $C_{ds} = 0.12pF$, $R_{ds} = 400\Omega$, $C_{gs} = 0.3pF$, $C_{gd} = 0$, $g_m = 30mS$
$f = 5GHz$.

$$Y_{11} = \frac{j\omega C_{gs}}{1+j\omega R_i C_{gs}} = 0.0094\angle 86^\circ = 0.00062 + j\,0.0094 \checkmark$$

$$Y_{21} = \frac{g_m}{1+j\omega R_i C_{gs}} = 0.03\angle 4^\circ \checkmark, \quad Y_{12} = 0$$

$$Y_{22} = \frac{1}{R_{ds}} + j\omega C_{ds} = 0.0025 + j\,0.00377 = 0.00452\angle 56.5^\circ$$

$$\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21} = 0.000515\angle 34^\circ$$

$$S_{11} = \frac{(Y_0 - Y_{11})(Y_0 + Y_{22})}{\Delta Y} = \frac{Y_0 - Y_{11}}{Y_0 + Y_{11}} = 0.95\angle{-50^\circ} \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{-2Y_{21}Y_0}{\Delta Y} = 2.33\angle 150^\circ$$

$$S_{22} = \frac{Y_0 - Y_{22}}{Y_0 + Y_{22}} = 0.785\angle{-22^\circ} \checkmark$$

6.2 CONTINUED.

If conjugately matched, the unilateral transducer gain is,

$$G_{TU} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2} = 148.8 = 21.7 \text{ dB}$$

Using the circuit model, (6.21) gives

$$G_{TU} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = 21.6 \text{ dB} \checkmark$$

(These results were also verified with Serenade)

$\boxed{6.3}$ The S-matrix is $[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$

For $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = 0$, $\Gamma_{out} = 0$

Using (6.15), (6.16), (6.11):

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$
$$G_T = |S_{21}|^2 = 0.5 \checkmark$$
$$G = |S_{21}|^2 = 0.5$$

For $Z_L = 25\Omega$: $\Gamma_L = -1/3$, $\Gamma_S = \Gamma_{out} = 0$, $\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$.

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$
$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444 \checkmark$$
$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{1-|\Gamma_{in}|^2} = 0.457 \checkmark$$

$\boxed{6.4}$ Using the μ-test of (6.33) gives

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.117\angle{-50^\circ}$$

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-\Delta S_{11}^*| + |S_{12}S_{21}|} = \frac{1-(0.34)^2}{|0.45\angle{-25^\circ} - (0.117\angle{-50^\circ})(0.34\angle{170^\circ})| + (0.06)(4.3)}$$

$= 1.19$ ✓ $\Rightarrow$ The device is unconditionally stable

Using the K-Δ test of (6.31)-(6.32):

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.35 > 1 \quad ✓$$

Since $K>1$ and $|\Delta|<1$, the device is unconditionally stable.

$\boxed{6.5}$ Using the K-Δ test of (6.31)-(6.32):

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 1.52\angle{-49^\circ}$$

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.75 \quad ✓$$

Since $K<1$ the device is potentially unstable. The stability circle parameters are,

$$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 0.66\angle{-70^\circ} \quad ✓$$

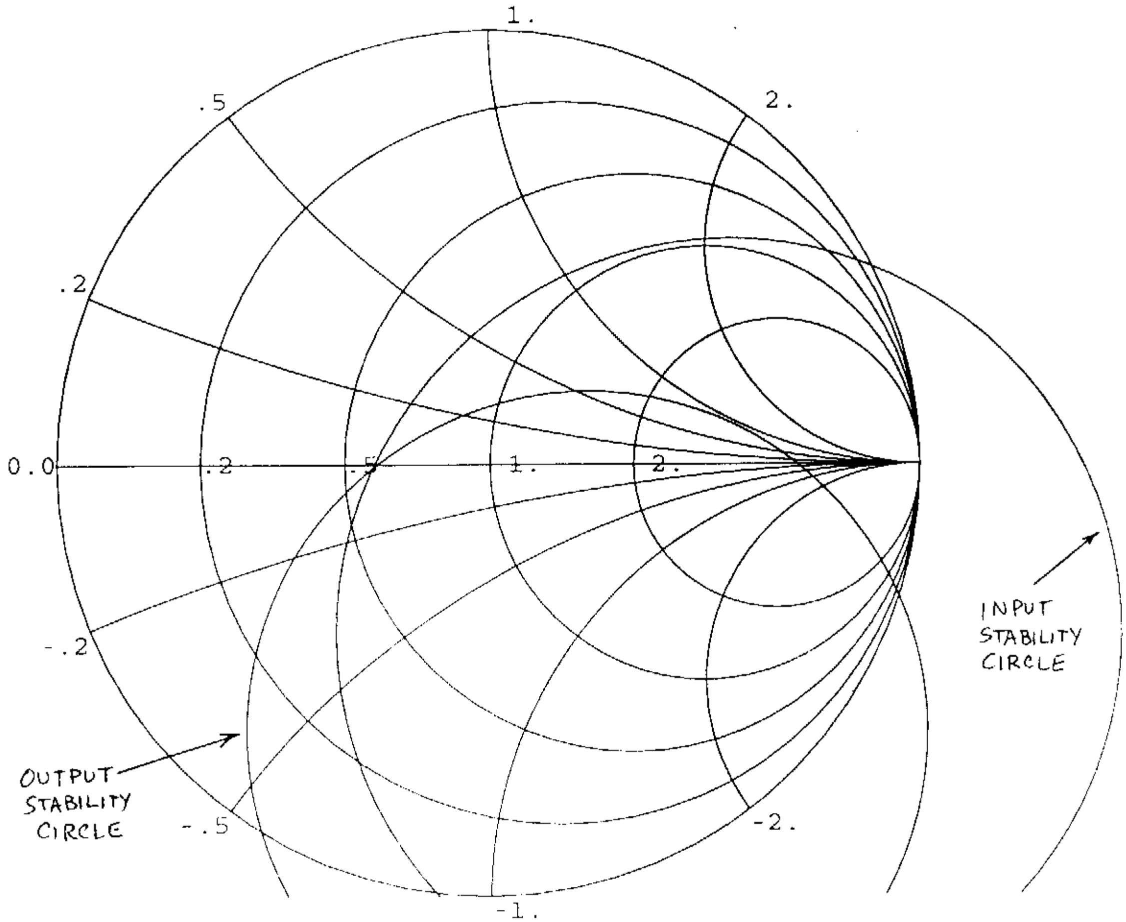
$$R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| = 0.79 \quad ✓$$

$$C_S = \frac{(S_{11}-\Delta S_{22}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 0.68\angle{-35^\circ}$$

$$R_S = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| = 0.91 \quad ✓$$

A plot of the stability circles is shown below:

6.5 CONTINUED.



6.6 Using (6.33) to compute μ:

DEVICE	μ	STABILITY
A	1.193	UNCOND. STABLE
B	0.283	POTEN. UNSTABLE
C	1.057	UNCOND. STABLE

Device A is the most stable.

$\boxed{6.7}$ From (6.33) the μ-parameter test is,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{12}S_{21}|} > 1 \quad \text{for unconditional stability}$$

If $S_{12}=0$ (unilateral) then we have $\Delta = S_{11}S_{22}$, so

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2 S_{22}|} = \frac{1-|S_{11}|^2}{|S_{22}||1-|S_{11}|^2|} > 1$$

Since the denominator is positive, and μ is positive, the numerator must also be positive, thus $|S_{11}|<1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1, \text{ so } |S_{22}|<1. \quad \text{(for uncond. stab.)}$$

$\boxed{6.8}$ From the definitions of (6.44),

$$B_1 = 1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2 \quad , \quad C_1 = S_{11} - \Delta S_{22}^* .$$

Similar to the expansion used after (6.38), it can be verified by direct expansion that,

$$|C_1|^2 = |S_{11}-\Delta S_{22}^*|^2 = |S_{12}S_{21}|^2 + (1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

So the condition that $B_1^2 - 4|C_1|^2 > 0$ implies that,

$$(1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2)^2 > 4|S_{12}S_{21}|^2 + 4(1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

$$1+2|S_{11}|^2-2|S_{22}|^2-2|\Delta|^2+|S_{11}|^4-2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$+2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2+4(|S_{11}|^2-|\Delta|^2-|S_{11}|^2|S_{22}|^2+|\Delta|^2|S_{22}|^2)$$

$$1-2|S_{11}|^2-2|S_{22}|^2+2|\Delta|^2+|S_{11}|^4+2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$-2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2$$

$$(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2 > 4|S_{12}S_{21}|^2$$

or,

$$K^2 = \frac{(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2}{4|S_{12}S_{21}|^2} > 1 \quad \checkmark$$

$\boxed{6.9}$ $S_{11} = 0.65\angle -140°$, $S_{21} = 2.4\angle 50°$, $S_{12} = 0.04\angle 60°$, $S_{22} = 0.70\angle -65°$

First check stability:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.393\angle 165°$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.26$$

Since $|\Delta| < 1$ and $K > 1$ the transistor is unconditionally stable at 5 GHz. For maximum gain, the transistor should be conjugately matched:

$\Gamma_S = \Gamma_{in}^* = 0.826\angle 147°$ ✓

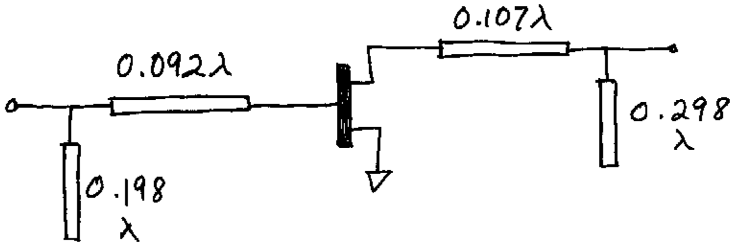
$\Gamma_L = \Gamma_{out}^* = 0.850\angle 71°$ ✓

The gain can then be calculated as

$G_S = 3.14$, $G_0 = 5.76$, $G_L = 1.64$

So the overall transducer gain is $G_T = 29.7 = 14.7$ dB ✓

Matching was done with a Smith chart. The final AC amplifier circuit is shown below:



Analysis by Serenade gives $G = 14.7$ dB ✓

$\boxed{6.10}$ $S_{11} = 0.61\angle{-170°}$, $S_{12} = 0$, $S_{21} = 2.24\angle{32°}$, $S_{22} = 0.72\angle{-83°}$, $f_0 = 1.8\,GHz$.

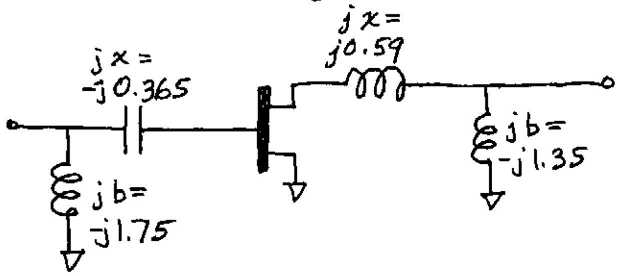
The transistor is unconditionally stable since $K = \infty$ and $|\Delta| < 1$. Since the transistor is unilateral ($S_{12} = 0$), we have,

$$\Gamma_S = S_{11}^* = 0.61\angle{170°} \checkmark$$
$$\Gamma_L = S_{22}^* = 0.72\angle{83°} \checkmark$$

and the maximum gain, from (6.45), is

$$G_{TU_{MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 16.6 = 12.2\,dB$$

Lumped element matching was done with a Smith chart:



at 1.8 GHz the matching element values are,

$$C = \frac{-1}{\omega Z_0 x_C} = 4.8\,pF \qquad L = \frac{x_L Z_0}{\omega} = 2.6\,nH$$

$$L = \frac{-Z_0}{\omega b_L} = 2.5\,nH \qquad L = \frac{-Z_0}{\omega b_L} = 3.3\,nH$$

analysis with Serenade gives $G = 12.2\,dB$ ✓, and $RL < 26\,dB$ at both ports.

$\boxed{6.11}$ $S_{11} = 0.61\angle{-170^\circ}$, $S_{12} = 0$, $S_{21} = 2.24\angle{32^\circ}$, $S_{22} = 0.72\angle{-83^\circ}$, $f = 2.4\,GHz$

$G = 10\,dB$, $G_S = 1\,dB$, $G_L = 2\,dB$.

Since $K = \infty$ and $|\Delta| < 1$, the transistor is unconditionally stable. From (6.50),

$$G_{S_{MAX}} = \frac{1}{1-|S_{11}|^2} = 1.59 \checkmark \qquad G_{L_{MAX}} = \frac{1}{1-|S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1\,dB = 1.26$ and $G_L = 2\,dB = 1.58$, we have

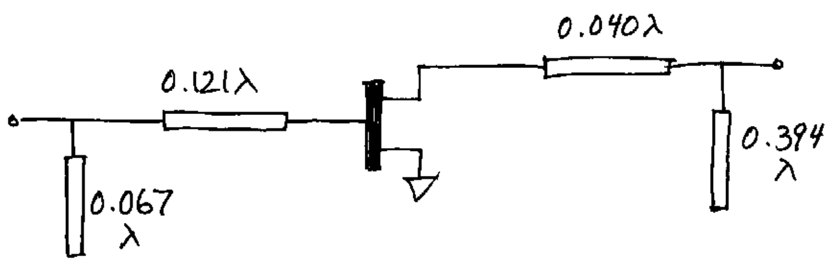
$$g_S = \frac{G_S}{G_{S_{MAX}}} = 0.792 \quad , \quad g_L = \frac{G_L}{G_{L_{MAX}}} = 0.760$$

Then the center and radii of the constant gain circles can be found from (6.54)-(6.55):

$$C_S = 0.524\angle{170^\circ} \checkmark \qquad C_L = 0.625\angle{83^\circ} \checkmark$$

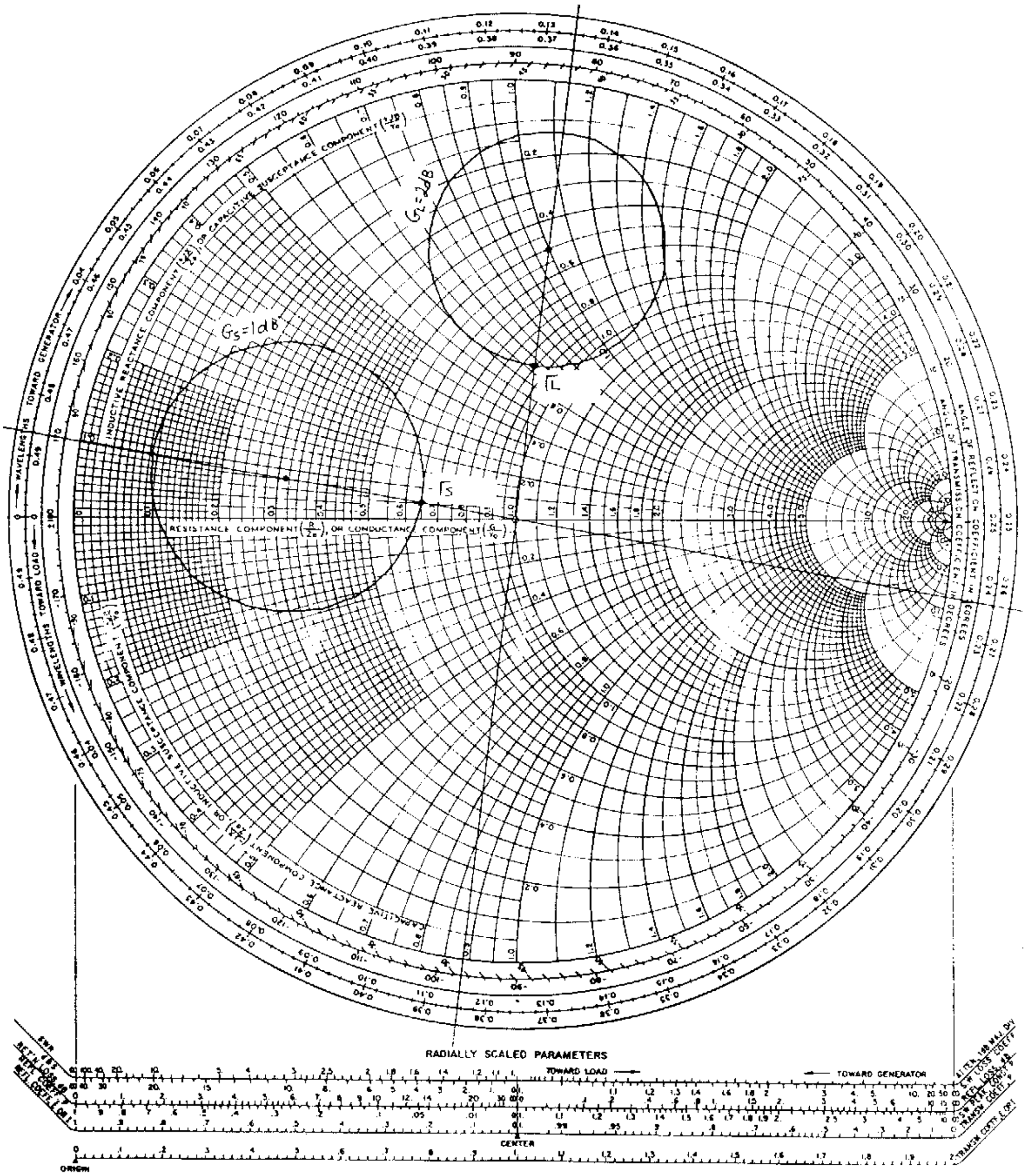
$$R_S = 0.310 \checkmark \qquad R_L = 0.269 \checkmark$$

Since $G_o = 10 \log |S_{21}|^2 = 7.0\,dB$, using the $G_S = 1\,dB$ and the $G_L = 2\,dB$ gain circles will give an overall gain of 10dB. We plot these circles on a Smith chart, and choose $\Gamma_S = 0.215\angle{170^\circ}$ ✓ and $\Gamma_L = 0.361\angle{83^\circ}$ ✓ to minimize the magnitude of these values. after matching, we have the following AC circuit:



6.11 CONTINUED

Serenade analysis gives $G = 10.1\,dB$ ✓ The return losses are $RL_1 = -7\,dB$, $RL_2 = -6\,dB$. These mismatches serve to reduce the gain to 10 dB. The Smith chart plot of the relevant constant gain circles ~~is~~ shown below:



$\boxed{6.12}$ $S_{11}=0.34\angle -170°$, $S_{12}=0.06\angle 70°$, $S_{21}=4.3\angle 80°$, $S_{22}=0.45\angle -25°$

From (6.49),

$$U=\frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}=\frac{(0.06)(4.3)(0.34)(0.45)}{[1-(0.34)^2][1-(0.45)^2]}=0.056 \checkmark$$

So from (6.48) the bounds on the error in G_T/G_{TU} are

$$0.897=\frac{1}{(1+U)^2}<\frac{G_T}{G_{TU}}<\frac{1}{(1-U)^2}=1.122$$

or,

$$-0.47\,dB<G_T(dB)-G_{TU}(dB)<0.5\,dB$$

$\boxed{6.13}$ From (6.50a) and (6.51a), with $G_S=1$, we have

$$g_s=\frac{1}{G_{S_{MAX}}}=1-|S_{11}|^2 \quad , \quad 1-g_s=|S_{11}|^2$$

So (6.54) reduces to

$$C_S=\frac{(1-|S_{11}|^2)S_{11}^*}{1-|S_{11}|^4}=\frac{S_{11}^*}{1+|S_{11}|^2}$$

$$R_S=\frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4}=\frac{|S_{11}|}{1+|S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left|\Gamma_S-\frac{S_{11}^*}{1+|S_{11}|^2}\right|=\frac{|S_{11}|}{1+|S_{11}|^2}$$

Since $\Gamma_S=0$ is a solution of this equation, the circle must pass through the center of the Smith chart.

$\boxed{6.14}$ $S_{11}=0.7\angle -110°$, $S_{12}=0.02\angle 60°$, $S_{21}=3.5\angle 160°$, $S_{22}=0.8\angle -70°$

$F_{MIN}=2.5dB$, $\Gamma_{OPT}=0.7\angle 120°$, $R_N=15\Omega$.

First check stability: $K=1.07$, $|\Delta|=0.53$.
Since $K>1$ and $|\Delta|<1$, the device is unconditionally stable.

Minimum noise figure occurs for $\Gamma_S=\Gamma_{OPT}=0.7\angle 120°$.
Then we maximize gain by conjugate matching the output:

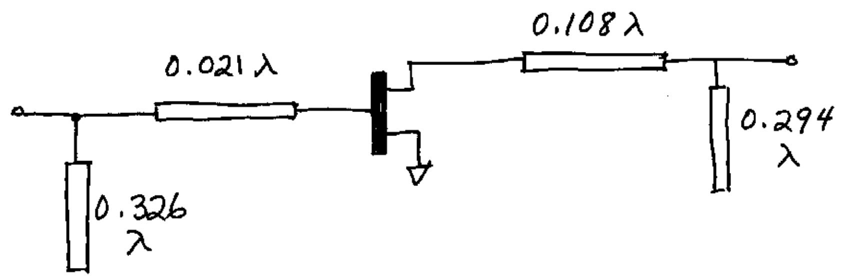
$$\Gamma_L=\left(S_{22}+\frac{S_{12}S_{21}\Gamma_S^*}{1-S_{11}\Gamma_S}\right)^*=0.873\angle 74° \checkmark$$

So the noise figure will be $F=F_{MIN}=2.5dB$, and the gain will be,

$$G_T=\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|S_{22}|^2}{|1-S_{22}\Gamma_L|^2}$$

$$=(1.85)(12.25)(3.81)=86.3=19.4dB$$

The final AC amplifier circuit is,



Analysis using Serenade gives $RL_1=10dB$, $RL_2=18dB$, $F=2.5dB \checkmark$, $G=19.7dB \checkmark$

$\boxed{6.15}$ $S_{11} = 0.6\angle{-60^\circ}$, $S_{12} = 0$, $S_{21} = 2.0\angle{81^\circ}$, $S_{22} = 0.7\angle{-60^\circ}$

$F_{MIN} = 2.0\,dB$, $\Gamma_{OPT} = 0.62\angle{100^\circ}$, $R_N = 20\,\Omega$.

Since $S_{12} = 0$ and $|S_{11}||S_{22}| < 1$, the device is unconditionally stable. The overall gain is $G_{TU} = G_S G_0 G_L$, where $G_0 = |S_{21}|^2 = 4 = 6\,dB$ ✓. So $G_S + G_L = 0\,dB$.

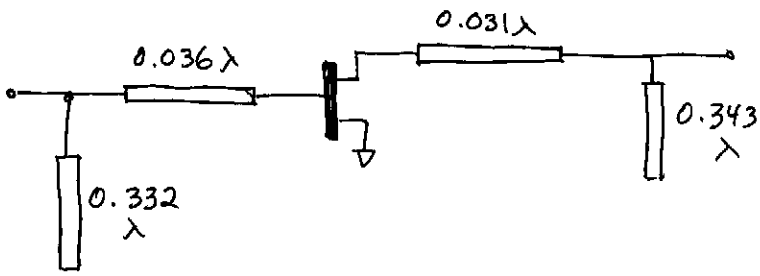
Plot noise figure circles for $F = 2.0, 2.05, 2.2,$ and $3.0\,dB$:

F(dB)	N	C_F	R_F
2.05	0.0134	$0.6\angle{100^\circ}$	0.09
2.20	0.055	$0.59\angle{100^\circ}$	0.18
3.00	0.30	$0.48\angle{100^\circ}$	0.40
2.00	0.	$0.62\angle{100^\circ}$	0

Now plot constant gain circles for $G_S = G_L = 0\,dB$:

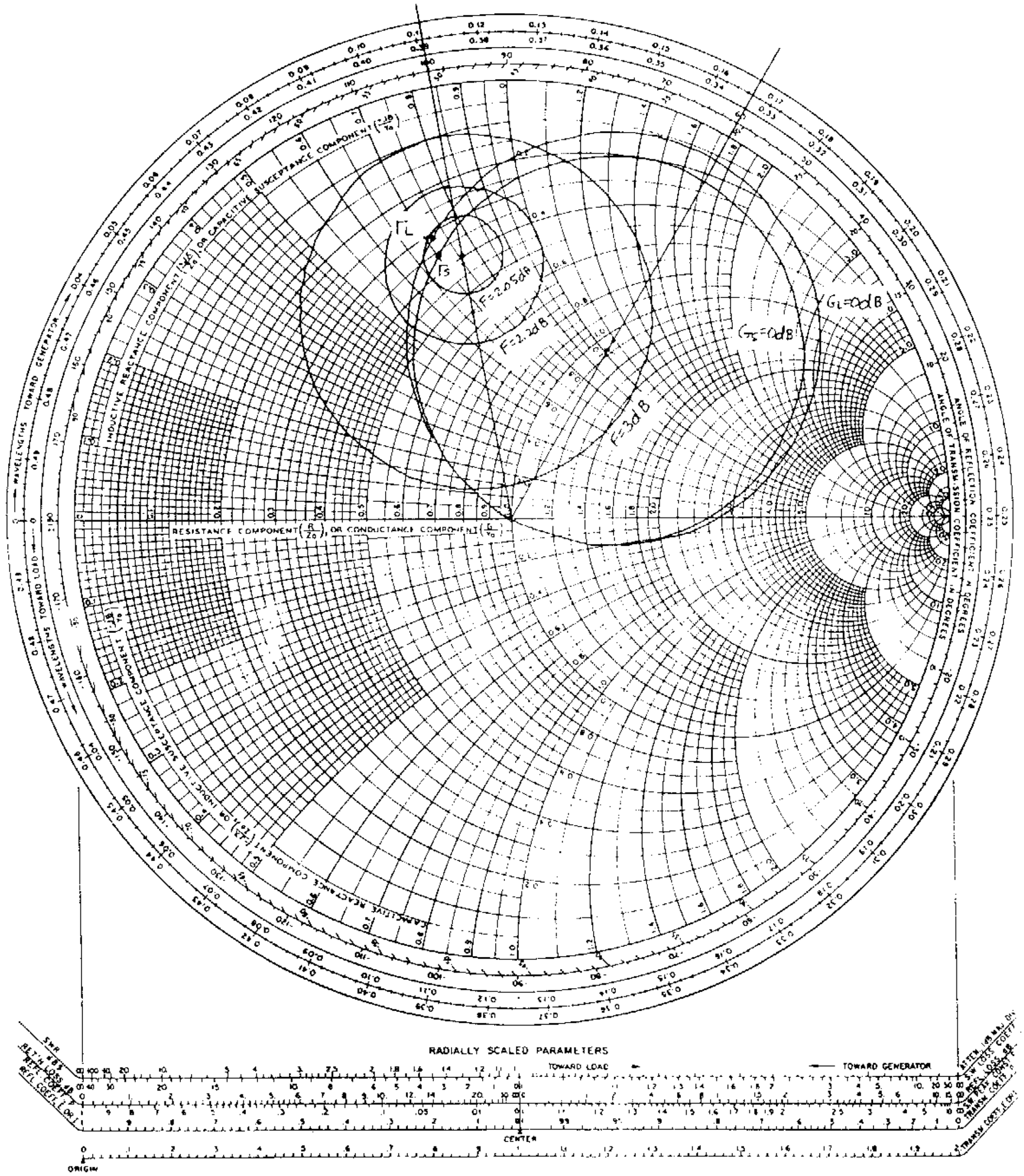
$G_{S_{MAX}} = 1.56$	$G_{L_{MAX}} = 1.96$
$g_S = 0.641$	$g_L = 0.510$
$C_S = 0.44\angle{60^\circ}$	$C_L = 0.47\angle{60^\circ}$
$R_S = 0.44$	$R_L = 0.47$

These two circles are close together near the $F = 2\,dB$ point. See attached Smith chart plot. We choose $\Gamma_L = 0.66\angle{105^\circ}$, $\Gamma_S = 0.62\angle{105^\circ}$. Then we should obtain $F \simeq 2.04\,dB$.



6.15 CONTINUED.

Analysis by Serenade gives $RL_1 = -4\,dB$, $RL_2 = -3.5\,dB$, $G = 6.1\,dB$, $F = 2.04\,dB$ ✓.



6.16 S-parameters and noise parameters are the same as for Problem 6.15.

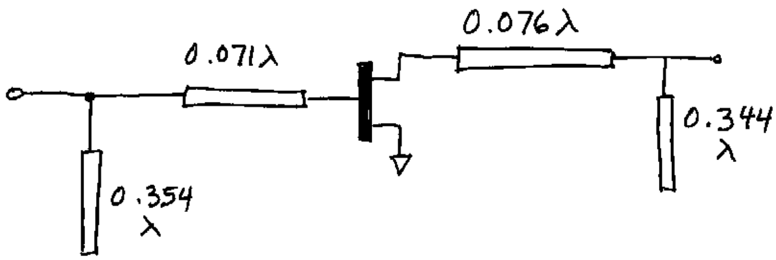
Plot the $F = 2.5\,dB$ constant noise figure circle:

$N = 0.141$, $C_F = 0.543\angle 100°$, $R_F = 0.286$

Now, $G_{S_{MAX}} = 1.56 = 1.93\,dB$, $G_{L_{MAX}} = 1.96 = 2.92\,dB$. But these points ($S_{11}^*$, S_{22}^*) do not lie on the $F = 2.5\,dB$ circle. We can plot some gain circles to just give intersections with the $F = 2.5\,dB$ noise circle:

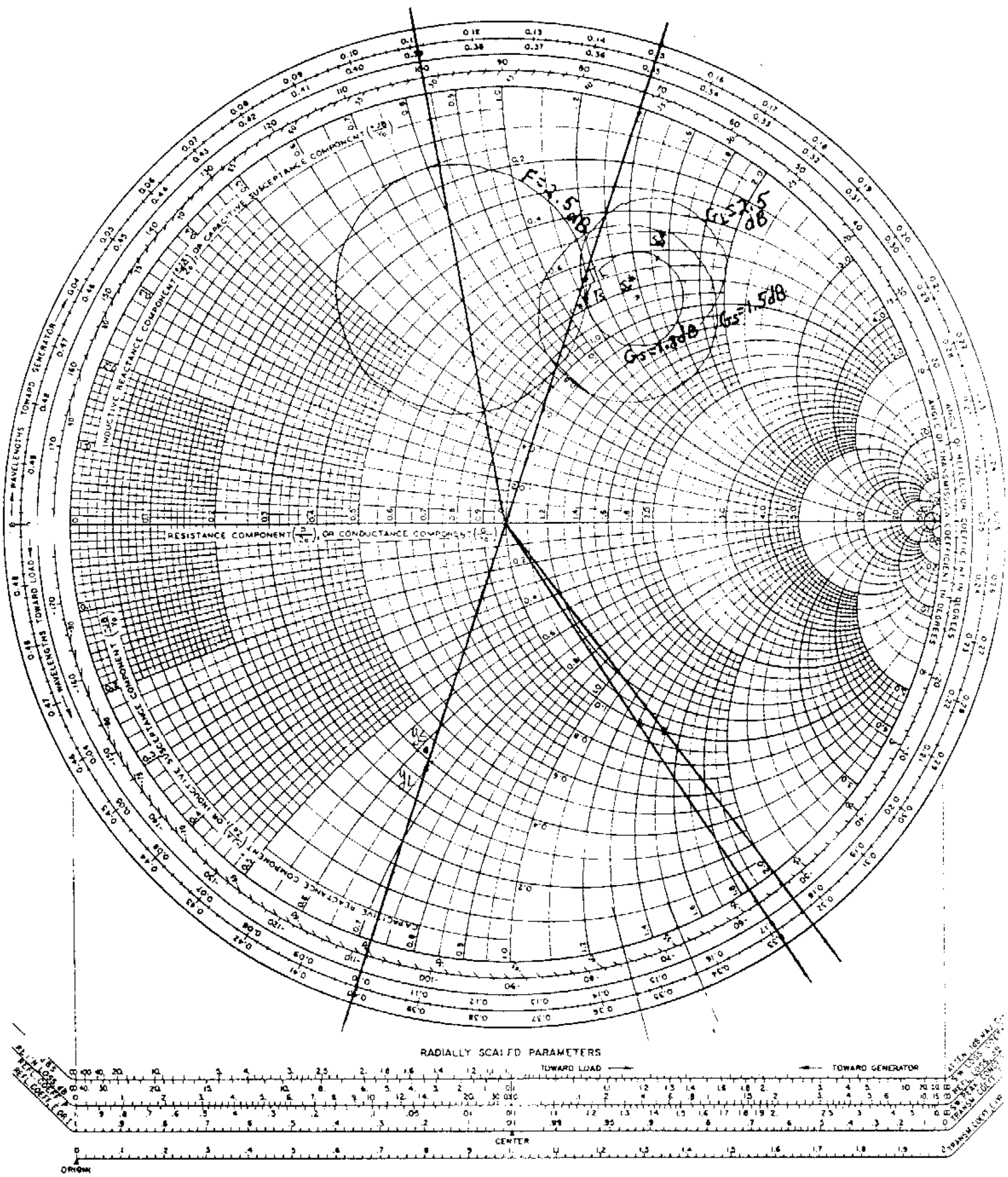
$G_S = 1.5\,dB$	$g_S = 0.905$	$C_S = 0.56\angle 60°$	$R_S = 0.204$
$G_L = 2.5\,dB$	$g_L = 0.907$	$C_L = 0.67\angle 60°$	$R_L = 0.163$
$G_S = 1.7\,dB$	$g_S = 0.948$	$C_S = 0.58\angle 60°$	$R_S = 0.149$
$G_S = 1.8\,dB$	$g_S = 0.970$	$C_S = 0.59\angle 60°$	$R_S = 0.112$

The $G_S = 1.8\,dB$ and $G_L = 2.5\,dB$ circles are close to optimum (the $F = 2.5\,dB$ noise circle). Thus we have $\Gamma_S = 0.545\angle 70°$, $\Gamma_L = 0.59\angle 72°$, which will yield a gain of $G_T = 1.8 + 2.5 + 6 = 10.3\,dB$. The final AC circuit is shown below:



analysis by Serenade gives $RL_1 = 14\,dB$, $RL_2 = 11\,dB$, $G = 10.4\,dB$, $F = 2.4\,dB$ ✓. Smith chart shown below.

6.16 CONTINUED.



$\boxed{6.17}$ $S_{11} = 0.76\angle 169°$, $S_{12} = 3.08\angle 69°$, $S_{21} = 0.079\angle 53°$, $S_{22} = 0.36\angle -169°$

$\Gamma_{Sp} = 0.797\angle -187°$, $\Gamma_{Lp} = 0.491\angle 185°$, $G_p = 10\,dB$. $f = 1\,GHz$.

Check stability using small-signal S-parameters:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.452\angle -27°$$

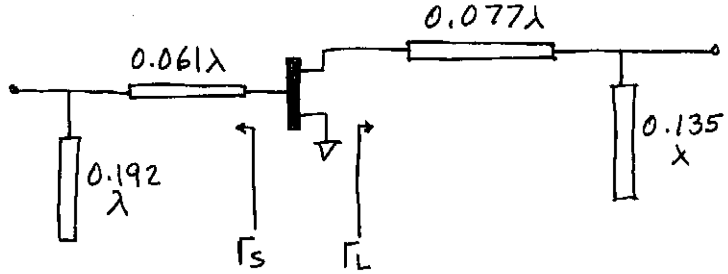
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.02$$

Since $|\Delta| < 1$ and $K > 1$, the device is unconditionally stable at this frequency.

Using the given large-signal source and load reflection coefficients gives,

$$\Gamma_S = 0.797\angle -187°, \quad \Gamma_{Lp} = 0.491\angle 185°$$

Then the matching circuits can be designed, resulting in the following AC circuit:



Since the gain with this Γ_S, Γ_p is 10 dB, the input power for a 1 W output is,

$$P_{in} = P_{out} - G_p = 30\,dBm - 10 = 20\,dBm = 100\,mW.$$

Chapter 7

$\boxed{7.1}$ $v_{RF}(t) = V_{RF}[\cos(\omega_{LO}-\omega_{IF})t + \cos(\omega_{LO}+\omega_{IF})t]$

$v_{LO}(t) = V_{LO}\cos\omega_{LO}t$

after mixing and LPF:

$$v_{OUT}(t) = \frac{KV_{RF}V_{LO}}{2}[\cos\omega_{IF}t + \cos\omega_{IF}t] = V_{RF}V_{LO}K\cos\omega_{IF}t$$

(both sidebands convert to same IF)

$\boxed{7.2}$ $f_{RF} = 600\ MHz$

$f_{IF} = 80\ MHz.$

Two possible LO frequencies are, from (7.4),

$$f_{LO} = f_{RF} \pm f_{IF} = 680\ MHz,\ 520\ MHz \checkmark$$

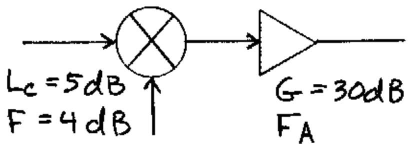
The image frequency for $f_{LO} = 680 MHz$ is,

$$f_{IM} = f_{LO} + f_{IF} = 680 + 80 = 760\ MHz \checkmark$$

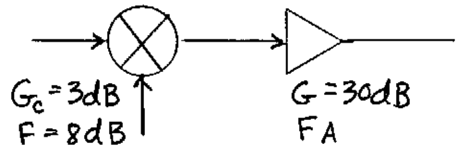
The image frequency for $f_{LO} = 520\ MHz$ is,

$$f_{IM} = f_{LO} - f_{IF} = 520 - 80 = 440\ MHz.$$

7.3



CASE #1



CASE #2

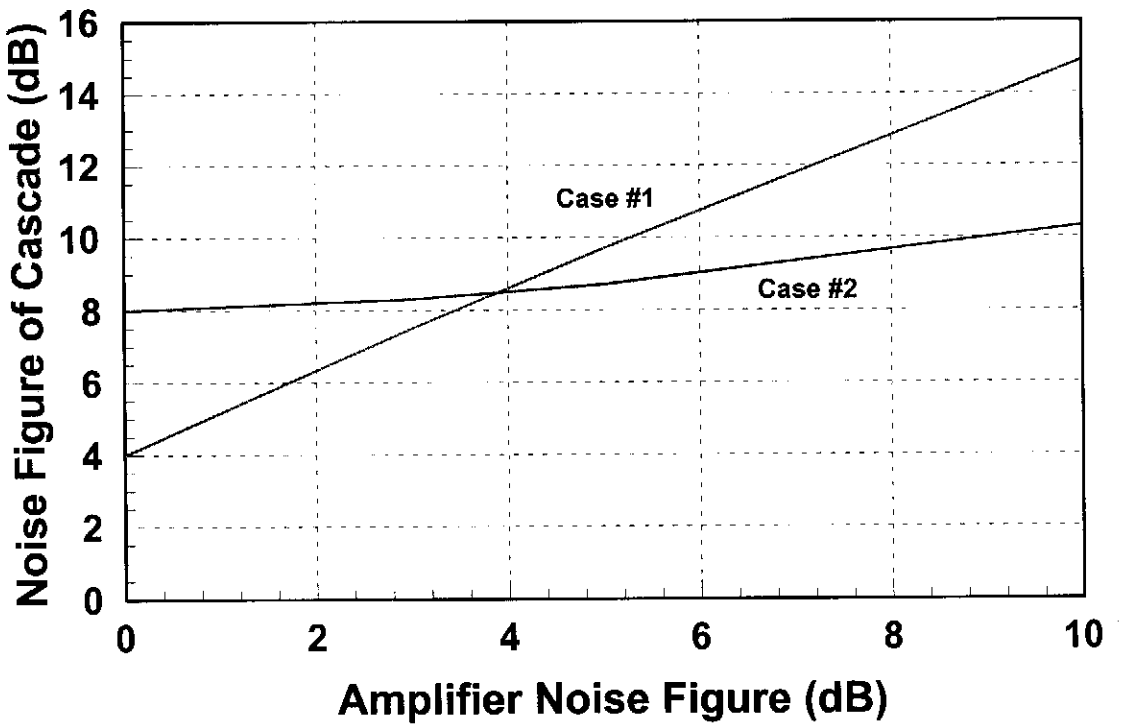
$$F_c = 2.51 + \frac{F_A - 1}{1/3.16}$$

$$F_c = 6.31 + \frac{F_A - 1}{2.0}$$

F_A (dB)	F_c (#1) dB	F_c (#2) dB
0	4.0	8.0
3	7.5	8.3
5	9.7	8.7
10	14.9	10.3

3 dB = 2.0
4 dB = 2.51
5 dB = 3.16
8 dB = 6.31

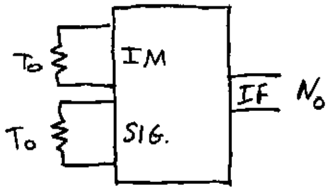
RESULTS ARE PLOTTED BELOW:



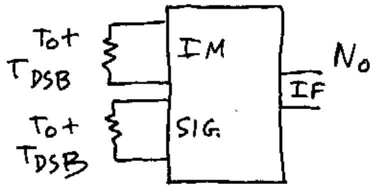
$\boxed{7.4}$

DSB

LOSSY



LOSSLESS



$$N_o = N_{ADDED} + \frac{kT_oB}{L} + \frac{kT_oB}{L} \qquad N_o = \frac{2kB}{L}(T_o + T_{DSB})$$

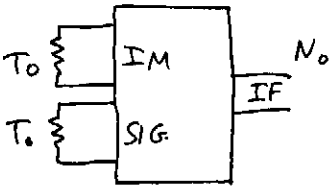
(B in SSB)

$$\therefore \quad N_{ADDED} = \frac{2kB}{L} T_{DSB}$$

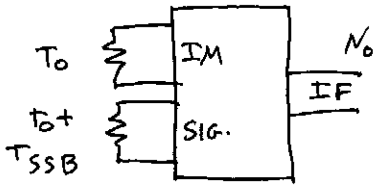
$$F_{DSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{2kT_o B} = 1 + \frac{T_{DSB}}{T_o} \qquad (\text{INPUT NOISE} - N_i = 2kT_oB)$$

SSB

LOSSY



LOSSLESS



$$N_o = N_{ADDED} + \frac{2kT_oB}{L} \qquad N_o = \frac{2kBT_o}{L} + \frac{kBT_{SSB}}{L}$$

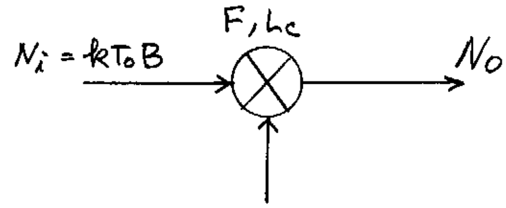
$$\therefore \quad T_{SSB} = \frac{L\, N_{ADDED}}{kB}$$

$$\underline{T_{SSB} = 2\,T_{DSB}} \quad \checkmark$$

$$F_{SSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{kT_oB} = 2 + \frac{T_{SSB}}{T_o} = 2\left(1 + \frac{T_{DSB}}{T_o}\right) = 2\,F_{DSB} \quad \checkmark$$

$$(\text{INPUT NOISE} - \ N_i = kT_oB)$$

$\boxed{7.5}$



$$N_o = \underbrace{\frac{kT_0B}{L_c}}_{\text{INPUT NOISE}} + \underbrace{\frac{kT_0B(F-1)}{L_c}}_{\text{MIXER NOISE}} = \frac{kT_0BF}{L_c} \checkmark$$

$\boxed{7.6}$ $\quad i(t) = I_S\left[e^{30v(t)} - 1\right]$, $\quad v(t) = 0.01\cos\omega_1 t + 0.01\cos\omega_2 t$

$$i(t) = i\Big|_{v=0} + \frac{di}{dv}\Big|_{v=0} v + \frac{d^2 i}{dv^2}\Big|_{v=0}\frac{v^2}{2} + \frac{d^3 i}{dv^3}\Big|_{v=0}\frac{v^3}{6} + \cdots$$

$$i\Big|_{v=0} = 0 \;,\quad \frac{di}{dv}\Big|_{v=0} = 30 I_S \;,\quad \frac{d^2 i}{dv^2}\Big|_{v=0} = 900\, I_S \;,\quad \frac{d^3 i}{dv^3}\Big|_{v=0} = 27{,}000\; I_S .$$

So,
$$i(t) = I_S(30v + 900v^2 + 27{,}000 v^3) + \cdots$$

$$v = 0.01\cos\omega_1 t + 0.01\cos\omega_2 t$$
$$v^2 = 10^{-4}\left[\cos^2\omega_1 t + 2\cos\omega_1 t\cos\omega_2 t + \cos^2\omega_2 t\right]$$
$$= 10^{-4}\left[1 + \tfrac{1}{2}\cos 2\omega_1 t + \tfrac{1}{2}\cos 2\omega_2 t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right]$$
$$v^3 = 10^{-6}\left[\cos^3\omega_1 t + 3\cos^2\omega_1 t\cos\omega_2 t + 3\cos\omega_1 t\cos^2\omega_2 t + \cos^3\omega_2 t\right]$$
$$= 10^{-6}\Big[\tfrac{1}{4}\cos 3\omega_1 t + \tfrac{3}{4}\cos\omega_1 t + \tfrac{3}{2}\cos\omega_2 t + \tfrac{3}{4}\cos(2\omega_1-\omega_2)t$$
$$+ \tfrac{3}{4}\cos(2\omega_1+\omega_2)t + \tfrac{3}{2}\cos\omega_1 t + \tfrac{3}{4}\cos(\omega_1-2\omega_2)t$$
$$+ \tfrac{3}{4}\cos(\omega_1+2\omega_1)t + \tfrac{3}{4}\cos\omega_2 t + \tfrac{1}{4}\cos 3\omega_2 t\Big]$$

1
1 1
1 2 1
1 3 3 1

ω	(AMPLITUDE)/I_S
ω_1, ω_2	$(30)(.01) + (27{,}000/6)(10^{-6})(\tfrac{3}{4}) = 0.310$ ✓
$2\omega_1, 2\omega_2$	$(900)(10^{-4})/4 = 0.0225$ ✓
$3\omega_1, 3\omega_2$	$(27{,}000)(10^{-6})(\tfrac{1}{4})(\tfrac{1}{6}) = 0.00113$ ✓
$\omega_1+\omega_2$	$(900)(10^{-4})/2 = 0.045$ ✓
$\omega_1-\omega_2$	" $= 0.045$ ✓
$2\omega_1-\omega_2$	$(27{,}000)(10^{-6})(3/4)(1/6) = 0.00338$ ✓
$2\omega_1+\omega_2$	" $= 0.00338$ ✓
$\omega_1-2\omega_2$	" $= 0.00338$ ✓
$\omega_1+2\omega_2$	" $= 0.00338$ ✓
$\omega=0$	0.045

$\boxed{7.7}$

$$v(t) = V_{RF}\cos\omega_{RF}t - I_{RF}R_g\cos\omega_{RF}t - I_{IF}R_{IF}\cos\omega_{IF}t - I_{IM}R_g\cos\omega_{IM}t$$

$$g(t) = g_0 + 2g_1\cos\omega_{LO}t + 2g_2\cos 2\omega_{LO}t$$

$$\begin{aligned}
i(t) &= g(t)\,v(t) \\
&= g_0\left(V_{RF}\cos\omega_{RF}t - I_{RF}R_g\cos\omega_{RF}t - I_{IF}R_{IF}\cos\omega_{IF}t - I_{IM}R_g\cos\omega_{IM}t\right) \\
&\quad + 2g_1\left(V_{RF}\cos\omega_{RF}t\cos\omega_{LO}t - I_{RF}R_g\cos\omega_{RF}t\cos\omega_{LO}t\right. \\
&\qquad\qquad \left. - I_{IF}R_{IF}\cos\omega_{IF}t\cos\omega_{LO}t - I_{IM}R_g\cos\omega_{IM}t\cos\omega_{LO}t\right) \\
&\quad + 2g_2\left(V_{RF}\cos\omega_{RF}t\cos 2\omega_{LO}t - I_{RF}R_g\cos\omega_{RF}t\cos 2\omega_{LO}t\right. \\
&\qquad\qquad \left. - I_{IF}R_{IF}\cos\omega_{IF}t\cos 2\omega_{LO}t - I_{IM}R_g\cos\omega_{IM}t\cos 2\omega_{LO}t\right)
\end{aligned}$$

$\omega_{RF} = \omega_{LO} + \omega_{IF}$

$\omega_{RF} + \omega_{LO} = $ "A"

$\omega_{IM} = \omega_{RF} - 2\omega_{LO}$

$\cos\omega_{RF}t\cos\omega_{LO}t = \frac{1}{2}\cos\omega_{IF}t + \frac{1}{2}\cos At$ $\qquad A = \omega_{RF} + \omega_{LO}$

$\cos\omega_{IF}t\cos\omega_{LO}t = \frac{1}{2}\cos\omega_{RF}t + \frac{1}{2}\cos\omega_{IM}t$

$\cos\omega_{IM}t\cos\omega_{LO}t = \frac{1}{2}\cos\omega_{IF}t + \frac{1}{2}\cos Bt$ $\qquad B = \omega_{RF} - 3\omega_{LO}$

$\cos\omega_{RF}t\cos 2\omega_{LO}t = \frac{1}{2}\cos\omega_{IM}t + \frac{1}{2}\cos Ct$ $\qquad C = \omega_{RF} + 2\omega_{LO}$

$\cos\omega_{IF}t\cos 2\omega_{LO}t = \frac{1}{2}\cos At + \frac{1}{2}\cos Bt$

$\cos\omega_{IM}t\cos 2\omega_{LO}t = \frac{1}{2}\cos\omega_{RF}t + \frac{1}{2}\cos Dt$ $\qquad D = \omega_{RF} - 4\omega_{LO}$

Equating terms with frequency $\omega_{RF}, \omega_{IF}, \omega_{IM}$ gives

ω_{RF}: $\quad i_{RF} = g_0(V_{RF} - I_{RF}R_g) - g_1 I_{IF}R_{IF} - g_2 I_{IM}R_g$

ω_{IF}: $\quad i_{IF} = -g_0 I_{IF}R_{IF} + g_1(V_{RF} - I_{RF}R_g) - g_1 I_{IM}R_g$

ω_{IM}: $\quad i_{IM} = -g_0 I_{IM}R_g - g_1 I_{IF}R_{IF} + g_2(V_{RF} - I_{RF}R_g)$

(Frequencies A, B, C, D are outside operating band)

In matrix form:

$$\begin{bmatrix} i_{RF} \\ i_{IF} \\ i_{IM} \end{bmatrix} = \begin{bmatrix} g_0 & g_1 & g_2 \\ g_1 & g_0 & g_1 \\ g_2 & g_1 & g_0 \end{bmatrix} \begin{bmatrix} V_{RF} - I_{RF}R_g \\ -I_{IF}R_{IF} \\ -I_{IM}R_g \end{bmatrix} \quad \checkmark$$

$\boxed{7.8}$ To find the short-circuit IF current, set $R_{IF}=0$. Then,

$$I_{RF}(1+g_0R_g) = g_0V_{RF} - g_2R_gI_{IM}$$
$$I_{IF} = g_1(V_{RF}-I_{RF}R_g) - g_1R_gI_{IM}$$
$$I_{IM}(1+g_0R_g) = g_2(V_{RF}-I_{RF}R_g)$$

Eliminate I_{IM}:

$$I_{RF}(1+g_0R_g) = g_0V_{RF} - g_2^2R_g\frac{(V_{RF}-I_{RF}R_g)}{1+g_0R_g}$$
$$I_{IF} = g_1(V_{RF}-I_{RF}R_g) - g_1g_2R_g\frac{(V_{RF}-I_{RF}R_g)}{1+g_0R_g}$$

Simplify:

$$I_{RF}\left[(1+g_0R_g) - \frac{g_2^2R_g^2}{1+g_0R_g}\right] = V_{RF}\left[g_0 - \frac{g_2^2R_g}{1+g_0R_g}\right]$$
$$I_{IF}\left[g_1 - \frac{g_1g_2R_g}{1+g_0R_g}\right]V_{RF} - \left[g_1R_g - \frac{g_1g_2R_g^2}{1+g_0R_g}\right]I_{RF}$$

and:

$$I_{RF}\left[(1+g_0R_g)^2 - g_2^2R_g^2\right] = V_{RF}\left[g_0(1+g_0R_g) - g_2^2R_g\right]$$
$$I_{IF} = \left[g_1 - \frac{g_1g_2R_g}{1+g_0R_g}\right]V_{RF} - \left[g_1R_g - \frac{g_1g_2R_g^2}{1+g_0R_g}\right]I_{RF}$$

eliminate I_{RF}:

$$\frac{I_{IF}}{V_{RF}} = g_1\left(1-\frac{g_2R_g}{1+g_0R_g}\right) - g_1R_g\left(1-\frac{g_2R_g}{1+g_0R_g}\right)\frac{[g_0(1+g_0R_g)-g_2^2R_g]}{[(1+g_0R_g)^2-g_2^2R_g^2]}$$

$$= g_1\left(1-\frac{g_2R_g}{1+g_0R_g}\right)\left[1 - R_g\frac{g_0(1+g_0R_g)-g_2^2R_g}{(1+g_0R_g)^2-g_2^2R_g^2}\right]$$

$$= g_1\left(1-\frac{g_2R_g}{1+g_0R_g}\right)\frac{1+g_0R_g}{(1+g_0R_g-g_2R_g)(1+g_0R_g+g_2R_g)}$$

$$= \frac{g_1}{1+g_0R_g+g_2R_g} \quad \checkmark$$

To find the open-circuit voltage at the IF port, set $I_{IF}=0$:

7.8 CONTINUED.

$$I_{RF} = g_0(V_{RF} - I_{RF}R_g) + g_1 V_{oc} - I_{IM} g_2 R_g$$

$$0 = g_1(V_{RF} - I_{RF}R_g) + g_0 V_{oc} - I_{IM} g_1 R_g$$

$$I_{IM} = g_2(V_{RF} - I_{RF}R_g) + g_1 V_{oc} - I_{IM} g_0 R_g$$

Eliminate I_{IM} using second equation:

$$I_{RF}(1 + g_0 R_g - g_2 R_g) = V_{RF}(g_0 - g_2) + V_{oc}(g_1 - g_0 g_2/g_1)$$

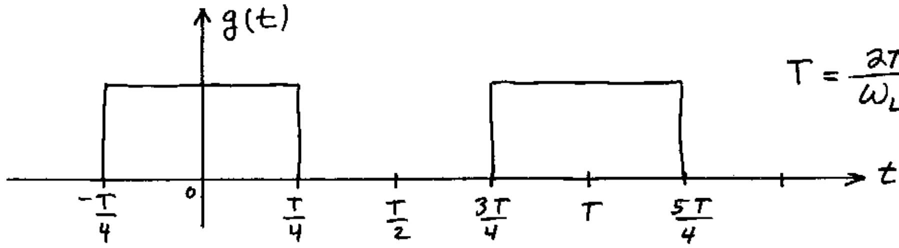
$$I_{RF}(g_2 R_g - 1 - g_0 R_g) = V_{RF}\left(g_2 - \frac{1+g_0R_g}{R_g}\right) + V_{oc}\left[g_1 - \frac{g_0}{g_1 R_g}(1+g_0R_g)\right]$$

Eliminate I_{RF}:

$$-V_{RF}\left(g_0 - \frac{1+g_0R_g}{R_g}\right) = V_{oc}\left[g_1 - \frac{g_0 g_2}{g_1} + g_1 - \frac{g_0(1+g_0R_g)}{g_1 R_g}\right]$$

$$V_{oc} = \frac{V_{RF}(1/R_g)}{\left[2g_1 - \dfrac{g_0g_2R_g + g_0(1+g_0R_g)}{g_1 R_g}\right]} = \frac{g_1 V_{RF}}{\left[2g_1^2R_g - g_0g_2R_g - g_0(1+g_0R_g)\right]} \checkmark$$

$\boxed{7.9}$



$T = \dfrac{2\pi}{\omega_{LO}}$

The general form of the Fourier series is,

$$g(t) = \sum_{n=-\infty}^{\infty} g_n e^{jn\omega_{LO}t} \, ,$$

with

$$g_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\, e^{-jn\omega_{LO}t}\, dt = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jn\omega_{LO}t}\, dt$$

$$= \frac{1}{T}\,\frac{e^{-jn\pi/2} - e^{jn\pi/2}}{-jn\omega_{LO}} = \frac{2\sin n\pi/2}{nT\omega_{LO}} = \frac{2\sin n\pi/2}{2\pi n}$$

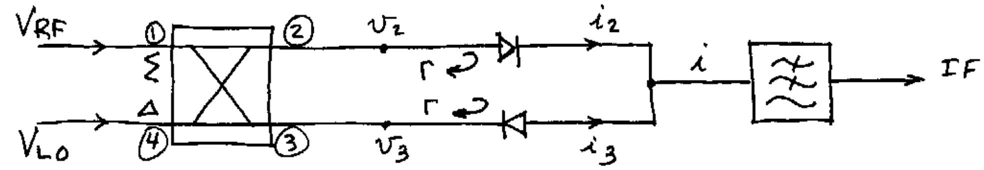
$$= \frac{1}{2}\,\frac{\sin n\pi/2}{n\pi/2}$$

For $n=0$, $g_0 = 1/2$. Thus,

$$g(t) = \frac{1}{2} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n\pi/2} e^{jn\omega_{LO}t} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n\pi/2} e^{-jn\omega_{LO}t}$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n\pi/2} \cos n\omega_{LO}t \quad \checkmark$$

$\boxed{7.10}$



$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

let $v_{RF}(t) = V_{RF} \cos \omega_{RF} t = v_1(t)$

$v_{LO}(t) = V_{LO} \cos \omega_{LO} t = v_4(t)$

Then the diode voltages are,

$$v_2(t) = \frac{1}{\sqrt{2}} V_{RF} \cos(\omega_{RF} t - 90°) + \frac{1}{\sqrt{2}} V_{LO} \cos(\omega_{LO} t + 90°)$$

$$v_3(t) = \frac{1}{\sqrt{2}} V_{RF} \cos(\omega_{RF} t - 90°) + \frac{1}{\sqrt{2}} V_{LO} \cos(\omega_{LO} t - 90°)$$

Assume $i_2 = k v_2^2$, $i_3 = -k v_2^2$. $\omega_{IF} = \omega_{RF} - \omega_{LO}$.

Then, after LP filtering, the diode currents are,

$$i_2 = \frac{k}{4} V_{RF} V_{LO} \cos(\omega_{RF} t - 90° - \omega_{LO} t - 90°) = \frac{-k}{4} V_{RF} V_{LO} \cos \omega_{IF} t.$$

$$i_3 = -\frac{k}{4} V_{RF} V_{LO} \cos(\omega_{RF} t - 90° - \omega_{LO} t + 90°) = \frac{-k}{4} V_{RF} V_{LO} \cos \omega_{IF} t.$$

So the IF output current is $i(t) = -\frac{k}{2} V_{RF} V_{LO} \cos \omega_{IF} t$ ✓

AT RF INPUT:

$V_2^+ = \Gamma V_2^- = \frac{-j}{\sqrt{2}} \Gamma V_{RF}^+$; $V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}} \Gamma V_{RF}^+$

$V_{RF}^{\Sigma} = V_1^- = V_2^+ (-j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{-\Gamma V_{RF}^+}$ ✓

$V_{RF}^{\Delta} = V_4^- = V_2^+ (j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{0}$ ✓

AT LO INPUT:

$V_2^+ = \Gamma V_2^- = \frac{j}{\sqrt{2}} \Gamma V_{LO}^+$; $V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}} \Gamma V_{LO}^+$

$V_{LO}^{\Sigma} = V_1^- = V_2^+ (-j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{0}$ ✓

$V_{LO}^{\Delta} = V_4^- = V_2^+ (j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{-\Gamma V_{LO}^+}$ ✓

Assume now that

$$v_{LO}(t) = V_{LO}^{(2)} \cos 2\omega_{LO} t.$$

7.10 CONTINUED

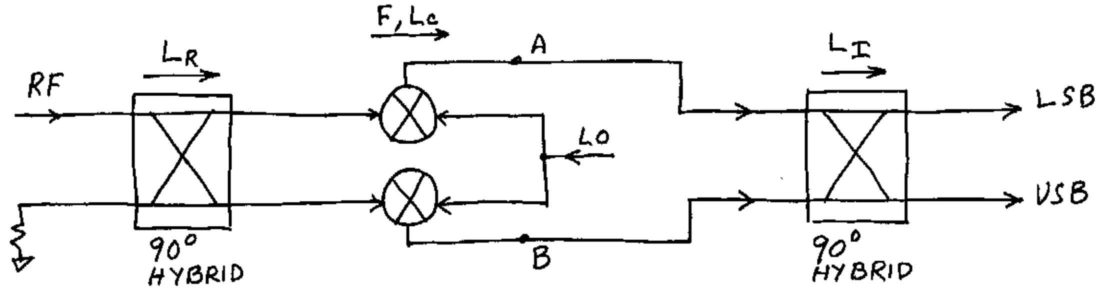
Then after LPF,

$$v_2^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

$$v_3^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

Then forming $i(t) = k(v_2^2 - v_3^2)\Big|_{LPF} = 0$ for $\omega_{RF} \pm 2\omega_{LO}$ frequencies.

7.11



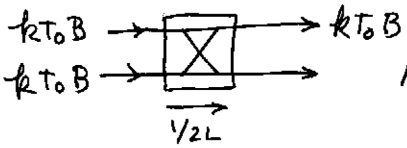
The noise power due to the RF hybrid and the mixer, ref. to IF output of mixer, is

$$N_A = N_B = \frac{kB}{L_c}\left[T_0 + (F-1)T_0\right] = \frac{kBT_0F}{L_c},$$

since the noise power output of the matched hybrid is kT_0B. The total noise power output is (at either LSB or USB),

$$N_o = \frac{N_A}{2L_I} + \frac{N_B}{2L_I} + \frac{N_{added}}{2L_I} = \frac{kBT_0F}{L_IL_c} + \frac{N_{added}}{2L_I}$$

Nadded is the output noise power of the IF hybrid when not terminated at second input port:



$$N_o = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{added}}{2L} = kT_0B$$

Thus $N_{added} = 2kT_0B(L-1)$

So, $N_o = \dfrac{kBT_0F}{L_IL_c} + kT_0B\left(1-\dfrac{1}{L_I}\right)$; $S_o = \dfrac{4S_i}{L_c}\dfrac{1}{4L_IL_R} = \dfrac{S_i}{L_cL_IL_R}$

$N_i = kT_0B$

and then,

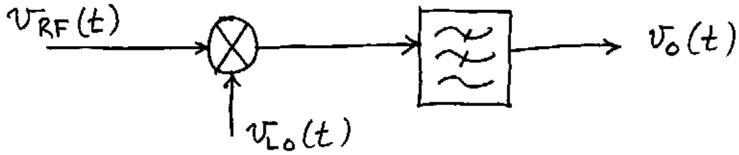
$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L_cL_IL_R}{kT_0B}\left[\frac{kBFT_0}{L_IL_c} + kT_0B\left(1-\frac{1}{L_I}\right)\right] = \underline{FL_R + L_cL_RL_I - L_cL_R}$$

CHECK: if $L_R = L_I = 1$, $F_{TOT} = F + 2L_c - 2L_c = F$ ✓ (mixer noise only)

CHECK: if $F = L_c$ (passive mixer loss only), $F_{TOT} = L_cL_IL_R$ ✓

(The cascade noise figure formula can be used to obtain the same result if we set $F_R = L_R$, $F_I = L_I$.)

$\boxed{7.12}$



$$v_{RF}(t) = V_U \cos(\omega_{LO}+\omega_{IF})t + V_L \cos(\omega_{LO}-\omega_{IF})t$$

$$v_{LO}(t) = V_{LO} \cos\omega_{LO}t$$

$$v_o(t) = k\,v_{RF}(t)\,v_{LO}(t)\Big|_{LPF} = \frac{k}{2} V_U V_{LO} \cos\omega_{IF}t + \frac{k}{2} V_L V_{LO} \cos\omega_{IF}t$$

$$= \frac{k}{2} V_{LO}(V_U + V_L)\cos\omega_{IF}t$$

$$P_{IF} = \frac{1}{2}\left[\frac{k}{2} V_{LO}(V_U+V_L)\right]^2 = \frac{k^2}{8} V_{LO}^2 (V_U+V_L)^2$$

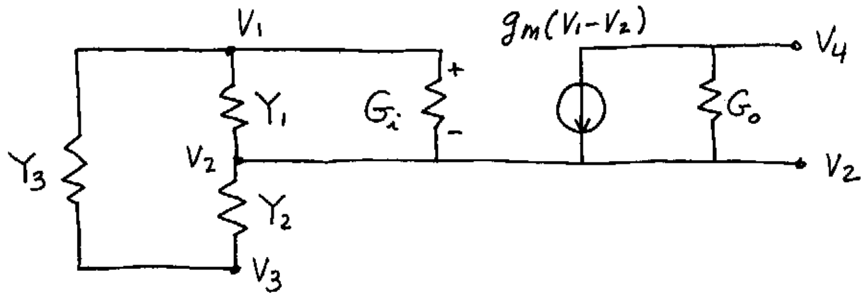
From (7.68),

$$P_1 + P_2 = \frac{K^2 V_{LO}^2}{8}(V_L^2 + V_U^2)$$

(It is not clear that these powers should be equal?)

Chapter 8

8.1



Writing KCL for nodes V_1, V_2, V_3, V_4:

V_1: $(V_3-V_1)Y_3 + (V_2-V_1)Y_1 + (V_2-V_1)G_i = 0$

V_2: $(V_1-V_2)Y_1 + (V_3-V_2)Y_2 + (V_1-V_2)G_i + g_m(V_1-V_2) + (V_4-V_2)G_o = 0$

V_3: $(V_1-V_3)Y_3 + (V_2-V_3)Y_2 = 0$

V_4: $(V_2-V_4)G_o - g_m(V_1-V_2) = 0$

Rearranging:

$$
\begin{aligned}
&V_1(Y_1+Y_3+G_i) + V_2(-Y_1-G_i) + V_3(-Y_3) + V_4(0) = 0\\
&V_1(-Y_1-G_i-g_m) + V_2(Y_1+Y_2+G_i+G_o+g_m) + V_3(-Y_2) + V_4(-G_o) = 0\\
&V_1(-Y_3) + V_2(-Y_2) + V_3(Y_2+Y_3) + V_4(0) = 0\\
&V_1(g_m) + V_2(-G_o-g_m) + V_3(0) + V_4 G_o = 0
\end{aligned}
$$

which agrees with the matrix of (8.3).

$\boxed{8.2}$ From (8.4),

$$\det\begin{bmatrix}(Y_1+Y_3+G_i) & -Y_3 \\ (g_m-Y_3) & (Y_2+Y_3)\end{bmatrix}=0 \quad \text{for oscillation.}$$

For a Colpitts oscillator, let $Y_1=j\omega C_1$, $Y_2=j\omega C_2$, $Z_3=R+j\omega L_3$.

Then,

$$\det[\cdot]=\left(j\omega C_1+\frac{1}{R+j\omega L_3}+G_i\right)\left(j\omega C_2+\frac{1}{R+j\omega L_3}\right)+\left(\frac{1}{R+j\omega L_3}\right)\left(g_m-\frac{1}{R+j\omega L_3}\right)=0$$

$$\left[1+(G_i+j\omega C_1)(R+j\omega L_3)\right]\left[1+j\omega C_2(R+j\omega L_3)\right]+g_m(R+j\omega L_3)-1=0$$

$$\cancel{1}+j\omega C_2\,\cancel{(R+j\omega L_3)}+(G_i+j\omega C_1)(R+j\omega L_3)+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)^{\cancel{2}}+$$
$$+\,g_m(R\cancel{+j\omega L_3})-\cancel{1}=0$$

$$j\omega C_2+G_i+j\omega C_1+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)+g_m=0$$

Re: $\quad G_i+g_m-\omega^2L_3G_iC_2-\omega^2C_1C_2R=0$

Im: $\quad \omega C_2+\omega C_1+\omega C_2G_iR-\omega^3C_1C_2L_3=0$

$$C_1+C_2+C_2G_iR-\omega^2C_1C_2L_3=0$$

$$\omega=\sqrt{\frac{C_1+C_2+C_2G_iR}{C_1C_2L_3}}=\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{G_iR}{C_1}\right)} \quad ✓$$

$\boxed{8.3}$ $f_o = 30\,MHz$, $\beta = 40$, $R_i = 800\,\Omega$

Choose $\underline{C_1' = C_2 = 500\,pF}$. Then (8.20) gives L_3 as,

$$L_3 = \frac{1}{\omega_o^2}\left(\frac{1}{C_1'} + \frac{1}{C_2}\right) = \underline{0.11\,\mu H}$$

From (8.22), the maximum value of inductor resistance is,

$$R_{MAX} = G_i\left[\frac{1 + g_m/G_i}{\omega_o^2 C_1 C_2} - \frac{L_3}{C_1}\right]$$

Since $G_i = 1/R_i$, $g_m/G_i = \beta$, and assuming $RG_i << 1$ so that $C_1 \simeq C_1'$, we have,

$$R_{MAX} = \frac{1}{R_i}\left(\frac{1+\beta}{\omega_o^2 C_1' C_2} - \frac{L_3}{C_1'}\right) = 5.5\,\Omega$$

So the minimum inductor Q is,

$$Q_{MIN} = \frac{\omega_o L_3}{R_{MAX}} = \underline{\underline{3.8}} \qquad \text{Then } C_1 \simeq 500\,pF.$$

$\boxed{8.4}$ $f_o = 10\,MHz$, $R = 30\,\Omega$, $C = 27\,pF$, $C_o = 5.5\,pF$.

From (8.23a), $L = \dfrac{1}{\omega_s^2 C} = 9.4\,mH$ $\quad (\omega_s = 2\pi \cdot 10\,MHz)$

From (8.23b), $L = \dfrac{C_o + C}{\omega_p^2 C_o C} = 9.4\,mH$ $\quad (\omega_p = 2\pi \cdot 10\,MHz)$

Using L in (8.23a,b) gives $f_s = 9.990\,MHz$, $f_p = 10.015\,MHz$, for a percentage difference of 0.25%.

The Q is

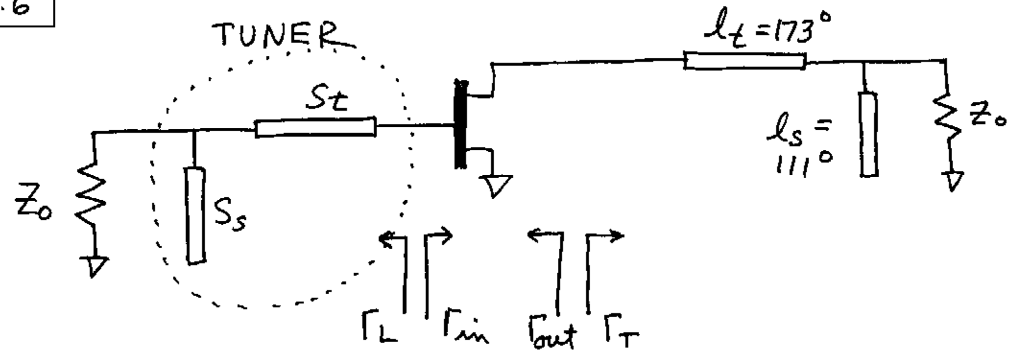
$$Q = \frac{\omega L}{R} = 20{,}000.$$

$\boxed{8.5}$ From (8.24), $Z_L + Z_{in} = 0$ for oscillation.

Then,
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{-Z_{in} - Z_o}{-Z_{in} + Z_o} = \frac{Z_{in} + Z_o}{Z_{in} - Z_o} = \frac{1}{\Gamma_{in}}$$

Thus $\Gamma_L \Gamma_{in} = 1$ ✓

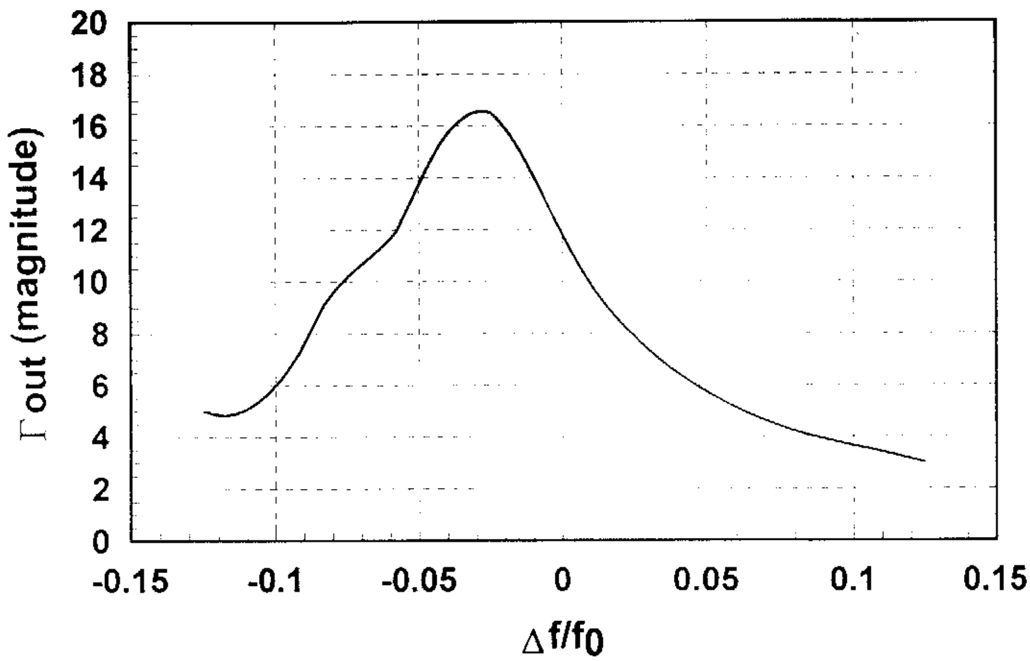
$\boxed{8.6}$



As in Example 8.4, choose $\Gamma_L = 0.6\angle{-130°}$. Then Γ_{out}, Z_{out}, Z_T, ℓ_t, and ℓ_s are unchanged. Then we have the matching problem of using a stub tuner to match 50Ω to Γ_L. The stub susceptance is $jb_s = +j1.56$, for a stub length of $S_s = 0.158\lambda$. The line length is $S_t = 0.004\lambda$.

Computer analysis gives Γ_{out} vs. f, which is plotted on the attached graph. The maximum does not occur at f_o, because the tuner is not resonant at f_o. The Q is much lower than in Example 8.4. This result shows the importance of using a high-Q resonator.

8.6 CONTINUED.



$\boxed{8.7}$ $S_{11} = 1.2\angle 150°$, $S_{12} = 0.2\angle 120°$, $S_{21} = 3.7\angle -72°$, $S_{22} = 1.3\angle -67°$

As in Example 8.4, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \simeq 1$, since,

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8\angle -150°$ ✓. Then $\Gamma_{out} = 15.9\angle -99.3°$, and

$$Z_{out} = Z_0 \frac{1+\Gamma_{out}}{1-\Gamma_{out}} = -7.6 + j1.9\,\Omega$$

$$Z_T = \frac{-R_{out}}{3} - jX_{out} = 2.53 - j1.9\,\Omega$$

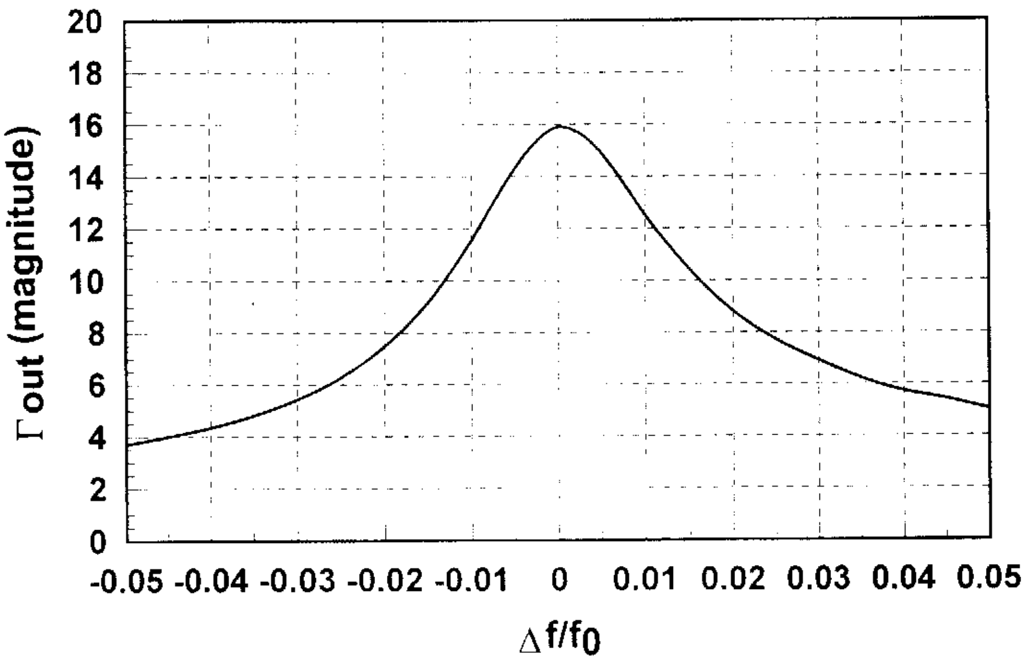
Matching Z_T to the load impedance gives $l_t = 0.031\lambda$, with a required stub susceptance of $+j4.0$. Thus, $l_s = 0.21\lambda$. At the dielectric resonator,

$$\Gamma_L' = \Gamma_L = e^{2j\beta l_r} = (0.8\angle -150°)\, e^{2j\beta l_r} = 0.8\angle 180°.$$

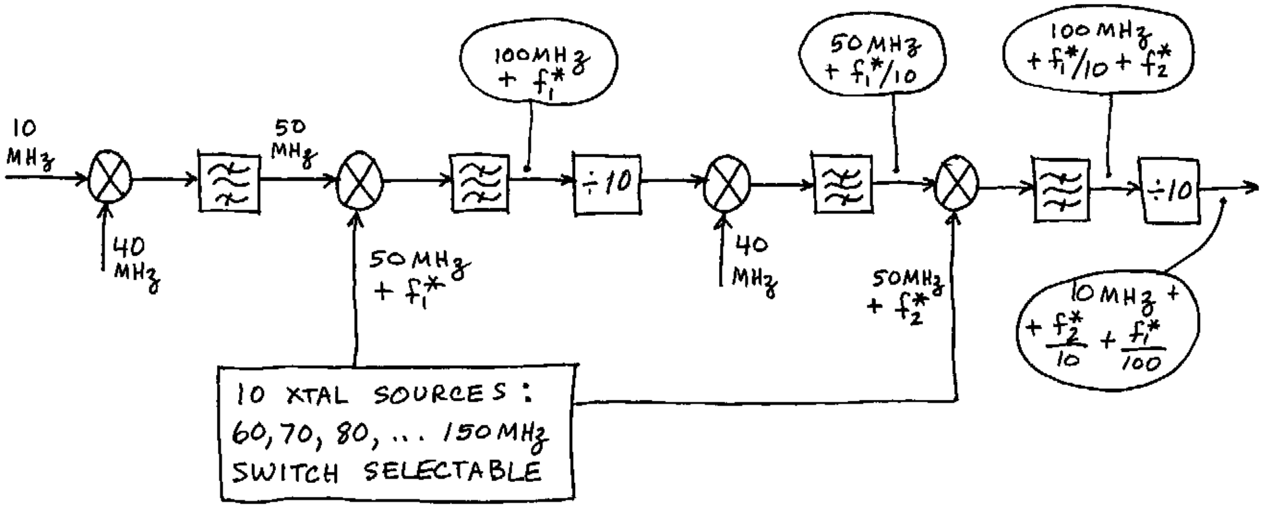
8.7 CONTINUED.

Then $\ell_r = 0.458\lambda$, and $Z_L' = Z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'} = 5.55\,\Omega = N^2 R$.

$|\Gamma_{out}|$ vs. f is plotted below. Note that resonance occurs at f_0, and the bandwidth is much narrower than the previous case. This is due to the much higher Q of the resonator.



$\boxed{8.8}$ The "double-mix and divide" method is preferred over the basic direct synthesis method because fewer sources are needed, and the filter requirements are much less stringent.



Two 10-position switches are used to select f_1^* and f_2^* individually as 10, 20, 30, ... 100 MHz. A 10 MHz and a 40 MHz source are also needed.

$\boxed{8.9}$ From (8.41), the clock frequency is $f_c = 4f_{MAX} = 40\,MHz$. From (8.42), the number of bits in the sine look-up table is N, where

$$2^N = \frac{f_c}{f_{MIN}} = \frac{40\,MHz}{1\,kHz} = 40{,}000. \quad \text{Thus } \underline{N=16}$$

From (8.43), the size of the DAC is M, where P_n is the spurious noise level:

$$P_n = -40\,dB = -6(M-1)\,dB \Rightarrow \underline{M = 8 \text{ bits}}$$

The memory size of the look-up table is then
$8 \text{ bits} \times 2^{16} = 0.5\,Mb.$

$\boxed{8.10}$ Using trig identities:

$$i_1 = K\left[\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_0 t + 2\theta_1) - \sin(\theta_2 - \theta_1) - \sin(2\omega_0 t + \theta_1 + \theta_2) + \tfrac{1}{2} - \tfrac{1}{2}\cos(2\omega_0 t + 2\theta_1)\right]$$

$$i_2 = -K\left[\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_0 t + 2\theta_2) - \sin(\theta_1 - \theta_2) - \sin(2\omega_0 t + \theta_1 + \theta_2) + \tfrac{1}{2} - \tfrac{1}{2}\cos(2\omega_0 t + 2\theta_1)\right]$$

Summing and low-pass filtering gives

$$\begin{aligned} v_o = i_1 + i_2\Big|_{LPF} &= K\left[-\sin(\theta_2 - \theta_1) + \sin(\theta_1 - \theta_2)\right] \\ &= 2K\sin(\theta_1 - \theta_2) \\ &\simeq 2K(\theta_1 - \theta_2) \qquad \text{for small } |\theta_1 - \theta_2|. \end{aligned}$$

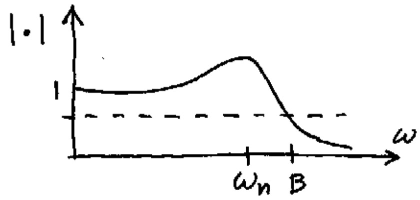
$\boxed{8.11}$

$$\frac{K_d \omega_f \Delta\omega}{\omega_n^2}\left[\frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}\right] =$$

$$= \frac{K_d \omega_f \Delta\omega}{\omega_n^2}\left[\frac{s^2 + 2\xi\omega_n s + \omega_n^2 - s^2 - 2\xi\omega_n s}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}\right] = \frac{K_d \omega_f \Delta\omega}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \checkmark$$

$\boxed{8.12}$ From (8.80) the normalized phase transfer function magnitude is,

$$\left|\frac{\theta_o(j\omega)}{\theta_i(j\omega)}\right| = \frac{1}{\sqrt{(1-\omega/\omega_n)^2 + (2\xi\omega/\omega_n)^2}}$$



The 3-dB bandwidth is defined as the frequency where the response has dropped by $1/\sqrt{2}$. Thus,

$$\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2} = 2$$

$$(\omega_n^2-\omega^2)^2 + 4\xi\omega^2\omega_n^2 = 2\omega_n^4$$

$$\omega^4 + \omega^2\omega_n^2(4\xi^2-2) - \omega_n^4 = 0$$

$$\omega^2 = \frac{(2-4\xi^2)\,\omega_n^2 \pm \omega_n^2\sqrt{(4\xi^2-2)^2+4}}{2}$$

$$= (1-2\xi^2)\omega_n^2 + \omega_n^2\sqrt{2-4\xi^2+4\xi^4}$$

We choose + for the upper frequency. Thus,

$$B = \omega_n\sqrt{(1-2\xi^2)+\sqrt{2-4\xi^2+4\xi^4}} \quad ✓$$

$\boxed{8.13}$ $\quad H(s) = \dfrac{1+sT_1}{sT_2} \quad , \quad \theta_i(s) = \dfrac{\Delta\omega}{s^2}$

From (8.59) the VCO control voltage is,

$$V_c(s) = \frac{s K_d H(s)}{s + K H(s)} \theta_i(s)$$

$$= \frac{K_d(1+s\tau_1)}{T_2\left[s + \frac{K(1+sT_1)}{sT_2}\right]} \frac{\Delta\omega}{s^2} = \frac{\Delta\omega K_d (1+sT_1)}{T_2 s\left(s^2 + \frac{KT_1}{T_2} s + \frac{K}{T_2}\right)}$$

Let $\omega_n^2 = \frac{K}{T_2}$, $\xi = \frac{T_1}{2}\sqrt{\frac{K}{T_2}}$. Then $2\omega_n \xi = \sqrt{\frac{K}{T_2}}\, \tau_1 \sqrt{\frac{K}{T_2}} = \frac{KT_1}{T_2}$

Then,

$$V_c(s) = \frac{\Delta\omega K_d (1+sT_1)}{T_2 s (s^2 + 2\omega_n \xi s + \omega_n^2)}$$

$$= \frac{\Delta\omega K_d}{\omega_n^2 T_2}\left[\frac{1}{s} - \frac{s + 2\xi\omega_n - \tau_1\omega_n^2}{(s^2 + 2\omega_n\xi s + \omega_n^2)}\right]$$

Using Laplace transform tables:

$$v_c(t) = \frac{\Delta\omega K_d}{\omega_n^2 T_2}\left\{1 - \left[\cos\sqrt{1-\xi^2}\,\omega_n t - \frac{\xi}{\sqrt{1-\xi^2}} \sin\sqrt{1-\xi^2}\,\omega_n t\right] e^{-\xi\omega_n t} - \right.$$

$$\left. -(2\xi\omega_n - \tau_1\omega_n^2)\frac{\sin\sqrt{1-\xi^2}\,\omega_n t}{\sqrt{1-\xi^2}\,\omega_n} e^{-\xi\omega_n t}\right\} U(t)$$

From (8.60) the loop phase error is,

$$\varepsilon(s) = \frac{s}{s + KH(s)}\theta_i(s) = \frac{\Delta\omega}{s^2 + \frac{K}{T_2}(1+sT_1)} = \frac{\Delta\omega}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\varepsilon(t) = \frac{\Delta\omega}{\omega_n\sqrt{1-\xi^2}} \sin\sqrt{1-\xi^2}\,\omega_n t\, e^{-\omega_n \xi t}\; U(t)$$

as $t \to \infty$, $\varepsilon(t) \to 0$.

$\boxed{8.14}$ From (8.92),

$$S_\phi = \frac{kT_0F}{P_0}\left(\frac{K\omega_\alpha \omega_h^2}{\Delta\omega^3} + \frac{\omega_h^2}{\Delta\omega^2} + \frac{K\omega_\alpha}{\Delta\omega} + 1\right)$$

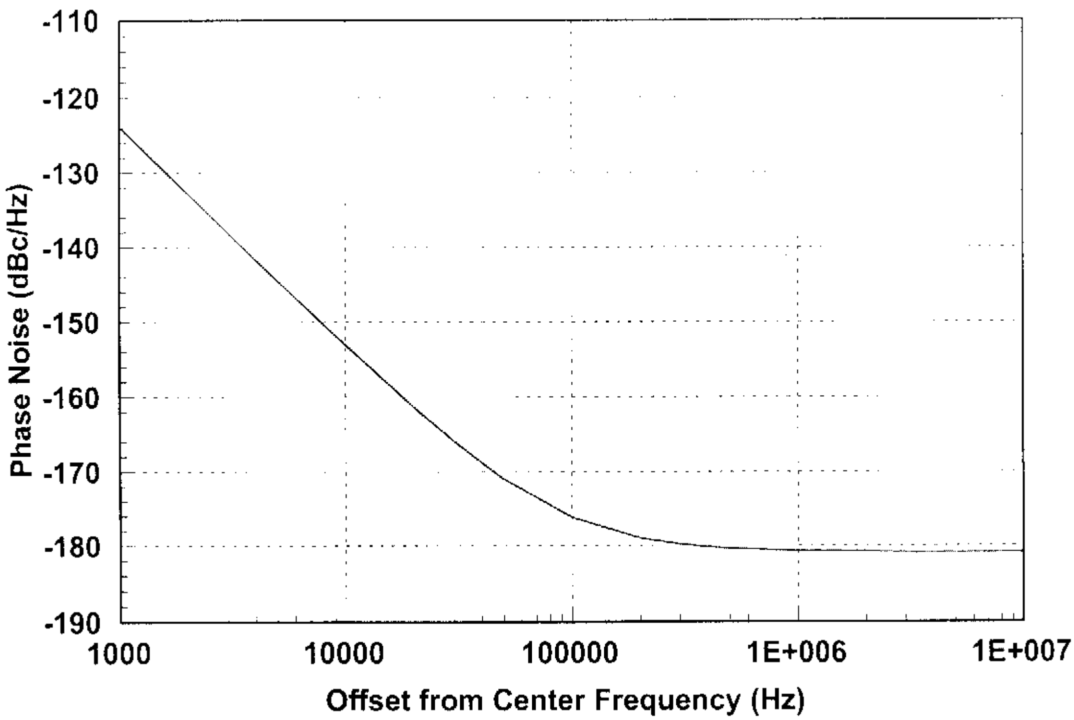
$$\mathcal{L}(f) = S_\phi/2 .$$

For $F = 6\,dB = 4$, $f_0 = 100\,MHz$, $Q = 500$, $P_0 = 10\,dBm = 10\,mW$, $K = 1$,
$f_\alpha = 50\,kHz$, $f_h = f_0/2Q = 100\,kHz$, $\Delta f = f - f_0$

a short computer program was written to compute data for the plot shown below.

(a) $\Delta f = 1\,MHz$, $S_\phi = -178\,dBm$, $\mathcal{L}(1MHz) = -181\,dBc/Hz$

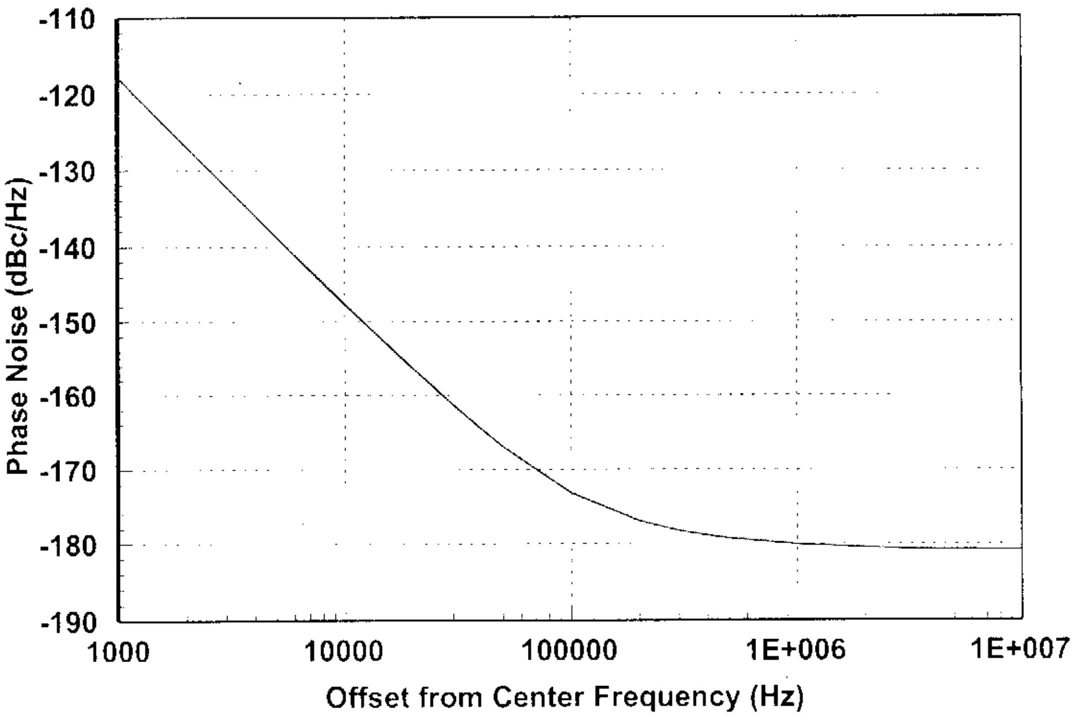
(b) $\Delta f = 10\,kHz$, $S_\phi = -150\,dBm$, $\mathcal{L}(10kHz) = -153\,dBc/Hz$



$\boxed{8.15}$ This calculation is similar to that of Problem 8.14, but with $f_\alpha = 200\,kHz$. Plot shown below.

(a) $\Delta f = 1\,MHz$, $S_\phi = -177\,dBm$, $\mathcal{L}(1\,MHz) = -180\,dBc/Hz$

(b) $\Delta f = 10\,kHz$, $S_\phi = -144\,dBm$, $\mathcal{L}(10\,kHz) = -147\,dBc/Hz$.



$\boxed{8.16}$

If C is the desired signal level, I is the undesired signal level, S is the desired rejection ratio, $\mathcal{L}(f)$ the phase noise, and B the filter bandwidth, then

$$S = \frac{C}{I B \mathcal{L}(f)}$$

In dB,

$$\mathcal{L}(f) = C(\text{dBm}) - I(\text{dBm}) - S(\text{dB}) - 10\log(B). \checkmark$$

$\boxed{8.17}$ $B = 12$ kHz, $S = 80$ dB, $C = I$.

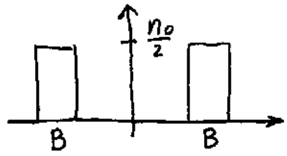
From (8.93),

$$\mathcal{L}(30\text{ kHz}) = C(\text{dBm}) - I(\text{dBm}) - S(\text{dB}) - 10\log(B)$$
$$= -80\text{ dB} - 10\log(12\times10^3)$$
$$= \underline{-121\text{ dBm}}$$

Chapter 9

9.1 $B = 30\text{ kHz}$, $\frac{n_o}{2} = 10^{-8}\text{ W/Hz}$, $SNR = 25\text{ dB}$

For SSB, $SNR = \frac{S_o}{N_o} = \frac{S_i}{N_i} = 25\text{ dB} = 316.2$



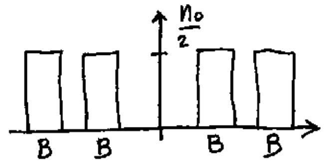
Then,

$$N_i = (2B)\left(\frac{n_o}{2}\right) = n_o B = 6\times10^{-4}\text{ W}$$

$$S_i = (SNR)N_i = 0.19\text{ W} = \underline{22.8\text{ dBm}} \checkmark$$

$$N_o = \frac{N_i}{4} = \frac{1}{4}(6\times10^{-4}) = 1.5\times10^{-4}\text{ W} = \underline{-8.2\text{ dBm}} \checkmark$$

For DSB-SC, $SNR = \frac{S_o}{N_o} = 2\frac{S_i}{N_i}$



Then,

$$N_i = 2n_o f_m = 1.2\times10^{-3}\text{ W} = \underline{0.79\text{ dBm}} \checkmark$$

$$S_i = \tfrac{1}{2}(SNR)N_i = -3\text{dB} + 25\text{dB} + 0.79\text{dBm}$$

$$= \underline{22.8\text{dBm}} \checkmark$$

$$N_o = \frac{N_i}{4} = 3\times10^{-4}\text{W} = \underline{-5.2\text{dBm}} \checkmark$$

9.2 For DSB-SC the received voltage is,

$$v_i(t) = \cos(\omega_{IF} - \omega_m)t + \cos(\omega_{IF}+\omega_m)t = 2\cos\omega_{IF}t\cos\omega_m t$$

After mixing with LO,

$$v_o(t) = \tfrac{1}{2}\cos[(2\omega_{IF} - \omega_m)t + \Delta\phi] + \tfrac{1}{2}\cos(\omega_m t + \Delta\phi)$$
$$+\tfrac{1}{2}\cos[(2\omega_{IF}+\omega_m)t + \Delta\phi] + \tfrac{1}{2}\cos(\omega_m t - \Delta\phi)$$

after LP filtering:

$$v_o(t)\big|_{LPF} = \tfrac{1}{2}\cos(\omega_m t + \Delta\phi) + \tfrac{1}{2}\cos(\omega_m t - \Delta\phi)$$
$$= \cos\omega_m t\cos\Delta\phi$$

Thus there is no distortion, but the amplitude is reduced, depending on the value of $\cos\Delta\phi$.

$\boxed{9.3}$ after mixing with LO:

$$v_o(t) = \tfrac{1}{2}\cos(2\omega_{IF} - \omega_m + \Delta\omega)t + \tfrac{1}{2}\cos(\omega_m + \Delta\omega)t$$
$$+ \tfrac{1}{2}\cos(2\omega_{IF} + \omega_m + \Delta\omega)t + \tfrac{1}{2}\cos(\omega_m - \Delta\omega)t$$

after LP filtering:

$$v_o(t)\Big|_{LPF} = \tfrac{1}{2}\cos(\omega_m + \Delta\omega)t + \tfrac{1}{2}\cos(\omega_m - \Delta\omega)t$$
$$= \cos\omega_m t \cos\Delta\omega t$$

This error will distort the signal by imposing an amplitude modulation.

$\boxed{9.4}$ $P_i = 30\text{ kW}$, $m = 0.7$

$$v(t) = A\left[1 + m\cos\omega_m t\right]\cos\omega_c t$$

$$P_i = \frac{A^2}{2} + \frac{m^2A^2}{4} = P_c + P_s$$

$$P_c = \frac{A^2}{2} = \frac{P_i}{1 + m^2/2} = \frac{30\text{ kW}}{1 + (0.7)^2/2} = \underline{24.1\text{ kW}} \checkmark$$

$$P_s = \frac{m^2A^2}{4} = \frac{m^2}{2}P_c = \frac{m^2}{2+m^2}P_i$$

$$= \frac{(0.7)^2(30\text{ kW})}{2 + (0.7)^2} = \underline{5.9\text{ kW}} \checkmark$$

$\boxed{9.5}$ The diode conducts only when $v_i(t) > 0$. The RC time constant is,

$$T = RC = 1\,\text{mS}.$$

For $0 \le t \le 1\,\text{mS}$, the output voltage is,

$$v_o(t) = 1 - e^{-t/T} \quad \text{V} \qquad (0 \le t \le 1\,\text{mS})$$

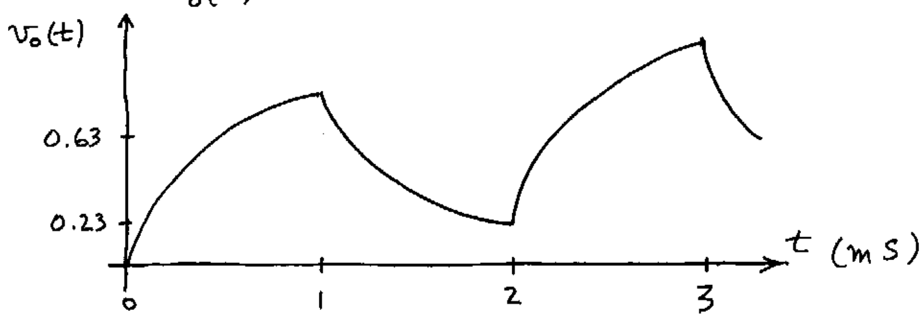
$$v_o(0) = 0$$

$$v_o(1) = 0.63 \text{ V}$$

For $1 \le t \le 2$ mS, the capacitor voltage discharges through the resistor:

$$v_o(t) = v_o(1)\, e^{-t/T} \qquad (1 \le t \le 2\,\text{mS})$$

$$v_o(2) = 0.23 \quad \text{V}$$



$\boxed{9.6}$ $v_i(t) = A[1+m(t)]\cos\omega_{IF}t + n(t)$.

after mixing with $\cos\omega_{IF}t$:

$$v_1(t) = A[1+m(t)]\cos^2\omega_{IF}t + x(t)\cos^2\omega_{IF}t - y(t)\cos\omega_{IF}t\sin\omega_{IF}t$$

after LPF:

$$v_o(t) = \frac{A}{2}[1+m(t)] + \frac{1}{2}x(t)$$

So,

$$S_i = \frac{A^2}{2} + \frac{m^2A^2}{4}$$

$$S_o = E\left\{\left[\frac{A}{2}m(t)\right]^2\right\} = A^2\frac{m^2}{8}$$

$$N_o = E\left\{\left[\frac{1}{2}x(t)\right]^2\right\} = \frac{N_i}{4}$$

Then the output SNR is,

$$\frac{S_o}{N_o} = \frac{A^2m^2}{8}\,\frac{4}{N_i}\,\frac{S_i}{\left(\frac{A^2}{2}+\frac{m^2A^2}{4}\right)}$$

$$= \frac{S_i}{N_i}\,\frac{2m^2}{2+m^2} \checkmark \qquad \text{(agrees with 9.28)}$$

$\boxed{9.7}$

$$v_{LO}(t) = \cos[(\omega + \Delta\omega)t + \Delta\phi]$$

a) ASK

$$v_i(t) = m(t)\cos\omega t \qquad m(t) = 0, 1$$

$$v_o(t) = v_i(t)v_{LO}(t)\Big|_{LPF} = \tfrac{1}{2}m(t)\cos(\Delta\omega t + \Delta\phi)$$

b) FSK

$$v_i(t) = \cos\omega t \qquad \omega = \omega_1 \text{ or } \omega_2.$$

For $\omega = \omega_1$: $v_o(t) = \cos[(\omega_1 + \Delta\omega)t + \Delta\phi]\cos\omega_1 t\Big|_{LPF}$

$$= \tfrac{1}{2}\cos(\Delta\omega t + \Delta\phi)$$

For $\omega = \omega_2$: $v_o(t) = -\cos[(\omega_2 + \Delta\omega)t + \Delta\phi]\cos\omega_2 t\Big|_{LPF}$

$$= -\tfrac{1}{2}\cos(\Delta\omega t + \Delta\phi)$$

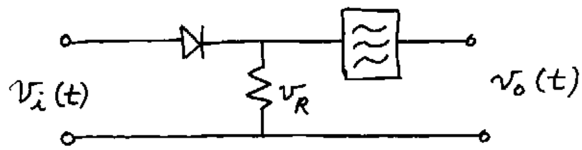
c) PSK

$$v_i(t) = m(t)\cos\omega t \qquad m(t) = -1, 1$$

$$v_o(t) = m(t)\cos\omega t\cos[(\omega + \Delta\omega)t + \Delta\phi]\Big|_{LPF}$$

$$= \tfrac{1}{2}m(t)\cos(\Delta\omega t + \Delta\phi)$$

$\boxed{9.8}$



For DSB-LC, $v_i(t) = A[1 + m(t)]\cos\omega_{IF}t$

Assuming $v_R = C v_i^2$, (SQUARE-LAW)

$$v_R(t) = CA^2[1 + 2m(t) + m^2(t)]\cos^2\omega_{IF}t$$

after LPF: $v_o(t) = \frac{CA^2}{2}[1 + 2m(t) + m^2(t)] \neq m(t).$

Distortion arises due to the $m^2(t)$ term.

$\boxed{9.9}$ $v(t) = 30\cos(2\pi f_{IF} + \beta\cos 2\pi f_m t)$

$f_{IF} = 90\ MHz,\ \beta = 5,\ f_m = 20\,kHz,\ Z_L = 50\Omega$

a) $\Delta f = \beta f_m = 5(20) = 100\ kHz$ $\beta = \Delta\omega/\omega_m$

b) $P_{TOT} = \frac{1}{2}(30)^2/50 = 9\ W$

c) $P_{90MHz} = \frac{1}{2}(30)^2(0.178)^2/50 = 0.285\ W$ $|J_0(5)| = 0.178$

d) $B = 2\omega_m(1+\beta) = 4\pi\ (20\,kHz)(6) = 1.508\ MHz$

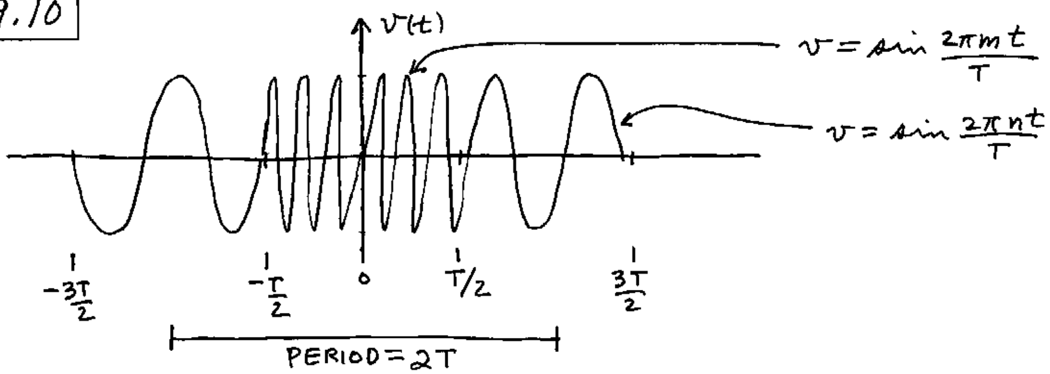
e) $P_B = 99.4\%$

n	$J_n(5)$
0	0.178
1	0.328
2	0.047
3	0.365
4	0.391
5	0.261
6	0.131

REQUIRE ± 6 SIDEBANDS

$$P = J_0^2(\beta) + 2\sum_{n=1}^{6} J_n^2(\beta) = 0.994$$

$\boxed{9.10}$



This is an odd function of t as defined above, so we can use a Fourier sine series:

$$v(t) = \sum_{k=1}^{\infty} V_k \sin\frac{\pi k t}{T},$$

9.10 CONTINUED.

$$V_k = \frac{1}{2T}\int_0^{2T} v(t)\sin\frac{\pi k t}{T}\,dt$$

$$= \frac{1}{2T}\int_{-T/2}^{T/2} \sin\frac{2\pi m t}{T}\sin\frac{\pi k t}{T}\,dt + \frac{1}{2T}\int_{T/2}^{3T/2}\sin\frac{2\pi n t}{T}\sin\frac{\pi k t}{T}\,dt$$

$$= \frac{1}{2T}\left[\frac{\sin\pi(2m-k)t/T}{2\pi(2m-k)/T} - \frac{\sin\pi(2m+k)t/T}{2\pi(2m+k)/T}\right]\Bigg|_{-T/2}^{T/2}$$

$$+ \frac{1}{2T}\left[\frac{\sin\pi(2n-k)t/T}{2\pi(2n-k)/T} - \frac{\sin\pi(2n+k)t/T}{2\pi(2n+k)/T}\right]\Bigg|_{T/2}^{3T/2}$$

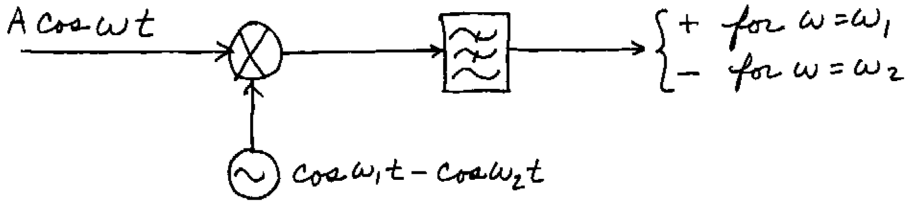
$$= \frac{1}{T}\left[\frac{\sin\frac{\pi}{2}(2m-k)}{2\pi(2m-k)/T} - \frac{\sin\frac{\pi}{2}(2m+k)}{2\pi(2m+k)/T}\right]$$

$$+ \frac{1}{T}\left[\frac{\sin\frac{3\pi}{2}(2n-k)}{2\pi(2n-k)/T} - \frac{\sin\frac{\pi}{2}(2n-k)}{2\pi(2n-k)/T} - \frac{\sin\frac{3\pi}{2}(2n+k)}{2\pi(2n+k)/T} + \right.$$

$$\left. + \frac{\sin\frac{\pi}{2}(2n+k)}{2\pi(2n+k)/T}\right]$$

The result does not seem to simplify much further.

$\boxed{9.11}$



after the LPF, the output is,

$$v_o(t) = \frac{A}{2}\cos(\omega-\omega_1)t - \frac{A}{2}\cos(\omega-\omega_2)t$$

If $\omega=\omega_1$, $v_o(t) = \frac{A}{2}$ ✓

If $\omega=\omega_2$, $v_o(t) = -\frac{A}{2}$ ✓

$\boxed{9.12}$

ASK $\quad s(t) = s_1(t) = V \;;\; s_o(T) = VT \;;\; v_o(T) = VT + n_o(T)$

$$P_e^{(1)} = P\{VT + n_o(T) < VT/2\} = P\{n_o(T) < -VT/2\}$$

$$= \int_{-\infty}^{-VT/2} \frac{e^{-n_o^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn_o = \int_{VT/2}^{\infty} \frac{e^{-n_o^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn_o = \frac{1}{\sqrt{\pi}}\int_{\frac{VT}{2\sqrt{2}\sigma}}^{\infty} e^{-x^2}dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{VT}{2\sqrt{2}\sigma}\right) \checkmark$$

PSK $\quad s(t) = s_1(t) = V \;;\; s_o(T) = VT \;;\; v_o(T) = VT + n_o(T)$

$$P_e^{(1)} = P\{VT + n_o(T) < 0\} = P\{n_o(T) < -VT\}$$

$$= \int_{-\infty}^{-VT} \frac{e^{-n_o^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn_o = \int_{VT}^{\infty} \frac{e^{-n_o^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn_o = \frac{1}{\sqrt{\pi}}\int_{\frac{VT}{\sqrt{2}\sigma}}^{\infty} e^{-x^2}dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{VT}{\sqrt{2}\sigma}\right) \checkmark$$

FSK $\quad s(t) = s_1(t) = V \;;\; s_o(T) = VT \;;\; v_o(T) = VT + n(T)$,
where $n(T) = n_1(T) - n_2(T)$. Variance of n is $2\sigma^2$.

$$P_e^{(1)} = P\{VT + n(T) < 0\} = P\{n(T) < -VT\}$$

$$= \int_{-\infty}^{-VT} \frac{e^{-n^2/4\sigma^2}}{\sqrt{4\pi\sigma^2}}\,dn = \int_{VT}^{\infty} \frac{e^{-n^2/4\sigma^2}}{\sqrt{4\pi\sigma^2}}\,dn = \frac{1}{\sqrt{\pi}}\int_{\frac{VT}{2\sigma}}^{\infty} e^{-x^2}dx$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{VT}{2\sigma}\right) \checkmark$$

as before, $\quad \dfrac{VT}{2\sigma} = \sqrt{\dfrac{E}{2\eta_o}}$.

$\boxed{9.13}$

ASK: $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/4n_o}\right)$

FSK: $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/2n_o}\right)$

PSK: $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/n_o}\right)$

P_e	$2P_e$	$\mathrm{erfc}^{-1}(2P_e)$	E_b/n_o (dB) ASK	FSK	PSK	
10^{-2}	2×10^{-2}	1.645	10.3	7.3	4.3	✓
10^{-8}	2×10^{-8}	3.968	18.0	15.0	12.0	✓

$\boxed{9.14}$ For coherent FSK, $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/2n_o}\right)$

$E_b = PT_b = (10^{-6})\left(\frac{1}{5\times10^{6}}\right) = 2\times10^{-13}$ W-sec.

$\frac{n_o}{2} = 10^{-14} \Rightarrow n_o = 2\times10^{-14}$ W/Hz $\quad (E_b/n_o = 10\,dB)$

So $\sqrt{E_b/2n_o} = 2.236$

$P_e = \frac{1}{2}\,\mathrm{erfc}(2.236) = \frac{1}{2}(1.56\times10^{-3}) = \underline{7.8\times10^{-4}}$ ✓

For noncoherent FSK, $P_e = \frac{1}{2}e^{-E_b/2n_o} = \underline{3.4\times10^{-3}}$ ✓

$\boxed{9.15}$ (the data for this problem are from the text by M. Schwartz)

$G_t = 27.6\,dB = 575.4$

$G_r = 61.3\,dB = 1.36\times10^{6}$ $\qquad \lambda = \frac{c}{f} = 0.1307\,m$

$E_b/n_o = 1.4\,dB = 1.38$

$R = 1.6\times10^{8}\,km = 1.6\times10^{11}\,m$

$n_o = kT_{sys} = (1.38\times10^{-23})(13.5) = 1.86\times10^{-22}$

$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} = 5.56\times10^{-17}$ W ✓

$\frac{E_b}{n_o} = \frac{P_r}{n_o R_b} \Rightarrow R_b = \frac{P_r}{n_o}\frac{n_o}{E_b} = \frac{5.56\times10^{-17}}{(1.38)(1.86\times10^{-22})}$

$= 2.17\times10^{5}$ bps $= \underline{217\text{ kbps}}$ ✓

$\boxed{9.16}$ let $s(t) = s_2(t) = -V\cos\omega t$ "0"

Then $s_0(T) = -\frac{V^2T}{2} = -E_b$

$$v_0(T) = s_0(T) + n_0(T)$$

$$n_0(T) = \int_0^T n(t)\, V\cos\omega t\, dt$$

$$\sigma^2 = N_0 = E\{n_0^2(T)\} = E\left\{\int_0^T\int_0^T n(t)n(s)V^2\cos\omega t\cos\omega s\, dt\, ds\right\}$$

$$= \int_0^T\int_0^T \frac{n_0}{2}\,\delta(t-s)V^2\cos\omega_0 t\cos\omega_0 s\, dt\, ds = \frac{V^2 n_0 T}{4} = \frac{n_0 E_b}{2}$$

$$P_e^{(0)} = P\{v_0 > 0\} = P\{n_0 > E\} = \int_E^\infty \frac{e^{-n_0^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\, dn_0 = \frac{1}{\sqrt{\pi}}\int_{E/\sqrt{2}\sigma}^\infty e^{-x^2}\, dx$$

$$\frac{E_b}{\sqrt{2}\,\sigma} = \frac{E_b\sqrt{2}}{\sqrt{2}\sqrt{n_0}\sqrt{E_b}} = \sqrt{\frac{E_b}{n_0}}$$

Thus, $P_e^{(0)} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{E_b}{\sqrt{2}\,\sigma}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{n_0}}\right)$ ✓

$\boxed{9.17}$ $$\mathrm{erfc}(x) \simeq \frac{e^{-x^2}}{\sqrt{\pi}\, x}$$

<u>ASK</u> $$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/4n_0}\right) \simeq \frac{e^{-E_b/4n_0}}{2\sqrt{\pi}\sqrt{E_b/4n_0}}$$

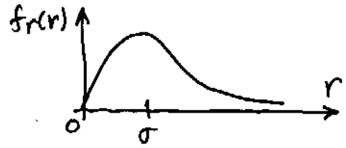
<u>FSK</u> $$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/2n_0}\right) \simeq \frac{e^{-E_b/2n_0}}{2\sqrt{\pi}\sqrt{E_b/2n_0}}$$

<u>PSK</u> $$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{E_b/n_0}\right) \simeq \frac{e^{-E_b/n_0}}{2\sqrt{\pi}\sqrt{E_b/n_0}}$$

E_b/n_0 (dB)	E_b/n_0	ASK		FSK		PSK	
		EXACT	L.A.	EXACT	L.A.	EXACT	L.A.
5	3.16	0.105	0.14	0.037	0.046	0.006	0.0067
15	31.6	3.5×10^{-5}	3.7×10^{-5}	9.4×10^{-9}	9.8×10^{-9}	9.4×10^{-16}	9.5×10^{-16}

(L.A. = large argument)

$\boxed{9.18}$ $$f_r(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad 0 \le r < \infty \qquad \text{(RAYLEIGH PDF)}$$



$$\int_{r=0}^{\infty} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} dr = -e^{-r^2/2\sigma^2}\Big|_0^{\infty} = 1 \checkmark$$

$$\frac{d f_r(r)}{dr} = \frac{e^{-r^2/2\sigma^2}}{\sigma^2} + \left(\frac{r}{\sigma^2}\right)\left(\frac{-r}{\sigma^2}\right) e^{-r^2/2\sigma^2} = 0$$

$$1 - r^2/\sigma^2 = 0 \Rightarrow r = \sigma \text{ at maximum of } f_r \checkmark$$

$\boxed{9.19}$ $$f_r(r) = \frac{r}{\sigma^2} e^{-(V^2+r^2)/2\sigma^2} I_0\left(\frac{Vr}{\sigma^2}\right), \quad 0 \le r < \infty \qquad \text{(RICIAN PDF)}$$

From Appendix B,

$$I_0(x) \simeq \frac{e^x}{\sqrt{2\pi x}} \quad \text{for large } x.$$

So,

$$f_r(r) \simeq \frac{r}{\sigma^2} e^{-(V^2+r^2)/2\sigma^2} \sqrt{\frac{\sigma^2}{2\pi V r}}\, e^{Vr/\sigma^2} \quad \text{for large } Vr.$$

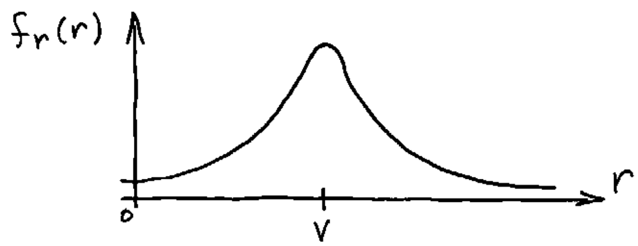
$$\simeq \sqrt{\frac{r}{2\pi V \sigma^2}}\, e^{-(V^2-2rV+r^2)/2\sigma^2}$$

$$\simeq \sqrt{\frac{r}{2\pi V \sigma^2}}\, e^{-(V-r)^2/2\sigma^2}$$

Since the peak of $f_r(r)$ occurs for $r \sim V$, we can simplify to,

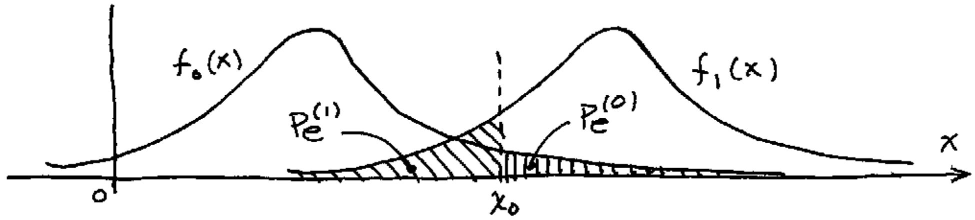
$$f_r(r) \simeq \sqrt{\frac{1}{2\pi\sigma^2}}\, e^{-(V-r)^2/2\sigma^2},$$

which is a gaussian distribution



$\boxed{9.20}$

$$P_e = P_0 \int_{x_0}^{\infty} f_0(x)\,dx + P_1 \int_{-\infty}^{x_0} f_1(x)\,dx$$

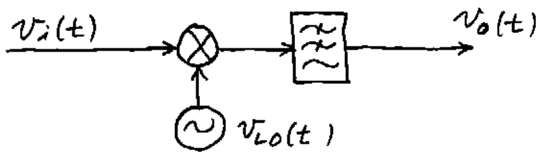


$$\frac{\partial P_e}{\partial x_0} = -P_0 f_0(x_0) + P_1 f_1(x_0) = 0 \quad , \text{ since } \frac{\partial}{\partial x}\int_a^x f(y)\,dy = f(x).$$

Thus, $\quad P_0 f_0(x_0) = P_1 f_1(x_0)$.

If $P_0 = P_1 = 1/2$, then optimum x_0 satisfies $f_0(x_0) = f_1(x_0)$.

$\boxed{9.21}$ $\quad v_{LO}(t) = \cos(\omega_0 t + \phi) \quad$ – PSK



a) $v_i(t) = \pm V \cos \omega_0 t + n(t)$

After LPF, $\quad v_0(t) = \pm VT\cos\phi + n_0$

$$P_e^{(0)} = P\{n_0(T) > VT\cos\phi\} = \int_{VT\cos\phi}^{\infty} \frac{e^{-n_0^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dn_0 = \frac{1}{2}\,\mathrm{erfc}\left(\frac{VT\cos\phi}{\sqrt{2}\,\sigma}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{n_0}}\cos\phi\right) \qquad \text{where } E = V^2T,\ \sigma^2 = \frac{n_0 T}{2}.$$

CHECK: if $\phi = 90°$, $P_e^{(0)} = \frac{1}{2}\mathrm{erfc}(0) = 1/2$ ✓

if $\phi = 270°$, $P_e^{(0)} = \frac{1}{2}\mathrm{erfc}(0) = 1/2$ ✓

if $\phi = 180°$, $P_e^{(0)} = \frac{1}{2}\mathrm{erfc}\left(-\sqrt{\frac{E}{n_0}}\right) = \frac{1}{2}\left[2 - \mathrm{erfc}\left(\sqrt{\frac{E}{n_0}}\right)\right]$

$= 1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E}{n_0}}\right)$ ✓ (complement of $\phi = 0$ case)

In the above we used the identity that $\mathrm{erfc}(-x) = 2 - \mathrm{erfc}(x)$.

9.21 CONTINUED.

$$P_e = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)d\phi = \frac{1}{2\pi}\int_0^{\pi}\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)d\phi \quad \text{(SYMMETRY)}$$

$$= \frac{1}{2\pi}\int_0^{\pi/2}\underbrace{\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)}_{(\cos\phi>0)}d\phi + \frac{1}{2\pi}\int_{\pi/2}^{\pi}\underbrace{\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)}_{(\cos\phi<0)}d\phi$$

let $\theta = \pi - \phi$
$d\theta = -d\phi$
$\cos\phi = \cos(\pi-\theta) = -\cos\theta$

$$= \frac{1}{2\pi}\int_0^{\pi/2}\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)d\phi + \frac{1}{2\pi}\int_0^{\pi/2}\mathrm{erfc}\left(-\sqrt{\frac{E}{\eta_0}}\cos\theta\right)d\theta$$

$$= \cancel{\frac{1}{2\pi}\int_0^{\pi/2}\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\phi\right)d\phi} + \frac{1}{2\pi}\int_0^{\pi/2}2\,d\theta - \cancel{\frac{1}{2\pi}\int_0^{\pi/2}\mathrm{erfc}\left(\sqrt{\frac{E}{\eta_0}}\cos\theta\right)d\theta}$$

$$= \frac{1}{2\pi}(2)\left(\frac{\pi}{2}\right) = \frac{1}{2} \quad \checkmark$$

$\boxed{9.22}$ RAYLEIGH FADED ASK:

From (9.76), $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{4\eta_0}}\right)$ for coherent ASK (non-faded).

Then the P_e for Rayleigh faded ASK is,

$$P_e = \int_{r=0}^{\infty}P_e(E_b|r)f_r(r)\,dr = \frac{1}{2}\int_{r=0}^{\infty}\mathrm{erfc}\left(\sqrt{\frac{r^2E_b}{4\eta_0}}\right)\frac{r}{\alpha^2}e^{-r^2/2\alpha^2}dr$$

This integral has the same form as (9.108), except that E_b is replaced by $E_b/4$. If we define $\Gamma = \frac{2\alpha^2E_b}{\eta_0}$ as before, then we can replace Γ in (9.112) with $\Gamma/4$:

$$P_e = \frac{1}{2}\left[1-\sqrt{\frac{\Gamma/4}{1+\Gamma/4}}\right] = \frac{1}{2}\left[1-\sqrt{\frac{\Gamma}{4+\Gamma}}\right]$$

RAYLEIGH FADED FSK:

From (9.81), $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2\eta_0}}\right)$ for coherent FSK (NON-FADED).

Then the P_e for Rayleigh fading FSK is,

$$P_e = \int_{r=0}^{\infty}P_e(E_b|r)f_r(r)\,dr = \frac{1}{2}\int_{r=0}^{\infty}\mathrm{erfc}\left(\sqrt{\frac{r^2E_b}{2\eta_0}}\right)\frac{r}{\alpha^2}e^{-r^2/2\alpha^2}dr$$

The above integral has the same form as (9.108) except that E_b is replaced by $E_b/2$. Define $\Gamma = \frac{2\alpha^2 E_b}{n_0}$ as before, then we can replace Γ in (9.112) with $\Gamma/2$:

$$P_e = \tfrac{1}{2}\left[1 - \sqrt{\frac{\Gamma/2}{1+\Gamma/2}}\right] = \tfrac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{2+\Gamma}}\right] \checkmark$$

P_e	FADED FSK Γ(dB)	NON-FADED FSK E_b/n_0(dB)
10^{-2}	16.9 ✓	7.3
10^{-5}	47.0 ✓	12.6
10^{-8}	77.0 ✓	15.0

$\boxed{9.23}$ For non-faded BPSK,

$$P_e = \tfrac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{n_0}}\right)$$

For $P_e = 10^{-2}, 10^{-5}, 10^{-8}$ solve for E_b/n_0 by trial-and-error.

For Rayleigh faded BPSK,

$$P_e = \tfrac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right], \quad \Gamma = \frac{2\alpha^2 E_b}{n_0}.$$

This can be solved directly for Γ:

$$\Gamma = \frac{(1-2P_e)^2}{1-(1-2P_e)^2}$$

P_e	FADED Γ(dB)	NON-FADED E_b/n_0 (dB)
10^{-2}	13.8 ✓	4.3
10^{-5}	44.0 ✓	9.6 ✓
10^{-8}	74.0 ✓	12.0 ✓

Note the 34dB increase in Γ for 10^{-5} for the faded case!

$\boxed{9.24}$

$$T_{sys} = T_A + (F-1)T_o = 200 + (5.01-1)(290) = 1363\,K \checkmark$$

$$N_o = kT_{sys}B = (1.38\times10^{-23})(1363)(30\times10^{3}) = 5.6\times10^{-16}\,W$$

For $P_e = 10^{-5}$, $\frac{E_b}{n_o} = 9.6\ dB = 9.12$ ✓

$$\frac{S_o}{N_o} = \frac{f_s}{B}\frac{E_b}{n_o} = \frac{23.3}{30}(9.12) = 7.1 = 8.5\ dB$$

$$S_o = \left(\frac{S_o}{N_o}\right)N_o = (7.1)(5.6\times10^{-16}) = 4.0\times10^{-15}$$

$$R^4 = \frac{P_t G_r G_t h_1^2 h_2^2}{S_o} = \frac{(30)(0.79)(40)^2(1.5)^2}{4\times10^{-15}} = 2.13\times10^{19}$$

$$\underline{R = 68\ km}$$

$\boxed{9.25}$ For alpha Centuri, $R = 4.35$ light yrs $= 4.1\times10^{16}\,m$

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi R)^2} = \frac{(10^3)(10^6)^2(0.15)^2}{(4\pi)^2(4.1\times10^{16})^2} = 8.5\times10^{-23}\ W \checkmark$$

For FSK with $P_e = 10^{-5}$, require $\frac{E_b}{n_o} = 12.6\,dB = 18.2$ ✓

$$\frac{E_b}{n_o} = \frac{P_r}{n_o R_b} = 18.2$$

$$n_o = kT = (4)(1.38\times10^{-23}) = 5.52\times10^{-23} \checkmark$$

$$R_b = \frac{P_r}{(18.2)n_o} = \frac{8.5\times10^{-23}}{(18.2)(5.52\times10^{-23})} = \underline{0.085\ bps} \checkmark$$

$\boxed{9.26}$ From (9.128), $P_e = \frac{1}{n}\, \text{erfc}\left(\sqrt{\frac{E_s}{N_0}\sin^2\frac{\pi}{M}}\right)$

for Gray-coded M-PSK, with $E_s = nE_b$, and $M = 2^n$ for $\underline{\underline{M>2}}$. For $P_e = 10^{-5}$,

USE $P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$ FOR $M=2$. $\longrightarrow$

M	n	$nP_e = \text{erfc}(x)$	x	E_b/N_0 (dB)		
2	1	2×10^{-5}	3.015	9.6	(BPSK)	✓
4	2	2×10^{-5}	3.015	9.6	(QPSK)	✓
8	3	3×10^{-5}	2.951	13.0	(8-PSK)	✓
16	4	4×10^{-5}	2.904	17.4	(16-PSK)	
32	5	5×10^{-5}	2.868	22.3	(32-PSK)	

$\left(x = \sqrt{\frac{nE_b}{N_0}\sin^2\frac{\pi}{M}}\right)$

$\boxed{9.27}$ $f = 600\,\text{Hz}$ to $3000\,\text{Hz}$; $S/N = 30\,\text{dB}$.

$$C = B\log_2\left(1 + \frac{S}{N_0 B}\right)$$

$B = 2400\,\text{Hz}$

$\frac{S}{N_0 B} = 30\,\text{dB} = 1000.$

Thus $C = 2400\log_2(1001) \simeq 24$ kbps.

(Current modems use data compression to allow an *effective* data rate greater than this value.)

$\boxed{9.28}$ $B = 30\,\text{kHz}$, $S = -60\,\text{dBm} = 10^{-9}\,\text{W}$, $\frac{\eta_0}{2} = 10^{-18}\,\text{W/Hz}$.

$$\frac{S}{\eta_0 B} = \frac{10^{-9}}{(2\times 10^{-18})(30\times 10^{3})} = 1.7\times 10^{4} = 42\,\text{dB}.$$

Thus,
$$C = B\log_2\left(1 + \frac{S}{\eta_0 B}\right) = 30{,}000\log_2(1.7\times 10^{4}).$$
$$\simeq 30{,}000(14) = \underline{420\,\text{kbps}}.$$

Chapter 10

$\boxed{10.1}$

$F = 8\,dB = 6.3$, $B = 50\,kHz$, $T_A = 1000K$, $SNR_{MIN} = 20\,dB = 100$.

$$S_{i_{MIN}} = kB\left[T_A + (F-1)T_0\right]\left(\frac{S_o}{N_o}\right)_{MIN}$$

$$= (1.38\times10^{-23})(50\times10^3)\left[1000 + (6.3-1)\right](290)(100)$$

$$= 1.75\times10^{-13}\ W = 1.75\times10^{-10}\ mW = -97.6\ dBm.$$

$$DR = -20\,dBm + 97.6\,dBm = \underline{77.6\,dB}$$

$\boxed{10.2}$

$P_t = 100\,mW$, $G_t = 3\,dB$, $G_r = 1\,dB$, $T_b = 100K$, $\eta = 70\%$, $f = 900\,MHz$.
$R_b = 1.6\,Mbps$, $P_e = 10^{-5}$, $F_{REC} = 12\,dB$.

$$T_A = \eta T_b + (1-\eta)T_0$$

$$= (0.7)(100) + (1-0.7)(290) = 157K.$$

$$\left(\frac{S}{N}\right) = \frac{E_b}{n_o}\frac{R_b}{B}, \quad \frac{E_b}{n_o} = 10, \quad F_{REC} = 15.8$$

$$S_{i_{MIN}} = kB\left[T_A + (F-1)T_0\right]\left(\frac{S}{N}\right) = k\left[T_A + (F-1)T_0\right]\left(\frac{E_b}{n_o}\right)R_b$$ (NOTE: INDEPENDENT OF B)

$$= (1.38\times10^{-23})\left[157 + (15.8-1)(290)\right](10)(1.6\times10^6)$$

$$= 9.8\times10^{-13}\ W$$

$$S_{i_{MIN}} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2}$$

$G_t = 2.0$

$G_r = 1.26$

$\lambda = \frac{300}{900} = 0.33\ m.$

$$R = \sqrt{\frac{P_t G_t G_r \lambda^2}{S_{i_{MIN}}(4\pi)^2}} = \frac{0.333}{4\pi}\sqrt{\frac{(0.1)(2)(1.26)}{(9.8\times10^{-13})}} = \underline{13{,}438\ M}$$

10.3

SINAD = 12 dB = 15.8 , SENSITIVITY = 3 μV (RMS, 50 Ω) , B = 30 kHz.

$$S_i = \frac{V^2}{Z_0} = \frac{(3\times10^{-6})^2}{50} = 1.8\times10^{-13}\,W = -97.4\,dBm \checkmark$$

$S+N = 15.8N \Rightarrow S = 14.8N$

$$F = \frac{S_i N_o}{S_o N_i} = \frac{(1.8\times10^{-13})}{kT_oB}\left(\frac{1}{14.8}\right) = \underline{20\,dB}$$

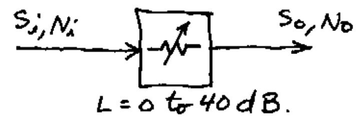
$N_i = kT_oB = 1.2\times10^{-16}$ W

10.4

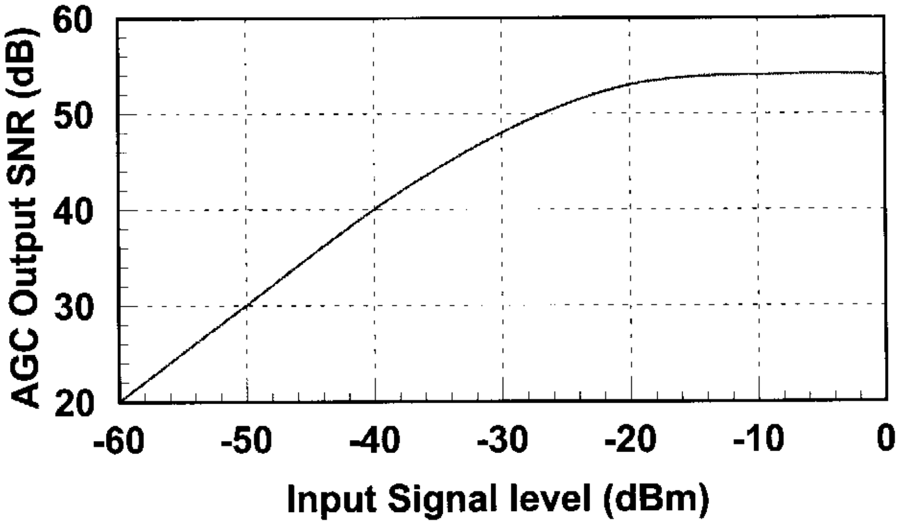
$S_i = -60$ dBm to -20 dBm , $S_o = -60$ dBm , $B = 1$ MHz.

$N_i = -80\,dBm = 1\times10^{-8}$ mW.

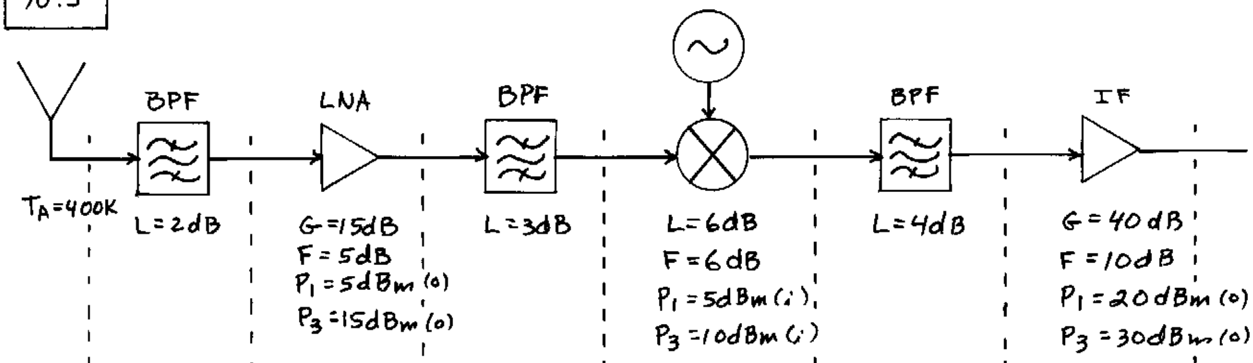
$$N_o = \frac{N_i}{L} + \frac{kT_oB(L-1)}{L}$$



S_i (dBm)	L (dB)	L	N_o (dBm)	S_o/N_o (dB)
−60	0	1	−80	20
−50	10	10	−90	30
−40	20	10^2	−100	40
−30	30	10^3	−108	48
−20	40	10^4	−113	53
−10	50	10^5	−114	54
0	60	10^6	−114	54



10.5



G		-2	13	10	4	0	40
F		2	7	7.04	7.29	7.75	11.8
P_3		100	100	95	10	6	20
S(dBm)	-90	-92	-77	-80	-86	-90	-50
S(dBm)	-30	-32	-17	-20	-26	-30	10

(a) overall noise figure is $F_{REC} = 11.8$ dB ✓ $= 15.1$

(b) if output SNR $= 12$ dB, and $T_A = 400$K, $B = 50$ kHz, $Z_0 = 50\,\Omega$, then,

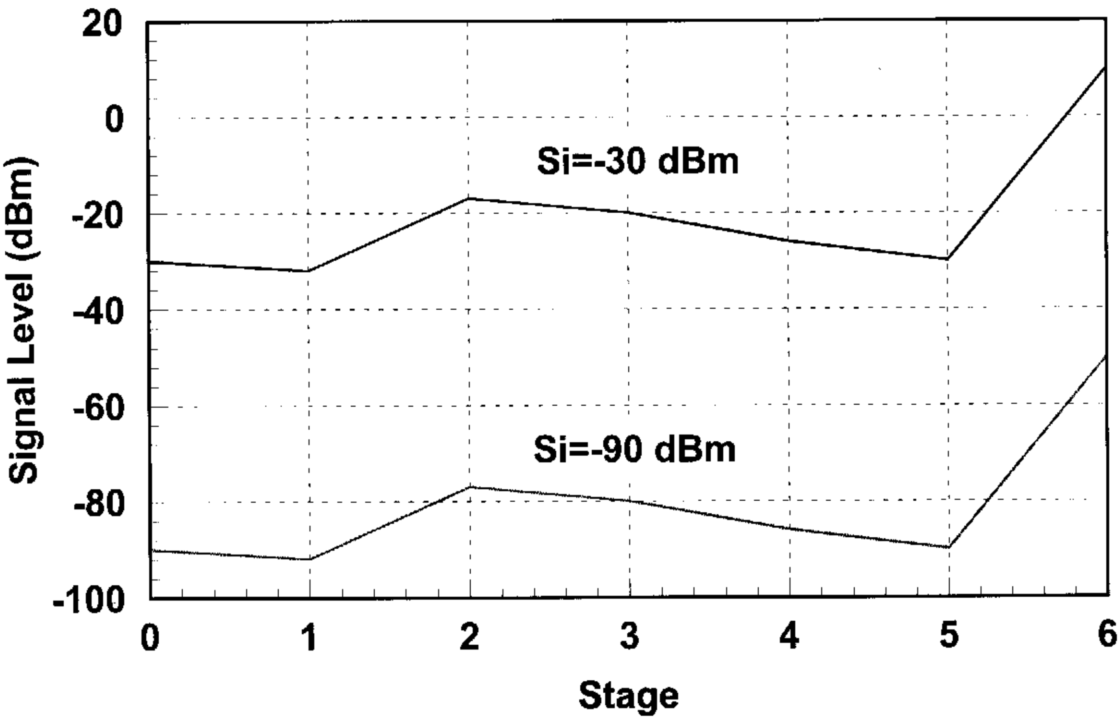
$$S_{i_{MIN}} = kB\left[T_A + (F-1)T_0\right]\left(\frac{S}{N}\right)_{MIN}$$
$$= (1.38\times10^{-23})(50\times10^{3})\left[400 + (15.1-1)(290)\right](15.8)$$
$$= 4.9\times10^{-14}\text{ W} = \underline{-103 \text{ dBm}} ✓$$

$$V_{i_{MIN}} = \sqrt{2Z_0 S_{i_{MIN}}} = \underline{2.2\,\mu\text{V}} ✓$$

(c) if $S_i = -90$ dBm, P_1 or P_3 is never exceeded
if $S_i = -30$ dBm, P_1 or P_3 is never exceeded.

See plot below:

10.5 CONTINUED.



$\boxed{10.6}$

$$SINAD = 1 + \frac{S_o}{N_o} \quad \text{(at output)}$$

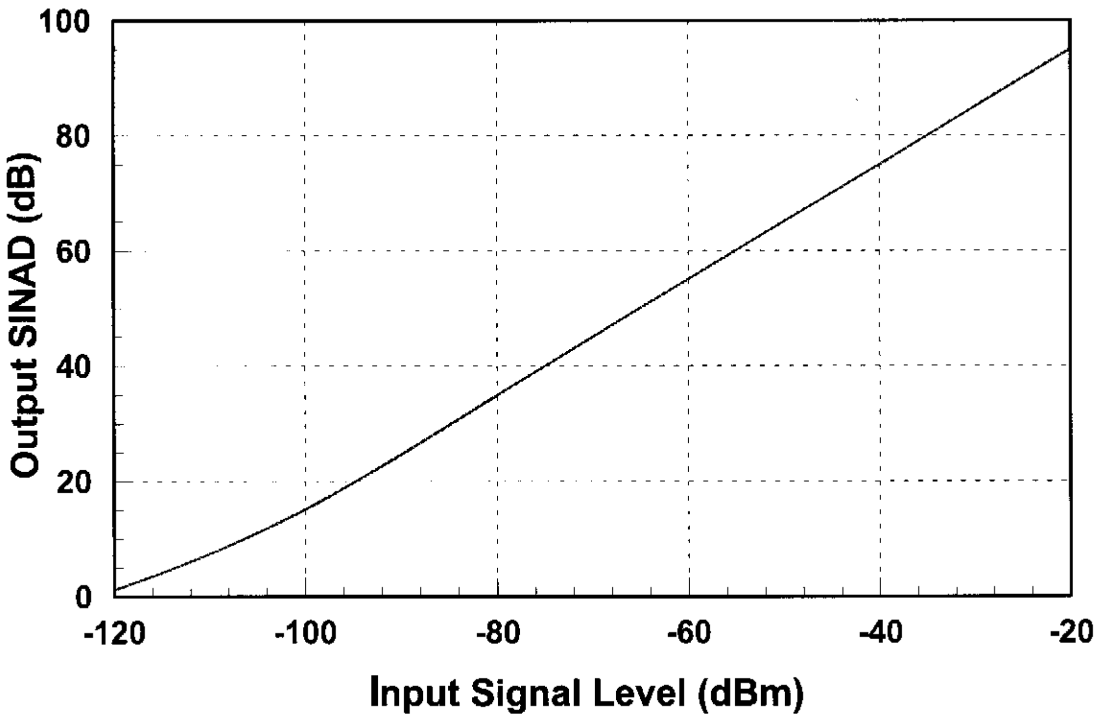
$$S_o(dB) = S_i(dB) + 40\,dB \quad \text{(overall gain} = 40\,dB)$$

$$N_o = kB[T_A + (F-1)T_0]G = 3.1 \times 10^{-11}\,W = \underline{-75\ dBm}$$

S_i (dBm)	SINAD (dB)
−120	1.2
−100	15.1
−80	35.0
−60	55.0
−40	75.0
−20	95.0

See graph below:

10.6 CONTINUED.



$\boxed{10.7}$

$$f_{IM} = f_{RF} - 2f_{IF}$$
$$= 880\text{ MHz} - 2(88)\text{ MHz}$$
$$= 704\text{ MHz} \quad (\text{if } f_{LO} = 792\text{ MHz}).$$

OR,

$$f_{IM} = f_{RF} + 2f_{IF}$$
$$= 880\text{ MHz} + 2(88)\text{ MHz}$$
$$= 1056\text{ MHz} \quad (\text{if } f_{LO} = 968\text{ MHz})$$

1) filtering
2) image rejection mixer
3) change IF.

$\boxed{10.8}$ $f = 900\,MHz,\ 910\,MHz,\ 920\,MHz.$

$BW = 1\,MHz,\quad f_{IF} = 10\,MHz.$

a) $f_{LO} = f_{RF} - f_{IF}$

$f_{IM} = f_{RF} - 2f_{IF}$

f (MHz)	f_{IM} (MHz)	RECV'D ?
900	880	NO
910	890	NO
920	900	YES

b) Say $f_{RF} = 900\,MHz.$

Then $f_{LO} = 900 - 10 = 890\,MHz.$

Spurs at $f = |m f_{RF} - n f_{LO}| = |900m - 890n|$

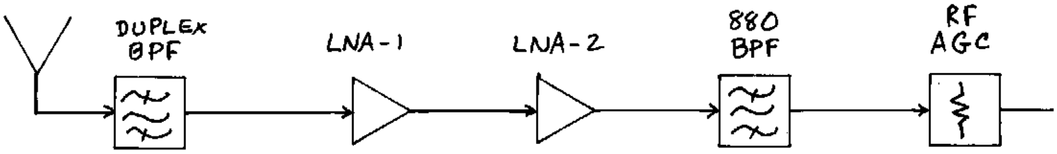
$= |890(m-n) + 10m|$

This is only within the passband for $m = n = 1$

m	n	900m − 890n.
1	1	10 ✓
1	2	−88 ×
2	1	910 ×
2	2	20 ×
3	1	1810 ×
3	2	920 ×
3	3	30 ×
1	3	−1770 ×
2	3	−870 ×

(data were verified with a short computer program)

$\boxed{10.9}$



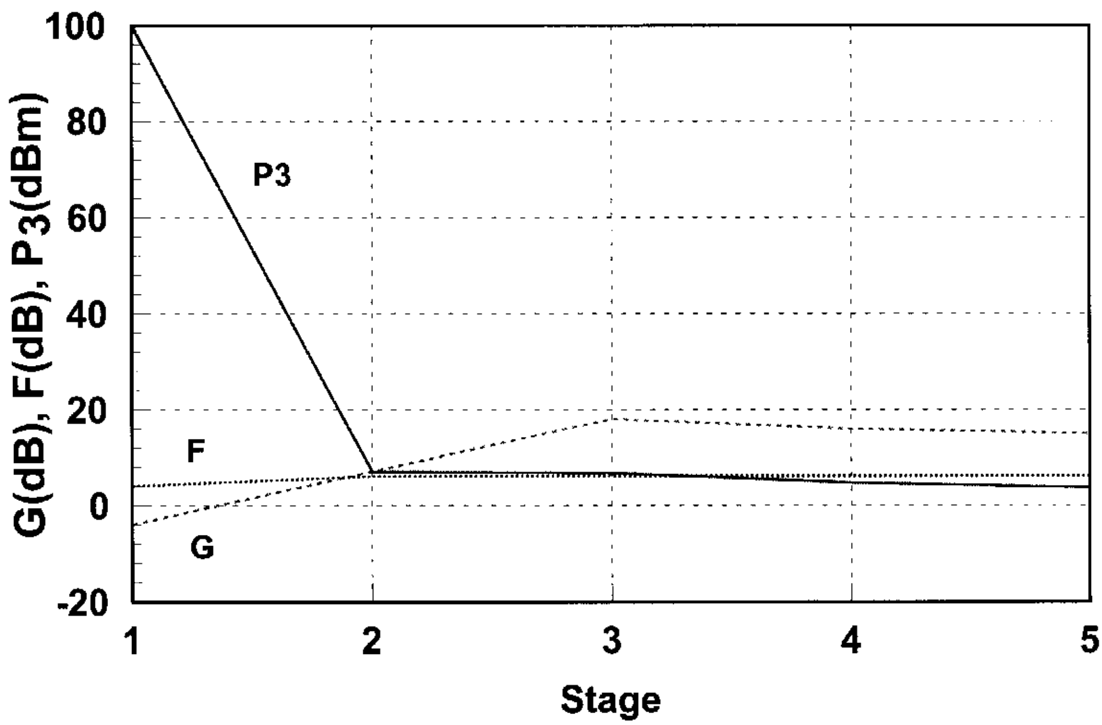
G(dB)	-4	11	11	-2	-1
F (dB)	4	2	2	2	1
P_3(dBm)	100	7	7	100	35

$G_T = 15$ dB

$F_T = 6.1$ dB

$P_{3_T} = 3.7$ dBm.

The attached FORTRAN code was used to compute the cascade values, and plotted below. The noise figure is about the same as for the original configuration, but P_3 is lower by about 2.4 dB.

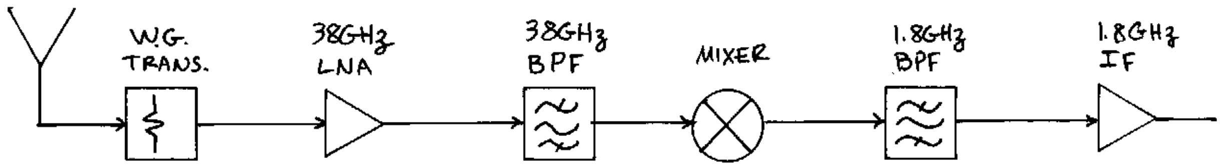


10.9 CONTINUED.

```
c compute cascade gain, noise figure and IP3 and write data file for plotting

      dimension g(30),f(30),p3(30)
      open(1,file='cascade.dat',status='unknown')
      write(6,5)
5     format('How many stages? ',$)
      read(5,*) n
c read in data
      do 10 i=1,n
      write(6,6) ,i
6     format('Enter G(dB), F(dB), IP3 (dBm) for stage #',i2,':',$)
      read(5,*) gdb,fdb,p3db
      g(i)=10.**(gdb/10.)
      f(i)=10.**(fdb/10.)
      p3(i)=10.**(p3db/10.)
10    continue
      write(6,*) 'Output G, F, and IP3 at each stage:'
c compute consecutive cascade results
      gt=1
      ft=1
      do 100 i=1,N
      gt=gt*g(i)
      ft=ft+g(i)*(f(i)-1.)/gt
          gi=1
          d=0
          do 200 j=1,i
          gi=gi*g(j)
200       d=d+gi/p3(j)
      pt=gt/d
      gtdb=10.*alog10(gt)
      ftdb=10.*alog10(ft)
      ptdb=10.*alog10(pt)
        write(6,*) i,gtdb,ftdb,ptdb
        write(1,*) i,gtdb,ftdb,ptdb
100   continue
      stop
      end
```

$\boxed{10.10}$



G (dB)	-1.0	20	-4.0	-7.0	0	13
F (dB)	1.0	3.5	4.0	7.0	0	2.5
P_3 (dBm)	100	15	100	10	100	25

The software code used in Problem 10.9 gives

$$G_T = \underline{21\text{ dB}} \checkmark$$
$$F_T = \underline{4.9\text{ dB}} \checkmark$$
$$P_{3T} = 15.5\text{ dBm at output}$$
$$= 15.5 - 21 = \underline{-5.5}\text{ dBm at input} \checkmark$$