

# Microwave Bandstop Filters with Minimum Through-Line Length

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**Abstract** — The theory of bandstop filters with minimum through-line length is developed in this paper. In contrast to conventional similar-resonator bandstop filters that employ odd multiples of 90-degree impedance inverters between resonators, the proposed design technique enables filters with through-line length restricted to only that which is required to obtain a desired coupling coefficient magnitude between a resonator and the through-line. Both even- and odd-order designs are shown to verify the concept. A 4<sup>th</sup>-order microstrip prototype with 45 degrees of through-line length per resonator was fabricated, measured, and compared to two conventional designs. It has half the through-line length of conventional designs.

**Index Terms** — Microwave filters, microstrip filters, filtering theory, resonator filters.

## I. INTRODUCTION

Conventional bandstop filter designs are often comprised of resonators that are coupled to a transmission line with odd multiples of 90 degrees between each pair of resonators [1]. These nominally 90-degree lengths of transmission line between resonators provide the impedance inversion required for symmetric filter responses. In practical filter implementations, part or all of the 90-degree length is used to couple to the resonator so that a relevant coupling coefficient for the desired filter response is obtained, and any remaining uncoupled length is required only for impedance inversion. Bandstop filters with less than 90 degrees between resonators have been designed with responses possessing varying degrees of asymmetry [2]. However, all previous similar-resonator bandstop filters with a symmetric frequency response either have uncoupled lengths of transmission line between each resonator that are only providing impedance inversion or resonators that are coupled along the entire 90-degree phase length between resonators [3]. The uncoupled lengths of line increase passband insertion loss and circuit size.

This paper demonstrates that the correct mixture of electric and magnetic coupling in a bandstop filter can reduce the phase length required between resonators to only that which is needed to obtain a desired coupling coefficient magnitude while retaining a symmetric filter response. Therefore, no uncoupled line length is required. The practical limit of the reduction of through-line length depends on manufacturing technology, resonator impedance, and desired coupling coefficient magnitude. The theory presented in this paper is applicable to all electromagnetic resonator technologies capable of both electric and magnetic coupling and all filter response types, including both even and odd order responses as well as elliptic responses for maximum selectivity [4].

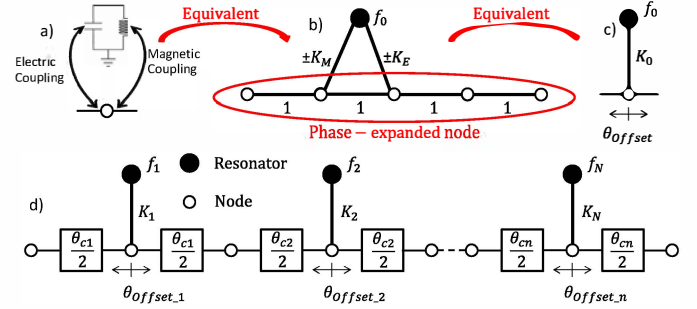


Fig. 1. Coupling routing diagrams. a) Resonator coupled to node with both electric and magnetic coupling. b) Phase-expanded node showing relative phase between electric and magnetic coupling to the node. c) Representation of the node in b) as a phase offset dependent on the signs and magnitudes of  $K_E$  and  $K_M$ . d) Filter with combination of physical phase shifts and coupling phase offsets.

## II. THEORY

Shown in Fig. 1a) is a resonator coupled to a node with a mix of electric and magnetic coupling. At resonance, the impedance looking into the node will be that of a short circuit when there is only electric coupling and that of an open circuit when there is only magnetic coupling. A mix of electric and magnetic coupling results in a corresponding phase shift at resonance. Fig. 1b) is an equivalent circuit of Fig. 1a) with the node expanded in phase to show the relative phase offset between electric ( $\pm K_E$ ) and magnetic ( $\pm K_M$ ) coupling to a resonator from the node. Each unit-impedance inverter has a phase of 90 degrees, so the entire physical node has a phase length of 360 degrees, which is functionally equivalent to 0 degrees. Note that in this structure, the insertion phase can be set to any value by adjusting the signs and relative magnitudes of  $K_E$  and  $K_M$ . The composite phase offset due to multiple types of coupling between a single node and a resonator can be reduced to the circuit in Fig. 1c), where

$$K_0 = \sqrt{K_E^2 + K_M^2} \quad (1)$$

and

$$\theta_{offset} = \frac{1}{2} \text{Arg} \left( \frac{2K_E}{K_E - jK_M} - 1 \right). \quad (2)$$

By adjusting the relative magnitudes of electric and magnetic coupling to the node, the phase offset shown in Fig. 1c) can be adjusted from  $-90^\circ$  (pure  $-K_M$  coupling) to  $0$  (pure  $K_E$  coupling) to  $+90^\circ$  (pure  $K_M$  coupling). If the phase offsets due to mixed  $K_E$  and  $K_M$  coupling for two serial bandstop resonators are such that they together with the electrical length

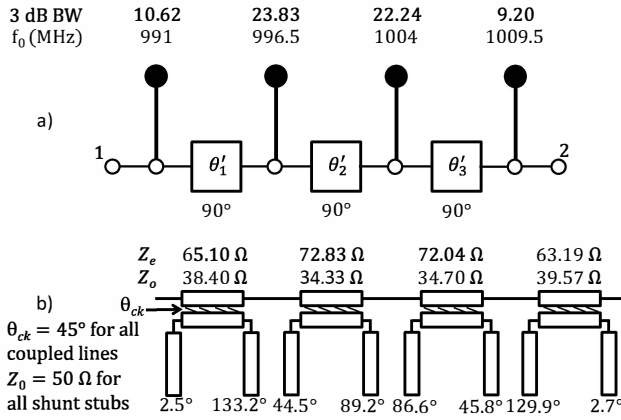


Fig. 2. Even-order prototype with all  $\theta_{ck} = 45^\circ$ . a) Coupling-routing diagram for 4<sup>th</sup>-order generalized Chebyshev filter. 3 dB bandwidth and  $f_0$  are for each resonator in isolation. b) Half-wavelength resonator implementation that corresponds to Fig. 1d). Electrical lengths are specified at 1 GHz.

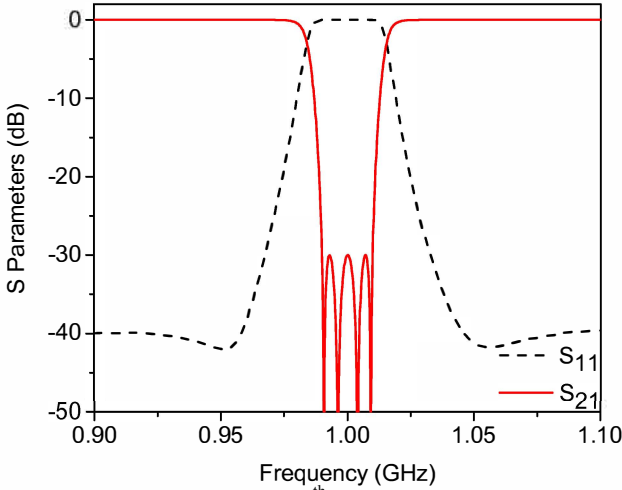


Fig. 3. Simulated response of 4<sup>th</sup>-order half-wavelength resonator filter in Fig. 2b).

of the physical line between the two resonators gives the total phase shift needed ( $\sim 90$  degrees for a conventional symmetric design [1]), a realization with minimum through-line length results. Fig. 1d) shows such a filter, where  $\theta_{ck}$ ,  $k = 1 \dots n$ , where  $n$  is the order, is the phase over which resonator  $k$  is coupled to the through-line to obtain coupling coefficient magnitude  $K_k$  and is the only physical length of line needed due to the proper design of  $\theta_{Offset, k}$  for each resonator.

### III. DESIGN PROCEDURE

A design procedure and two examples are shown below:

1. Design a conventional prototype bandstop network. An even-order example with half-wavelength resonators and an odd-order example with quarter-wavelength resonators are shown here to demonstrate the concept's applicability to a variety of implementations. The first example is a 4<sup>th</sup>-order generalized Chebyshev filter that is shown in Fig. 2a). The procedure in [4] was used to obtain the second example, which is the 5<sup>th</sup>-order

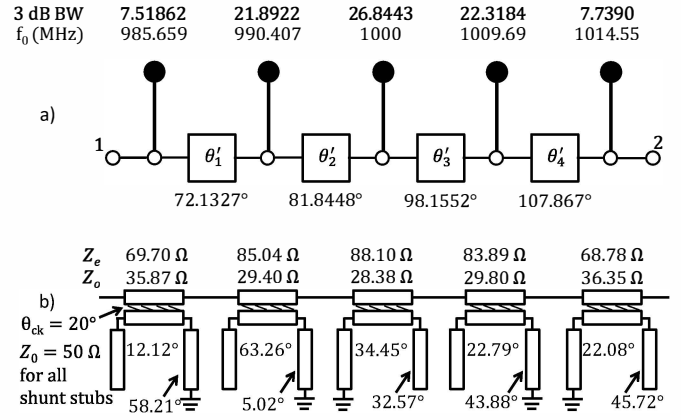


Fig. 4. Odd-order prototype with all  $\theta_{ck} = 20^\circ$ . a) Coupling-routing diagram for 5<sup>th</sup>-order elliptic filter. 3 dB bandwidth and  $f_0$  are for each resonator in isolation. b) Quarter-wavelength resonator implementation that corresponds to Fig. 1d). Electrical lengths are specified at 1 GHz.

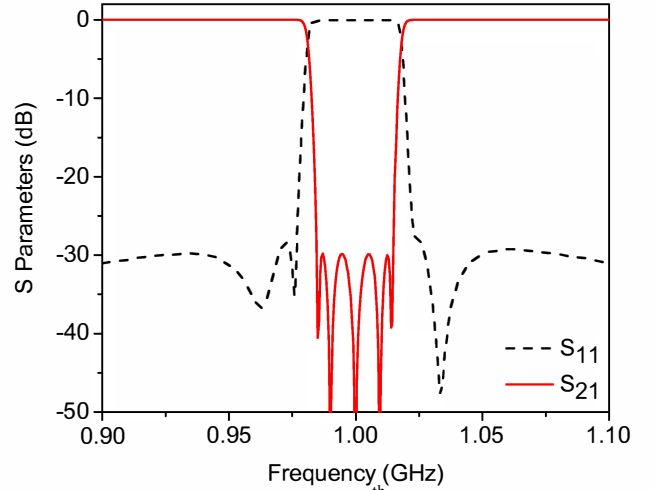


Fig. 5. Simulated response of 5<sup>th</sup>-order quarter-wavelength resonator filter in Fig. 4b).

elliptic filter shown in Fig. 4a). The center frequency of both filters is 1 GHz. The 4<sup>th</sup>-order filter has a 3-dB bandwidth of 31 MHz and a 30-dB bandwidth of 20 MHz. The 5<sup>th</sup>-order filter has a 3-dB bandwidth of 37 MHz and a 30-dB bandwidth of 30 MHz. Note that the average phase length between resonators is 90 degrees for both designs.

2. Assign values to  $\theta_{c1} - \theta_{cn}$ . For minimum through-line length,  $\theta_{c1} - \theta_{cn}$  should be set to the smallest values that will provide the required coupling coefficient magnitudes for the desired filter response within fabrication limits for the employed resonator technology.  $45^\circ$  is chosen for all  $\theta_{ck}$  values of the 4<sup>th</sup>-order design, and  $20^\circ$  is chosen for all  $\theta_{ck}$  values of the 5<sup>th</sup>-order design. Multiple  $\theta_{ck}$  values can be chosen for a single filter to further minimize length, but all  $\theta_{ck}$  values were chosen to be the same in the examples for simplicity.

Filter	$\theta_{\text{offset}_1}$	$\theta_{\text{offset}_2}$	$\theta_{\text{offset}_3}$	$\theta_{\text{offset}_4}$	$\theta_{\text{offset}_5}$
4 <sup>th</sup> -order	-153.295°	-110.825°	-68.465°	-23.75°	N/A
5 <sup>th</sup> -order	-158.73°	-105.345°	-42.485°	-144.52°	-54.785°

Table 1.  $\theta_{\text{Offset}_k}$  values for the 4<sup>th</sup>- and 5<sup>th</sup>-order example designs.

Filter	$\text{Arg}(S_{11_1})$	$\text{Arg}(S_{11_2})$	$\text{Arg}(S_{11_3})$	$\text{Arg}(S_{11_4})$	$\text{Arg}(S_{11_5})$
4 <sup>th</sup> -order	81.59°	-3.35°	-88.07°	-177.5°	N/A
5 <sup>th</sup> -order	117.46°	10.69°	-115.03°	89.04°	-90.43°

Table 2.  $\text{Arg}(S_{11_k})$  values for the 4<sup>th</sup>- and 5<sup>th</sup>-order example designs.

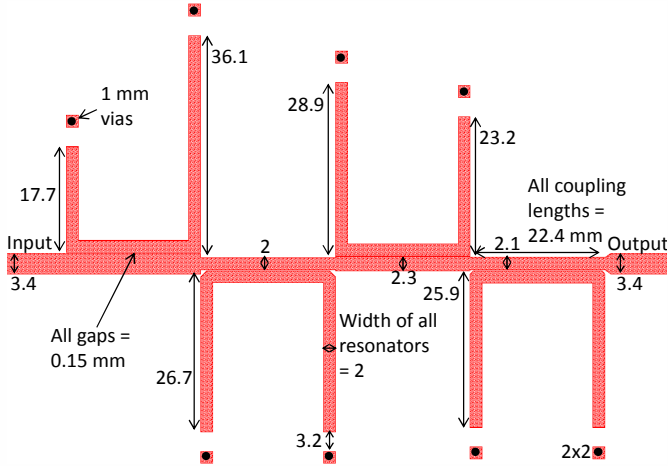


Fig. 6. 4<sup>th</sup>-order fabricated bandstop filter. Dimensions are in mm. 0.5-2.5 pF tuning capacitors are placed between the ends of the resonators and the ground pads.

- Calculate  $\theta_{\text{Offset}_1} - \theta_{\text{Offset}_n}$ . For a given value of  $\theta_{ck}$ , the range of achievable  $\theta_{\text{Offset}_k}$  values is restricted to:

$$-\left(180^\circ - \frac{\theta_{ck}}{2}\right) < \theta_{\text{Offset}_k} < -\frac{\theta_{ck}}{2}. \quad (3)$$

Setting  $\theta_{\text{Offset}_1}$  to the lower limit in (3) will yield the largest design margin.  $\theta_{\text{Offset}_2} - \theta_{\text{Offset}_n}$  can be calculated using (4).

$$\theta_{\text{Offset}_k} = \theta'_{k-1} - \frac{\theta_{ck}}{2} - \frac{\theta_{ck-1}}{2} + \theta_{\text{Offset}_{(k-1)}} \quad (4)$$

Note that  $\theta'_k$  is the through-line length from the original bandstop prototype filter, as shown in Figs. 2a) and 4a) for the examples. Ensure that the values obtained for  $\theta_{\text{Offset}_2} - \theta_{\text{Offset}_n}$  are achievable according to (3). If they are outside of the range in (3),  $\theta_{\text{Offset}_1}$  can be shifted closer to the lower limit in (3), or  $\theta_{ck}$  can be reduced to increase the range in (3) as long as the reduced value of  $\theta_{ck}$  can be used to obtain the desired coupling coefficient between resonator  $k$  and the through line. If these options are exhausted without obtaining  $\theta_{\text{Offset}_k}$  values that are bound by the range in (3), a different fabrication technology or resonator impedance capable of stronger coupling coefficients is needed to meet the filter specification. Table 1 shows the  $\theta_{\text{Offset}_k}$  values for both example filters.

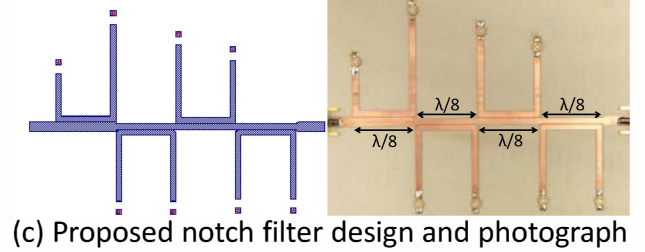
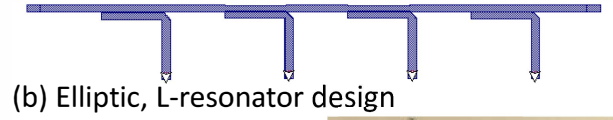
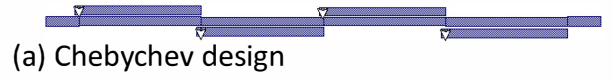


Fig. 7. a) Parallel-coupled Chebyshev design. b) L-resonator elliptic design [3]. c) Model and photograph of proposed filter. All filters are drawn to the same scale and have similar specifications.

- Design each  $k^{\text{th}}$  1<sup>st</sup>-order resonator section to provide the required resonant frequency  $f_k$ , coupling coefficient magnitude  $K_k$ , coupling length  $\theta_{ck}$ , and coupling reference plane offset  $\theta_{\text{Offset}_k}$ .  $\theta_{\text{Offset}_k}$  can be determined analytically by using the techniques derived in [5]. Alternatively, simulation of each individual resonator may be used to evaluate the phase of  $S_{11}$  at its center frequency, and  $\theta_{\text{Offset}_k}$  can be calculated using:

$$\theta_{\text{Offset}_k} = -\frac{\text{Arg}(S_{11_k}) + \theta_{ck} + 180^\circ}{2}. \quad (5)$$

- Series cascade the resonators and coupled line lengths. Table 2 shows the  $\text{Arg}(S_{11_k})$  values for both examples for reference. Note that phase offsets in multiples of  $360^\circ$  were applied to some of the values in Tables 1 and 2 so that the displayed values would fall between  $-180^\circ$  and  $180^\circ$ . Figs. 2b) and 4b) show the resulting designs after using the above procedure for the prototype networks in Figs. 2a) and 4a). The coupled lines are shown with even- and odd-mode impedances for generality. The simulated response of the 4<sup>th</sup>-order filter in Fig. 2b) is shown in Fig. 3, and the simulated response of the 5<sup>th</sup>-order filter in Fig. 4b) is shown in Fig. 5. The response in Fig. 3 is identical to the response of the circuit in Fig. 2a), and the response in Fig. 5 is identical to the response of the circuit in Fig. 4a). However, the total through-line length of the circuit in Fig. 2b) is half as long as the total through-line length of the circuit in Fig. 2a), and the total through-line length of the circuit in Fig. 4b) is less than 28% as long as the total through-line length of the circuit in Fig. 4a).

#### IV. MICROSTRIP PROTOTYPE DESIGN

A 4<sup>th</sup>-order microstrip prototype with half-wavelength resonators was designed and fabricated using the design procedure in Section III. Half-wavelength resonators were chosen so that tuning capacitors could be used on both ends of

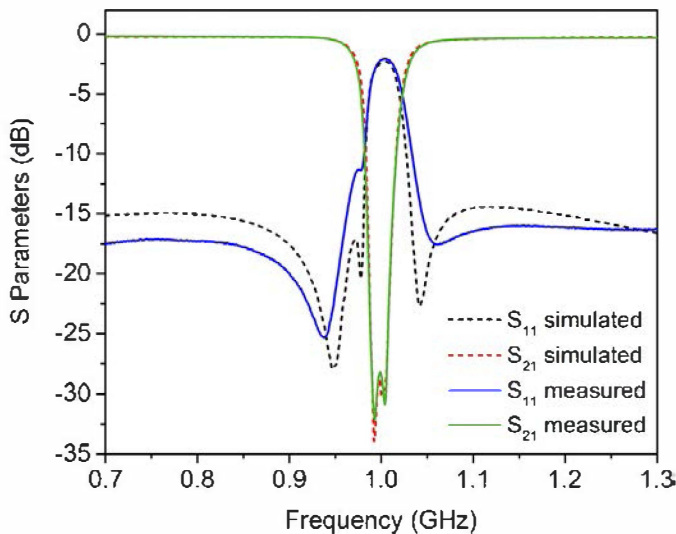


Fig. 8. Narrowband measured vs. simulated equi-ripple response.

the resonators in order to tune the resonant frequency and  $\theta_{Offset, k}$  values after fabrication. To account for the effect of the finite quality factor of microstrip resonators on the 3-dB bandwidth of the bandstop filter, a slightly different prototype filter response than the one shown in Section III was selected. A quasi-elliptic shape with three reflection zeroes, a center frequency of 1 GHz, 3-dB bandwidth of 50 MHz, and 30-dB bandwidth of 18 MHz was designed using CMS software from Guided Wave Technology. All  $\theta_{ck}$  values were set to 45 degrees based on the desired filter shape and bandwidth, as well as the minimum coupled-line gap possible with available fabrication equipment. A model of the filter can be seen in Fig. 6. The widths of the through-line sections were slightly adjusted at each resonator to achieve the desired coupling coefficients and passband matching. The substrate material is Rogers' 4003C ( $\epsilon_r = 3.38$ ,  $\tan(\delta) = 0.0021 @ 2.5$  GHz), and it is 1.524 mm thick. 0.5 pF to 2.5 pF mechanical tuning capacitors from Johanson Manufacturing (part number: 9702-0) were placed at both ends of each microstrip resonator. Fig. 7 shows a size comparison of Chebyshev, elliptic, and the proposed design with similar electrical specifications on the same size scale. It can be seen that the proposed design can accomplish similar performance with a much shorter through-line length. Note that the Chebyshev and elliptic filters use quarter-wavelength resonators, while the proposed design uses half-wavelength resonators for easier post-fabrication tuning. As shown in Fig. 4b), the proposed concept also works with quarter-wavelength resonators. Therefore, the total size of the proposed filter can be made even smaller at the cost of more difficult post-fabrication tuning if quarter-wavelength or capacitance-loaded resonators are used. A photograph of the fabricated filter is also shown.

## V. MEASURED RESULTS

Complementary simulated and measured results of the 4<sup>th</sup>-order prototype can be seen in Fig. 8. Simulations were

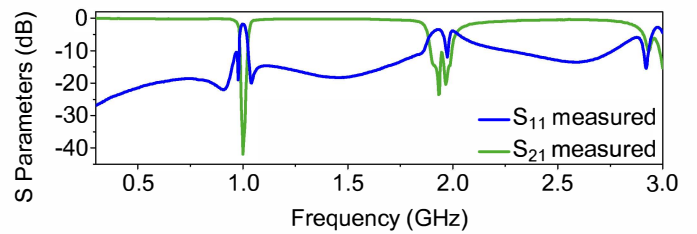


Fig. 9. Measured wide-band maximum attenuation response.

performed using Sonnet software. Sonnet models for the substrate and conductor were used, and the Johanson Manufacturing mechanical tuning capacitors were assumed to have a quality factor of 200 at 1 GHz in simulation. Measurements were performed on an Agilent N5222A network analyzer. A quasi-elliptic response with three reflection zeroes near the stopband can be seen in both simulated and measured results. The measured 28.2-dB equi-ripple attenuation bandwidth is 18 MHz, and the 3-dB bandwidth is 54 MHz, which are close to the simulated values but slightly degraded due to lower quality factor in the measured filter. The use of half-wavelength resonators for easier post-fabrication tuning results in the first spurious resonance appearing at approximately 2 GHz as shown in Fig. 9, where the notch is tuned to a maximum attenuation response by slightly adjusting the tuning capacitors. The upper passband could be extended further through the use of quarter-wavelength resonators as shown in Fig. 4b) or more heavily loaded resonators at the cost of quality factor.

## VI. CONCLUSION

The theory of bandstop filters with minimum through-line length is developed in this paper. It is applicable to any bandstop prototype topology, including absorptive and inline networks. Quarter- and half-wavelength resonator designs of even and odd order were shown. Minimum through-line length bandstop filters can significantly reduce the physical size and passband insertion loss of bandstop filters, enabling smaller, lighter systems with higher sensitivity.

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