

# A General Circuit Model for Defected Ground Structures in Planar Transmission Lines

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**Abstract**—This paper presents a general circuit model for planar transmission line defected ground structures (DGS) that exhibits multi electromagnetic band gaps or stop bands in frequency responses. The proposed equivalent circuit consists of lumped elements that can be easily extracted from full-wave EM simulations. It is simple and accurate for modeling a wide range of DGS in microstrip and coplanar waveguide over a broad bandwidth. Excellent agreement with EM simulations/measurements is demonstrated.

**Index Terms**—Circuit modeling, defected ground structures (DGS), electromagnetic band gaps (EBGs).

## I. INTRODUCTION

IN RECENT years, there has been a growing interest for defected ground structures (DGS) in planar transmission lines that are exhibiting multi electromagnetic band gaps (EBGs) or stop bands in frequency responses [1]–[7]. Novel types of compact filtering devices can be designed based on periodic or non-periodic DGS. However, a direct optimum design of an array of DGS using full-wave EM simulations is really time consuming particularly when the number of DGS in a device is large. In this case, the optimization based on an equivalent circuit of the device is highly desirable. To this end, the key issue is to obtain a simple and accurate circuit model for unit cells of DGS. A lumped-element circuit model has been proposed to model the first two resonant modes of a very specific slot resonator in the ground plane for microstrip structures [1]. More recently, circuit models based on slotlines have been reported for microstrip DGS with slotted ground plane [2], [3]. Although this type of circuit model using slotline elements can present a periodic frequency response, it requires accurate models for slotline effective dielectric constant and characteristic impedance since both parameters are highly frequency-dependent. Hence, this type of circuit model would be more suitable for simple transverse slots which do not include many irregular discontinuities as in [3].

This paper proposes a more general circuit model that is able to represent varieties of DGS in either microstrip or coplanar waveguide (CPW). Some examples of DGS are illustrated in Fig. 1. The structures of Fig. 1(a) and (c) may be referred to as the CPW and microstrip L-shaped DGS, respectively. While the structures of Fig. 1(b) and (d) can be seen as the metal-loaded dumbbell-shaped DGS in CPW and microstrip, respectively. These unit cells of DGS possess some interesting characteristics

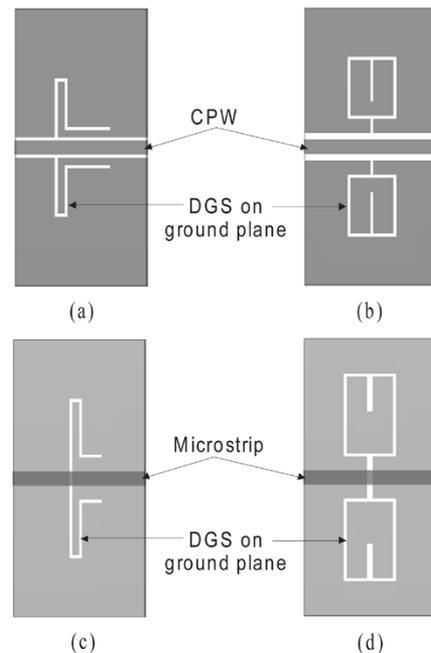


Fig. 1. Various defected ground structures: (a) CPW L-shaped DGS, (b) CPW metal-loaded dumbbell-shaped DGS, (c) microstrip L-shaped DGS, and (d) microstrip metal-loaded dumbbell-shaped DGS.

for developing DGS-based filtering devices. For instance, the floating metals inside the dumbbell-shaped DGS in Fig. 1(b) and (d) can be used not only to control the separation of the first two resonant modes, but also to facilitate dc bias for electronic tuning. This paper will focus on equivalent circuit modeling as described below.

## II. CIRCUIT MODEL

Many DGS including those depicted in Fig. 1 exhibit multi stop bands in frequencies. Fig. 2 shows the typical frequency response up to the second stop band for such DGS, where  $f_{01}$  and  $f_{02}$  are the first and second resonant frequencies, respectively, and  $f_T$  denotes a transit frequency. For many DGS device designs a frequency range up to the second stop band is most of interest. Therefore, the proposed circuit model as shown in Fig. 3 will only consider the first and second resonant modes. Thus, a unit cell DGS, either in microstrip or CPW, is modeled by the two  $LC$  resonators, i.e.,  $L_1$  and  $C_1$ ,  $L_2$  and  $C_2$ , in connection with a T-network consisting of  $C_p$ ,  $L_{s1}$ , and  $L_{s2}$ . The T-network is essential here to represent the interaction between the two resonators. Below the transit frequency  $f_T$  the first resonator dominates the frequency characteristics, whereas the second resonator is dominant for the frequency above  $f_T$ .

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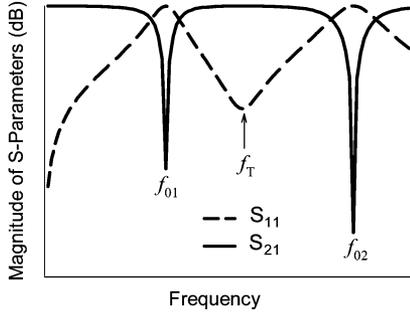


Fig. 2. Typical frequency response of the DGS that exhibit multi stop bands.

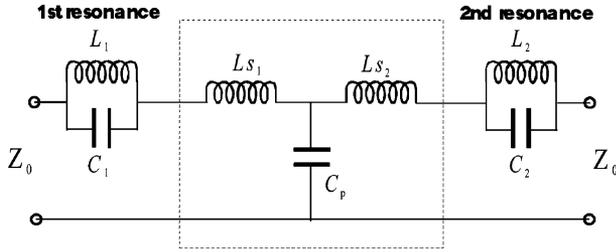


Fig. 3. General circuit model for the DGS.

In Fig. 3,  $Z_0$  is the characteristic impedance of the transmission line. To extract all other circuit parameters, the following data are sufficient and can be easily found from EM simulations:  $f_{01}$ ,  $f_{02}$ ,  $f_T$ ,  $\Delta f_{3dB-1}$  (the 3-dB bandwidth at  $f_{01}$ ),  $\Delta f_{3dB-2}$  (the 3-dB bandwidth at  $f_{02}$ ),  $X_{11}$ ,  $X_{22}$  and  $X_{21}$  which are the imaginary parts of three  $Z$ -parameters at  $f_T$ . It can be shown by circuit theory that

$$C_i = \frac{1}{Z_0} \cdot \frac{1}{4\pi\Delta f_{3dB-i}} \quad (1)$$

$$L_i = \frac{1}{(2\pi f_{0i})^2 C_i} \quad \text{for } i = 1, 2$$

$$C_p = -\frac{1}{2\pi f_T X_{21}}$$

$$L_{S_i} = \frac{X_{ii} - X_{21}}{2\pi f_T} + \frac{L_i}{\left(\frac{f_T}{f_{0i}}\right)^2 - 1} \quad \text{for } i = 1, 2 \quad (2)$$

where (1) can be derived from the transmission parameter, i.e.,  $S_{21}$  of the individual two-port resonator network when  $S_{21}$  is expressed in terms of the admittance of resonator. While (2) can be obtained by matching  $Z$ -parameters of the two-port T-network.

### III. MODELING RESULTS

We have applied the circuit model presented in the last section to model various DGS, and excellent agreement with EM simulations and/or measurements have been obtained. Some examples are described in this section.

#### A. Case 1

The CPW DGS shown in Fig. 4(a) is on a 1.27 mm thick dielectric substrate with a relative dielectric constant of 10.8, where all the dimensions are in millimeters. For the modeling, two reference planes are specified as indicated by the arrows shifting from the ports. The CPW line has a characteristic

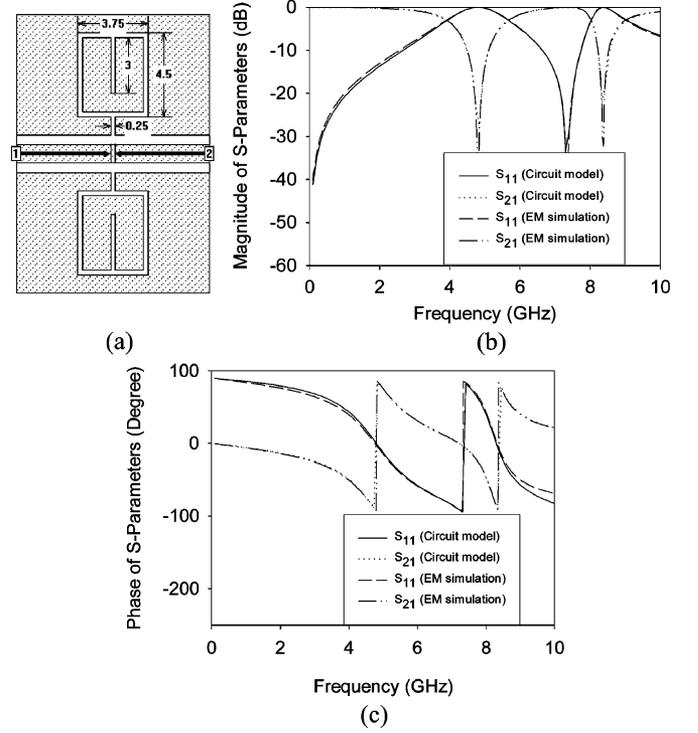
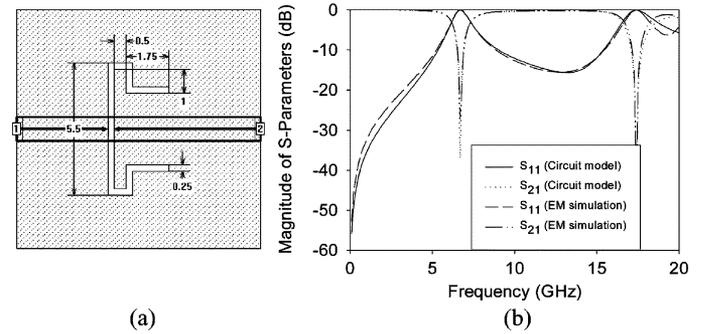

 Fig. 4. (a) Layout of the CPW DGS modeled. (b) Magnitude and (c) phase responses of theoretical (circuit model) and full-wave EM (Sonnet *em*) results.


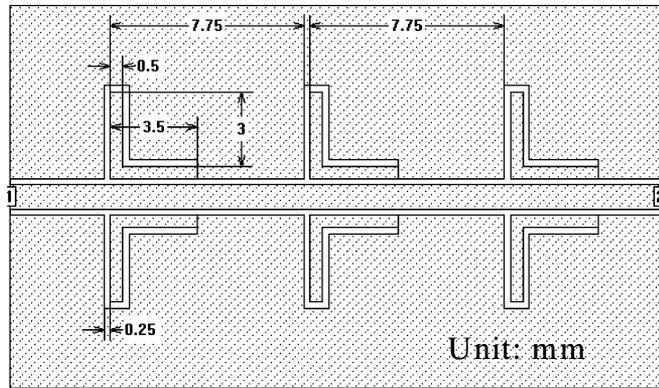
Fig. 5. (a) Layout of the microstrip DGS modeled. (b) Magnitude responses of theory (circuit model) and full-wave EM simulation.

impedance of  $50 \Omega$  at the reference planes. The circuit parameters extracted from the EM simulation are  $C_1 = 0.8966$  pF,  $L_1 = 1.226$  nH,  $C_2 = 1.632$  pF,  $L_2 = 0.2212$  nH,  $C_p = 0.033$  pF,  $L_{S1} = 0.995$  nH, and  $L_{S2} = -0.919$  nH. Note that the negative value for  $L_{S1}$  and  $L_{S2}$  is perfectly allowed for the circuit modeling. This is similar to a lumped-element inverter with negative elements in which the negative elements physically may be absorbed by the adjacent reactance components in the circuit [8].

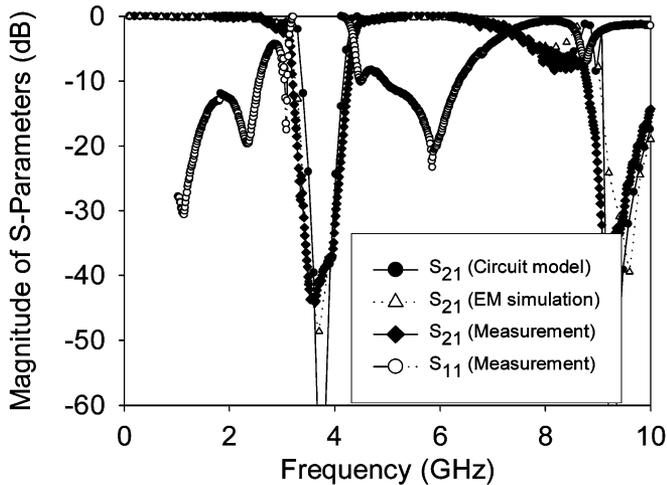
Fig. 4(b) and (c) show the theoretical (circuit model) and full-wave simulated  $S$ -parameter results for magnitude and phase, respectively. Excellent agreement can be observed.

#### B. Case 2

The second modeling example is for a microstrip L-shaped DGS shown in Fig. 5(a), where all the dimensions are in millimeters. The substrate used has a thickness of 1.27 mm and a relative dielectric constant of 10.8. The microstrip line has



(a)



(b)

Fig. 6. (a) Layout of the three-cell period DGS in CPW. (b) Magnitude responses of theory (circuit model), full-wave EM simulation, and measurement.

a characteristic impedance of  $50 \Omega$  at the reference planes. In this case, the circuit parameters extracted from the EM simulation are  $C_1 = 1.349 \text{ pF}$ ,  $L_1 = 0.4184 \text{ nH}$ ,  $C_2 = 1.02 \text{ pF}$ ,  $L_2 = 0.082 \text{ nH}$ ,  $C_p = 0.028 \text{ pF}$ ,  $L_{s1} = -0.749 \text{ nH}$ , and  $L_{s2} = 0.5745 \text{ nH}$ . This DGS has two widely separated resonant modes, nevertheless, excellent agreement between the theory (circuit model) and full-wave EM simulation can be observed from dc to 20 GHz in Fig. 5(b). Again, this agreement can also be shown for phase.

### C. Case 3

This example is to demonstrate the application of the proposed circuit model for modeling a period CPW GDS as shown in Fig. 6(a), which is on a 1.27 mm-thick dielectric substrate with a relative dielectric constant of 10.8. The unit cell of DGS is that of Fig. 1(a). With the dimensions shown, the circuit parameters extracted from the EM simulation are  $C_1 = 2.059 \text{ pF}$ ,  $L_1 = 0.8746 \text{ nH}$ ,  $C_2 = 1.457 \text{ pF}$ ,  $L_2 = 0.2303 \text{ nH}$ ,  $C_p = 0.0385 \text{ pF}$ ,  $L_{s1} = -1.009 \text{ nH}$ , and  $L_{s2} = 1.093 \text{ nH}$ . The periodicity of the unit cells is 7.75 mm. An equivalent circuit for this three-cell DGS device is simply obtained by cascading the unit cell circuit model in Fig. 3 and the 7.75 mm-long transmission line alternatively. Fig. 6(b) show the circuit modeled, full-wave EM simulated and measured  $S_{21}$ -parameters, where very good agreement can be seen.

## IV. CONCLUSION

A general circuit model for modeling a wide range of DGS has been proposed. The simple formulation for circuit parameter extraction has been presented. The applications of the proposed circuit model have been demonstrated with good agreement with full-wave EM simulations and measurements.

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