Electromagnetic Radiation under Explicit Symmetry Breaking

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We report our observation that radiation from a system of accelerating charges is possible only when there is explicit breaking of symmetry in the electric field in space within the spatial configuration of the radiating system. Under symmetry breaking, current within an enclosed area around the radiating structure is not conserved at a certain instant of time resulting in radiation in free space. Electromagnetic radiation from dielectric and piezoelectric material based resonators are discussed in this context. Finally, it is argued that symmetry of a resonator of any form can be explicitly broken to create a radiating antenna.

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Symmetries in physical systems are characterized by conserved quantities [1]. For example, in electromagnetic systems, there is symmetry of the electromagnetic field which is defined by global conservation of charges and current and gauge invariance [2]. Symmetry breaking can be interesting as is the context of spontaneous symmetry breaking which leads to generation of Goldstone bosons [3–4]. Such bosons were first observed by Nambu in the context of superconductors [5–6]. Breaking of $U(1)$ gauge symmetry in superconductors results in cooper pairs [7]. Spontaneous breaking of local symmetry results in Goldstone bosons getting mass [8]. There are other instances of symmetry breaking, for example, translational and rotational symmetry breaking of the crystal structure results in the generation of Goldstone phonons and in magnets, Goldstone modes are propagating spin waves [9].

Besides spontaneous symmetry breaking, explicit symmetry breaking in some localized region of space and time can offer novel physical insights. Explicit symmetry breaking is associated with a condition where the dynamic equations and the Lagrangian of the system are not invariant due to some terms which break its symmetry [10]. In electromagnetism, for static isolated charges, the electric field lines hold rotational symmetry which is expressed in Gauss’s law given by $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$ where $\mathbf{E}$ is the electric field, $\rho$ is the volume charge density, and $\varepsilon_0$ is permittivity of free space. The schematic representation of electric lines of field of a static point charge emerging radially from its center having a rotational symmetry is shown in Fig. 1(a). As the charge is subjected to a periodic acceleration, the symmetry of the electric field is explicitly broken within a localized region of space and time resulting in rotation of the electric field which generates magnetic field resulting in emission of electromagnetic radiation from the charge center. The rotation of the electric field and generation of magnetic field is described by Maxwell’s equations, $\nabla \cdot \mathbf{E} = 0$ and $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$, where $\mathbf{B}$ is magnetic flux density which is dependent on time $t$. The phenomenon is further illustrated in the context of a two wire parallel transmission line excited by a time varying voltage source at one end. Charges developed along the two wire parallel transmission line generate electric fields having translational symmetry [Fig. 1(b)]. Although the charges are accelerating, there is translational symmetry in the electric field lines as the two wire transmission line along which the charges are moving is symmetrical in structure. The radiation fields get canceled due to symmetry and there is no net radiation [11]. When the symmetry of the transmis-

FIG. 1. Dependence of electric field symmetry on charge configuration. (a) Rotational symmetry in the electric field distribution of a static positive charge. (b) Electric field has got translational symmetry in a parallel transmission line but the symmetry is explicitly broken towards its flared end. (c) The currents of amplitude $I$ in the wires (represented by arrows) are in the same direction in the flared section of the transmission line and the amplitude of electric fields (represented by the curves) add up resulting in radiation. The length of the flared ends are $L_0/2$. Thus, an antenna is made by explicitly breaking the symmetry of the transmission line. (d) The pattern of the electric field shown by curved lines in the electromagnetic radiation from the flared end of a transmission line.
transmission line structure of Fig. 1(b) is broken by opening the wire at its ends [Fig. 1(c)], the translational symmetry of the electric field is broken due to rotation of electric field lines as illustrated in Fig. 1(d) resulting in generation of electromagnetic radiation.

According to Noether’s theorem, whenever there is a symmetry, some physical quantity is conserved [1]. In the context of static charges, we can easily associate the symmetry of electric field lines with the conservation of charges within a localized region of space and time. The rotational symmetry of the electric field of point charges (Fig. 1) can be directly mapped to rotational symmetry of its Lagrangian which has angular invariance. For example, the potential energy of a test charge \( q \) at a distance \( r \) from another charge \( Q \) is given by \( U = Qq/(4\pi\varepsilon_0 r) \). Here, \( d\mathcal{L}/dr \) and \( d\mathcal{L}/d\theta \) are equal to zero where \( \theta \) is angular displacement from the center of the charge.

In the case of a parallel two wire transmission line, excited by a time varying voltage source, the charges are in motion but the translational symmetry of the electric field is maintained and this can be associated with the conservation of current at a certain instant of time. Here, the Lagrangian of the system also has translational symmetry. The current along a surface perpendicular to the transmission line is conserved at all instants of time in the sense that current flowing in one direction has an equal and opposite amount of current flowing in the opposite direction. If we consider that current in the top wire as \( I_1 \), magnetic field at a distance \( r \) from its center is \( B_1 = \mu_0 I_1/2\pi r \), where \( \mu_0 \) is permeability of free space; for the lower wire having a current \( I_2 \) the magnetic field is \( B_2 = \mu_0 I_2/2\pi r \). As the wires are placed close to each other, for a distance \( r \gg s \), the distance between the wires, the net magnetic field is \( B = B_1 - B_2 = \mu_0 (I_2 - I_1)/2\pi r \). The electric field at a distance \( r \) outside the upper wire is \( E_1 = \sigma_1/\varepsilon_0 \) where \( \sigma_1 \) is the charge density on the upper wire. The value for the lower wire is \( E_2 = \sigma_2/\varepsilon_0 \) where \( \sigma_2 \) is the charge density on the lower wire. The net electric field in the region outside the wire is \( E = (E_2 - E_1) = (\sigma_2 - \sigma_1)/\varepsilon_0 \). The net Lagrangian measured at a point outside the wires is

\[
\mathcal{L} = \frac{1}{2} L (I_1 - I_2)^2 - \frac{1}{2C} (Q_1 - Q_2)^2, \tag{1}
\]

where \( L \) is the net inductance and \( C \) the capacitance of the two wires, \( Q_1 \) and \( Q_2 \) are charges associated with the wires. If currents in the two wires are similar in magnitude, the total Lagrangian is zero and hence it is invariant with time and space. The Lagrangian density is given by

\[
\mathcal{L}_d = \frac{B^2}{2\mu_0} - \frac{1}{2} \varepsilon_0 E^2, \tag{2}
\]

which also remains zero. However, the Lagrangian has a finite value in the space within the wires due to the finite value of the electric field and the related potential, but it has translational symmetry as well as temporal invariance linked to the symmetries of the electric field.

Under explicit symmetry breaking, some physical quantity must lose its conserved value within a localized region of space and time. In the context of a two wire transmission line, the symmetry of the structure is broken with wires being open ended as shown in Figs. 1(b) and 1(c). Figure 1(d) shows the electric lines of field of the resulting radiation pattern. In this case, the current is not conserved at a certain instant of time through a plane perpendicular to the flared end of the transmission line as there is unidirectional current and charges corresponding to it which results in radiation. Here, the electric field loses translational and temporal symmetries and variation of Lagrangian with space \( \partial\mathcal{L}/\partial r \) and time, \( \partial\mathcal{L}/\partial t \) are not equal to zero within a localized region as the current is transformed into electromagnetic waves which are radiated out into free space. Thus, there is an absence of current conservation in the local region of the radiating structure at a certain instant of time due to explicit symmetry breaking resulting in non-conservation of current and charge in the localized region of the radiating element at that particular instant of time which is the key mechanism of radiation. We can easily argue that explicit symmetry breaking and non-conservation of Noether current within a local region at a certain instant of time is the primary driver of radiation from the electrodynamic system.

Electromagnetic radiation under symmetry breaking in the context of a Hertzian dipole antenna whose electric lines of field are similar to Fig. 1(d), is also extendable to infrared and optical frequencies. In a free electron laser, an electron beam at relativistic speed is accelerated in alternative magnetic flux of north and south poles of a wiggler magnet which results in periodic oscillation of the electron and emission of bremsstrahlung radiation [12]. If the undulator wavelength is \( \lambda_u \), then the laser wavelength is given by \( \lambda = \lambda_u/2\pi \). Here, as the electron is accelerated under the impact of periodically varying Lorentz force associated with the Wiggler magnet, translational symmetry of the electric field of the electron beam in the space surrounding it is broken resulting in generation of a rotating magnetic field as is the case with a Hertzian dipole antenna. The nonconserved Noether current which results in radiation slows down the electron beam which needs to be periodically energized for continuous emission.

A related aspect of the phenomenon of radiation in a free electron laser is symmetry breaking of the magnetic field associated with the electron beam. At constant velocity, the electron beam has a magnetic field intensity \( H \) given by \( I/2\pi r \), where \( I \) is the current and \( r \) is the radial distance from the charge in space. At any given distance \( r \), \( H \) is constant and it has translational symmetry along the
direction of the electron beam. When the electron beam is deflected under the influence of Lorentz force caused by the Wiggler magnet, translational symmetry breaking of the magnetic field results and is correlated to the symmetry breaking of the electric field resulting in electromagnetic radiation.

In order to bring about explicit local symmetry breaking in an electrodynamic system, we can create structural changes in symmetric resonators which behave like filters and introduce asymmetry, transforming them into asymmetric resonators which behave like radiating antennas under symmetry breaking. The transformation of the system having symmetry to a system where the symmetry is lost with an associated Lagrangian having terms due to which the Lagrangian does not remain invariant is the condition associated with explicit symmetry breaking [10]. This aspect has been explored in detail from an empirical perspective in the present work.

The simplest example of a symmetric resonator is a capacitor-inductor circuit where charges keep oscillating between inductive and capacitive elements under thermal interaction which allows selective transfer of energy from its excitation end to the ground terminal. Here, the Lagrangian has temporal invariance as potential energy from the capacitor is transferred to kinetic energy of charges in the inductor in a periodic manner. Because of idealized assumptions associated with lumped elements which are concentrated in certain points in space, we cannot describe the spatial invariance of the Lagrangian with space from a physical perspective.

All resonators connected in symmetric mode behave like filters which are two port devices and can selectively transfer energy from input to the output side [14]. They have translational symmetry and the system is invariant to an interchange of input and output ports, terminals, or related connecting leads. The translational symmetry of filters is also integrated with temporal as well as spatial invariance of the Lagrangian of the particles passing through them. Filters act as conduits for selective transfer of energy having a set of input and output ports, terminals, or leads through which energy is fed and collected, but when either of the input or output terminals are closed, symmetry is broken, there is storage of energy. If energy is repeatedly fed into the system, the system radiates energy whose amplitude becomes high under resonance. A two wire transmission line under symmetric excitation is a symmetric resonator which acts like a filter of electrical signals where its Lagrangian is invariant with time and space. When we break its symmetry, it can radiate energy and we can create an antenna where the Lagrangian loses its temporal as well as spatial invariance. Thus, we can look at radiating structures or antennas as symmetric resonators whose symmetry has been explicitly broken.

An electrical resonator whose symmetry is explicitly broken by leaving one of the terminals floating is shown in Fig. 2(a). The device is an asymmetric resonator and works as an antenna. Instead of just being a conduit for selective energy transfer, it starts storing energy and radiating it. The concept is generic in the sense that all electrical resonators having an asymmetric form of excitation can be used as antennas. The asymmetric resonator of Fig. 2(a) resembles some of the early inventions on antennas as shown in Fig. 2(b), which shows the antenna invented by Marconi during the early days of radio communication [15]. Here the transmitting antenna is an inductor floating in free space with one terminal grounded. As time varying currents are fed to the inductor, it is transmitted to free space after being transformed into current, unlike a filter where the current traverses the circuit and flows to the ground. In all such systems, the symmetry breaking can be correlated to temporal variance of its Lagrangian and loss of Noether current from the system.

Another example of asymmetric resonators as antennas is that of dielectric resonator antennas (DRA). The field of DRA started with a 1939 paper by Richtmyer where it was argued that a dielectric material can act as an electromagnetic resonator [16]. The author offered a mathematical proof of the possibility that such a device can radiate depending on the boundary conditions associated with the dielectric material and air. The theoretical basis of the working of a DRA is based on the development of models...
of radiating sources associated with a measured value of electromagnetic field pattern [17]. Some early experimental and theoretical work in the field of DRA was done by Long, McAllister, and Shen [18]. They used the resonator shown in Fig. 2(c) where a cylindrical DRA of length $D_L$ and radius $R$ was mounted on a ground plane. Based on the radiation pattern of the device, the authors made an assumption that the boundary of dielectric material could be considered as a perfect magnetic conductor and expressed the electromagnetic field pattern generated by the DRA using Bessel functions. Thus, they argued that the DRA behaved like a magnetic dipole antenna. Related work on DRAs has further validated its magnetic dipole antennalike behavior [19]. However, some of the fundamental questions related to physics of DRAs remain unclear. For example, when a time varying excitation is fed to a dielectric material, there are oscillations of bound charges, which is essentially oscillation of electrical dipoles. The components of electric field $E$ associated with a Hertzian dipole antenna in spherical coordinates along the dimensions $r$, $\theta$, and $\phi$ in spherical coordinate system are given by [11]

$$E_r = \eta I_0 D_L \cos \theta \left[ 1 + \frac{1}{jkr} \right] e^{-jkr}, \quad \text{(3)}$$

$$E_\theta = j\eta I_0 D_L \sin \theta \left[ 1 + \frac{1}{jkr} \right] \frac{1}{kr^2} e^{-jkr}, \quad \text{(4)}$$

where $I_0$ is the current in the antenna, $D_L$ is its length, $\eta$ is the intrinsic impedance of space ($120\pi \Omega$), and $k$ is the wave number associated with the wave. The other component of electric field, $E_\phi = 0$. In the above equations, the $1/r^3$ term is the electric field generated by the oscillating dipoles. If electrical dipoles were responsible for radiation, then the power radiated by a DRA must fall off at a high rate and should have negligible value in the far field region. The origin of radiation field due to electron acceleration [1/r term in Eq. (4)] in the context of DRAs is not evident from related literature as there are no free currents in the material as is the case with metallic antennas and the magnetic wall model cannot be applied for the related values of dielectric constants [18]. Magnetic conductors are mathematical abstractions created in order to create symmetry in Maxwell’s equations and they do not have any physical reality [11].

A related issue with existing literature in this field is that DRAs are considered as waveguides which support electromagnetic modes where radiation is caused by material-air interface [18–19]. Waveguides cannot generate electromagnetic waves and they need to be coupled to microwave sources which have a distinct means of transforming current into electromagnetic waves using an active source with a feedback mechanism [20]. Horn antennas having waveguides as feed points are in turn powered by dipole or monopole antennas which transform current into electromagnetic waves resulting in radiation [21]. Thus, the mechanism of transformation of current into electromagnetic waves in the dielectric medium of a DRA which has bound charge has not been addressed so far as in order to generate electromagnetic waves, acceleration of free electrons in the cavity of the resonator at one of the resonant frequencies is an essential requirement.

The resonant frequency of a DRA considering the dielectric material as a resonator and applying the magnetic wall model which assumes that the surfaces of DRAs are perfect magnetic conductors and related boundary conditions associated with a typical cylindrical resonator where the structure is oriented in free space as shown in Fig. 2(c), is given by

$$f = \frac{1}{2\pi R \sqrt{\mu \varepsilon}} \sqrt{\left\{ \frac{x_{np}^2}{x_0'^{2np}} \right\} + \left[ \frac{\pi R}{2D_L} (2m + 1) \right]^2}, \quad \text{(5)}$$

where $m$ is an integer, $D_L$ is antenna length, $R$ is radius, $\mu$ is permeability of the dielectric material, $\varepsilon$ is its permittivity, $x_{np}$ and $x_0'^{np}$ are the parameters of Bessel function of the first kind which express the standing electromagnetic wave in the resonator in cylindrical coordinates [18]. A major problem with the magnetic wall model is that it does not lead to any closed form expressions which could predict input impedance of a DRA excited by a particular feed [22]. In addition to this, the model works under the assumption of infinite ground plane which fails for practical DRAs [23]. Different shapes and mounting arrangements of DRAs along with feeding configurations result in radiation patterns similar to electric dipole, magnetic dipole, and quadrupole antennas [24].

The working of DRAs can be understood only in the context of symmetry breaking of dielectric resonators. The DRA shown in Fig. 2(c) has a finite value of capacitance and it is mounted on a ground plane having an inductance which changes the boundary condition of the dielectric resonator giving an additional resonant mode given by $f_n = 1/2\pi \sqrt{LC}$ where $C$ is the capacitance of dielectric resonator and $L$ is the inductance of feeding wire and ground plane. Thus, the dielectric resonator works like an extended capacitor facilitating swing of electrons from its capacitive element to the ground plane which behaves like an inductor. The asymmetric excitation of the capacitor-inductor system of dielectric resonator on a ground plane where one of the input terminals of the system is subjected to time varying electrical excitation and the other is left floating in free space [Fig. 2(c)] results in a back and forth swing of electrons between its capacitive and inductive elements. Under these conditions, there is field enhancement under symmetry breaking and generation of electromagnetic waves which leaks out of the dielectric waveguide resulting in an antennalike behavior. Thus, the
A general theoretical model which predicts the empirically measured electric and magnetic field values associated with the radiation pattern of a DRA can be found by a superposition of the radiation pattern associated with the conduction current flowing in the ground plane, exciting cables, strip lines, or probes and electromagnetic modes supported by the dielectric resonator. Solving basic equations of electromagnetism, i.e., \( \nabla^2 \mathbf{A} + \alpha^2 \mu \varepsilon \mathbf{A} = -\mu \mathbf{J} \) where \( \mathbf{A} \) is the magnetic vector potential, \( J \) is the complex current density flowing in the LC circuit, \( \omega \) is angular frequency of current, \( \mu \) is permeability, and \( \varepsilon \) is permittivity of the given medium, and related equations such as \( \mathbf{E} = -j\omega \mathbf{A} - \nabla(\nabla \cdot \mathbf{A})/(\omega \mu \varepsilon) \) and \( \nabla \times \mathbf{B}/\mu = \mathbf{J} + j(\omega \varepsilon) \mathbf{E} \) for the specific structural geometry can lead to values of electric and magnetic field associated with the dielectric resonator and ground plane which gets superposed over the existing electromagnetic modes in the dielectric resonator whose frequencies are expressed by Eq. (5).

In order to further explore the effect of radiation under explicit symmetry breaking, thin film piezoelectric material was tested for its ability to radiate under asymmetric excitation. Such filters have a set of interdigital electrodes which transform an electrical signal into surface acoustic waves (SAW) which propagate along the film and are selectively collected by a similar set of electrodes at the output [25]. The electrical equivalent model of a SAW filter is similar to a two port resistor-inductor-capacitor filter as it has equivalent resistive, capacitive, and inductive elements [26]. A SAW filter connected in an asymmetric configuration is shown in Fig. 2(d) where the upper electrodes of SAW devices have been connected together to make a long monopole antenna and the input ground has been left floating. Under symmetric connection of the filter where the two input and output electrodes are part of a circuit, the device selectively transfers signal from the input to the output, but in the present context, when the signal is fed to the filter excited asymmetrically, it stores and drives current back into the ground plane resulting in radiation. This phenomenon was further investigated by using SAW devices mounted on ground planes of various sizes. It was found that by increasing the length of the ground plane, radiation efficiency increased.

The initial test was mainly focused on SAW devices at GPS frequencies. A SAW device from Epcos (B3525) was chosen as it is used as a filter for GPS frequencies centered at 1575 MHz. A set of two SAW devices (dimensions 7 mm \( \times \) 3 mm \( \times \) 3 mm) were mounted on an FR4 board of dimensions 7 cm \( \times \) 3 cm as shown in Fig. 3(a). The electrode diagrams are shown in Fig. 3(b). The objective was to increase the overall surface area of the piezoelectric film by using two SAW devices in series. The experiments were done in a Satimo, Stargate 64 anechoic chamber owned by Antenova Ltd., Cambridge. The chamber was screened in an enclosure to reduce reflection and was lined with absorbers.

The SAW device appears as a capacitive load having very low impedance when the input ground is left floating, in order to create a monopole antenna. To match it to 50 Ohm impedance at 1575 MHz, a shunt capacitor of 2.2 pF and a series inductor of 12 nH were used. The impedance matching elements together with the piezoelectric film result in a finite operational bandwidth of the device with selective radiation characteristics which is reflected in the drop in efficiency around 1554, 1578, and 1594 MHz. The efficiency of the device was measured around GPS frequency to be 42% [Fig. 3(c)]. The efficiency dropped to 22% when the SAW devices were removed leaving the bare exciting electrode, impedance matching elements were adjusted to create a monopole antenna matched at 1590 MHz [Fig. 3(d)]. The radiation efficiency drop of 20% confirms the role of SAW devices in the overall radiation. Further asymmetry is created by shorting both the input terminals of the SAW device [Fig. 4(a)]. This leads to a further increase in efficiency to 60% [Fig. 4(b)].

The results of efficiency measurements for such a SAW filter with a bandwidth between 1800–1885 MHz is shown in Fig. 4(c). The area of the SAW device had the dimensions of 3 mm \( \times \) 3 mm (thickness was 1 mm with packaging and 0.1 mm without it) and it was mounted over
a ground plane of 10 cm × 3 cm. The results obtained with Epcos SAW devices were repeated with tests using custom made SAW devices fabricated by Euroquartz. The device had an array of piezoelectric electrodes over an area of 3 mm × 3 mm acting as a resonator. However, its effective dimensions while including the connecting leads was 9 mm × 7 mm × 1 mm. Efficiency above 60% was obtained using matched elements and in the range of 40% when the device was removed and replaced with a wire of length 0.9 cm with matched elements [Fig. 4(d)]. The drop in efficiency when the SAW device was removed is indicated by the red curve in Fig. 4(d), confirming the role of the SAW device in electromagnetic radiation while highlighting the fact that the impedance matching elements also have a role in overall radiation.

The efficiency in all these measurements drops significantly when both the electrodes are excited in a symmetric manner; i.e., input and input ground electrodes are connected to the voltage source. In this configuration, the system does not work as efficient antennas, although, there are measurable levels of radiation [27]. Electromagnetic coupling between piezoelectric material and radio frequency magnetic fields through electromagnetic induction in the piezoelectric stack at medium frequencies (300–500 KHz) was observed in the near field region [28]. In the present work, ultrahigh frequency has been used and asymmetric excitation is introduced in a thin film of piezoelectric material which results in increased radiation efficiency. The efficiency is comparable to other high permittivity dielectric material antennas having similar physical dimensions. However, piezoelectric materials have some advantages considering the fact that they can be made in thin film form using deposition techniques.

The resonant frequency of a piezoelectric material based antenna can be calculated using Eq. (5) while incorporating the dimensions of the ground plane which influence the boundary conditions. When the size of the antenna is much smaller than the wavelength of electromagnetic radiation, the resonant frequency can be calculated by considering the capacitance and inductance of the system.

We have shown that explicit symmetry breaking in the structural configuration of charges leads to symmetry breaking of the electric field which results in electromagnetic radiation due to nonconservative current within a localized region of space and time. These observations are discussed in the context of a two wire transmission line where, as the lines are flared, radiation occurs as the translational symmetry of electric field is broken and current in the wire is not conserved. Radiation from a resonator comprising an inductor and a capacitor under asymmetric excitation is also presented.

The phenomenon of radiation from DRAs is discussed to further validate the concept of radiation under explicit symmetry breaking. The theoretical observations are further verified by using thin film based piezoelectric film based structures which were excited in the symmetric and asymmetric mode. It was found that there is significant enhancement of radiation under asymmetric excitation. This offers a general way of designing antennas using ultrasmall structures which may eventually pave the way towards integrating antennas with semiconductor fabrication technology.

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