# Design of Microstrip Cross-Coupled Bandpass Filter With Multiple Independent Designable Transmission Zeros Using Branch-Line Resonators

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Abstract—This letter proposes a novel branch-line resonator to design a fourth-order cross-coupled bandpass filter (BPF). Theoretically, the proposed branch-line resonator plays two crucial roles. First, the arbitrary number transmission zeros can be designed in the stopband of a fourth-order cross-coupled BPF. Second, the required external quality factor can also be designed without an impedance transformer. This study examines a novel fourth-order cross-coupled BPF with a single pair of transmission zeros produced by a cross-coupled mechanism near the passband and five independent designable transmission zeros created by the proposed resonator in the stopband.

*Index Terms*—Branch-line resonator, cross-coupled bandpass filter (BPF), external quality factor, transmission zeros.

## I. INTRODUCTION

N conventional non-0° feed fourth-order cross-coupled bandpass filters (BPFs) [1] designed using a coupled-resonator method, selectivity is favorable because a single pair of transmission zeros can be produced near the passband by the cross-coupled mechanism. However, the stopband rejection level may be degraded because the signal can directly pass from the input resonator to the output resonator. Furthermore, the two transmission zeros must be designed simultaneously; that is, each transmission zero can not be designed independently, and a small electric coupling is required if the two transmission zeros are designed far from the passband. However, it may be difficult to extract a precise small electric coupling; that is, the frequencies of the two transmission zeros cannot be easily predicted in a small electric coupling. Consequently, the two zeros caused by the cross-coupled mechanism are unsuitable for the stopband frequencies, which are far from the passband. Therefore, the  $0^{\circ}$  feed fourth-order cross-coupled filter [2] was proposed to improve the stopband response by creating two additional transmission zeros in the stopband. Although the stopband levels can be considerably improved near the two additional transmission zeros, designing the frequency of the two zeros to match the desired stopband is difficult when the specific external quality factor is determined. In addition, a filter that involves using dissimilar resonators [3] was designed to extend the stopband

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response, but the desired suppression frequencies were unable to be indicated easily. Transmission zeros have been designed in a filter by [4] and [5], thus improving the required stopband frequency responses. To increase the number of zeros in the stopband, the net-type filter [6] with multiple quarter-wavelength lines at the input/output resonator for out-of-band rejections was proposed, but the locations of zeros is difficult to design when the external quality factor is fixed.

In this study, the proposed branch-line resonator was used to design the input and output resonators of a novel fourth-order cross-coupled BPF. The proposed branch-line resonator is composed of N section open stubs. Theoretically, each branch-line resonator in the proposed filter can design N - 1 independent transmission zeros in the stopband. In other words, the proposed filter can design an arbitrary number of transmission zeros in the stopband. Based on literature review, previous studies have not developed a method to design arbitrary number transmission zeros in the stopband of a fourth-order cross-coupled filter based on the coupled-resonator method. This study focused on the detailed design of multiple independent transmission zeros. In addition, by properly designing the proposed branch-line resonator, the required external quality factor can also be met without additional transformers, such as in [7] and [8].

# II. DESIGN OF TRANSMISSION ZEROS BASED ON THE PROPOSED BRANCH-LINE RESONATOR

The proposed branch-line resonator with multiple transmission zeros can be composed of N shunt stubs, as shown in Fig. 1. The branch-line resonator has N - 1 open stubs for the design of multiple independent transmission zeros; that is, each length of the N - 1 open stubs is designed to equal quarter wavelength  $(\lambda/4)$  near the unwanted frequency, and the remnant stub is used for coupling between adjacent resonators. Theoretically, an arbitrary number of zeros can be created by the shunt stubs. However, the unwanted coupling is introduced when a large number of stubs are used. Thus, zeros can be designed at different frequencies to reduce the undesired couplings between the stubs. The manner of calculating the external quality factor of the resonator has been presented in [7] and [9].

The input admittance  $Y_{in}$  and the external quality factor  $Q_e$ , defined at the port of Fig. 1 can be written as

$$Y_{in} = \sum_{i=1}^{N} \frac{j \tan \theta_i}{Z_i} \tag{1}$$

$$Q_e = \frac{R_L}{2} \sum_{i=1}^{N} \frac{\theta_i \sec^2 \theta_i}{Z_i}.$$
 (2)

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Fig. 1. Equivalent circuit model of the proposed branch-line resonator

In (1) and (2),  $Z_i$  and  $\theta_i$ , i = 1, 2, ..., or N are the characteristic impedance and the electric length of each open stub, and  $R_L$  is the load impedance from the branch-line resonator to the load. In the filter structures used by [7] and [8], each input or output resonator can design one additional independent transmission zero to increase the required stopband suppression level. However, to meet the required external quality factor of each port (i.e., the required impedance match of each port), an additional transformer may be added before the input resonator or after the output resonator, which may require a larger circuit area. In the proposed filter design, the branch-line resonator can design its external quality factor without an additional impedance transformer. For example, as shown in Fig. 1, the electrical length of Stub  $T_i$  (or  $T'_i$ ), i = 1, 2, ..., or N - 1 is fixed when Stub  $T_i$ (or  $T'_i$ ), i = 1, 2, ..., or N - 1 is designed for N - 1 transmission zeros, for which each stub is equal to  $\lambda/4$  of each desired transmission zero frequency. The variables  $Z_i$ , i = 1, 2, ...,or N, and  $\theta_N$  select two unknown variables, and the remnant variables can be arbitrarily designed. Subsequently, the two unknown variables can be solved using (1) = 0 ( $Y_{in} = 0$ ) and (2), where  $Y_{in} = 0$  is the resonant condition of the branch-line resonator. Because the external quality factor  $Q_e$  in (2) is selected for the required value of filter specification, the additional transformers used in [7] and [8] can be avoided based on the proposed branch-line resonator design. The branch-line resonator using the conditions (1) = 0 (resonant condition) and (2) to solve the two remnant unknown variables is similar to the method used in [10], which can avoid the use of an additional impedance transformer.

## III. DESIGN OF THE PROPOSED BANDPASS FILTER WITH MULTIPLE INDEPENDENT DESIGNABLE TRANSMISSION ZEROS

Fig. 2 shows the conventional non-0° feed fourth-order crosscoupled BPF [1], which can usually produce two transmission zeros to increase passband selectivity. Theoretically, as shown in Fig. 2, one additional transmission zero can be created when designing a short circuit in both signal transmission paths (Paths 1 and 2) simultaneously. Therefore, placing the short circuit at the feeding point of the input or output resonator (Point A or Point B in Fig. 2) may be a favorable choice because the short circuit can be shared by both paths (Paths 1 and 2). As stated in [7] and [8], by adjusting the feeding position of the input or output resonator, one additional short circuit may be easily designed at the feeding point of the input or output resonator; however, it increases the size of the transformer. Therefore, this study proposes a new fourth-order cross-coupled filter to obtain additional independent controllable transmission zeros



Fig. 2. Conventional microstrip non-0° feed fourth-order cross-coupled BPF structure.



Fig. 3. Layout of the proposed microstrip BPF using branch-line resonators near input and output ports.

without requiring additional transformers. The design replaces the input and output resonators of the conventional non-0° feed cross-coupled filter (Fig. 2) with the proposed branch-line shape resonators presented in Section II. Fig. 3 shows the layout of the filter with five additional independent transmission zeros compared with a conventional non-0° feed cross-coupled filter (Fig. 2). In addition, the cross-coupled filter structure [1] was used to design the proposed filter, which can produce one pair of transmission zeros near the passband to improve selectivity.

This BPF (Fig. 3) was designed for a 3 dB fractional bandwidth (FBW) of approximately 6% and a center frequency of approximately 2 GHz. Based on the specification, the external quality factor and coupling coefficients were  $Q_e = 15.91$ ,  $M_{12} = M_{34} = 0.052, M_{23} = 0.046$ , and  $M_{14} = 0.01$ , where  $Q_e$  is the external quality factor, and  $M_{ij}$  is the coupling coefficient between  $R_i$  and  $R_j$ . For the input resonator  $R_1$ (stub number N = 4), the electrical lengths of open Stubs  $T_1$ ,  $T_2$ , and  $T_3$  were approximately 90° at 2.8 GHz, 4.2 GHz, and 3 GHz, respectively; that is,  $\theta_1 = 64.3^\circ$ ,  $\theta_2 = 42.9^\circ$ , and  $\theta_3 = 60^\circ$  at approximately 2 GHz, respectively. Therefore, three transmission zeros can be created near 2.8 GHz, 4.2 GHz, and 3 GHz. In addition, this study set  $Z_1 = Z_2 = Z_3 = 70 \Omega$ , and the remnant unknown variables,  $Z_4$  and  $\theta_4$ , can be calculated using (1) = 0 ( $Y_{in} = 0$ ) and (2), where (1) = 0 is the resonant condition of the proposed resonator and (2) is the



Fig. 4. Measured and simulated results of Fig. 3.

 TABLE I

 Comparison Between Previous Works and This Work

References	Operation	Insertion loss	Number of	Stopband response
and this	frequency	(dB)	predicted zeros	
work	$(f_0)$		in stopband	
[3]	1.51 GHz	2.7	2	-30 dB up to 8.2f <sub>0</sub>
[4]	2.4 GHz	0.78	3	-20 dB up to 4f <sub>0</sub>
[5]	1.5 GHz	2.52	3	-23.7 dB up to 10.6f <sub>0</sub>
This work	2 GHz	2.6	5	-27 dB up to 2.88f <sub>0</sub>

specific external quality factor of the filter. Subsequently, this study obtained  $Z_4 = 50 \ \Omega$  and  $\theta_4 = 106.5^\circ$  at approximately 2 GHz. For the output resonator R<sub>4</sub> (stub number N = 3), the electrical lengths of open Stubs  $T'_1$  and  $T'_2$  were approximately  $90^\circ$  at 1.6 and 3.6 GHz, respectively; that is,  $\theta_1 = 112.5^\circ$  and  $\theta_2 = 50^\circ$  at approximately 2 GHz, respectively. Therefore, two transmission zeros can be created near 1.6 and 3.6 GHz. In addition, this study set  $Z_2 = Z_3 = 50 \ \Omega$ , and the remnant unknown variables,  $Z_1$  and  $\theta_3$ , can be calculated using (1) = 0 $(Y_{in} = 0)$  and (2). Subsequently, this study obtained  $Z_1 = 33 \ \Omega$ and  $\theta_3 = 67.9^\circ$  at approximately 2 GHz. The remnant uniform-impedance resonators, R<sub>2</sub> and R<sub>3</sub>, were designed to resonate at 2 GHz, as shown in Fig. 3.

The substrate of the implemented filter has a relative dielectric constant of 3.65, a thickness of 0.508 mm, and a loss tangent of 0.0095. The detailed dimensions are shown in Fig. 3, and Fig. 4 shows the simulated and measured results of the proposed filter. The measured center frequency, 3 dB FBW, and minimal insertion loss are approximately 2 GHz, 5.9%, and 2.85 dB, respectively. By using cross-coupled BPF structure [1], one pair of transmission zeros was produced near the passband to improve selectivity. The five independent designable transmission zeros were measured at approximately 1.65 GHz (TZ<sub>1</sub>), 2.71 GHz (TZ<sub>2</sub>), 3.05 GHz (TZ<sub>3</sub>), 3.6 GHz (TZ<sub>4</sub>), and 4.21 GHz (TZ<sub>5</sub>), respectively. The five additional independent

transmission zeros contributed by the proposed branch-line resonators (R1 and R4) can result in five frequency depths to improve the rejection levels of the desired stopbands. Here, the zero generated by the stub  $T'_1$  at approximately 5 GHz is also observed. Table I shows a comparison between this work and some harmonic-suppressed filters without a transformer at the input/output port.

### IV. CONCLUSION

This letter proposes a novel microstrip fourth-order crosscoupled filter that involves using the proposed branch-line resonator with five independent designable transmission zeros in the stopband. The proposed branch-line resonator can design the external quality factor and transmission zeros separately. Theoretically, the number of transmission zeros in the proposed filter is unlimited. Therefore, by properly designing the proposed branch-line resonator, more transmission zeros can be obtained in the stopband of a fourth-order cross-coupled filter than in those of the implemented filter.

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