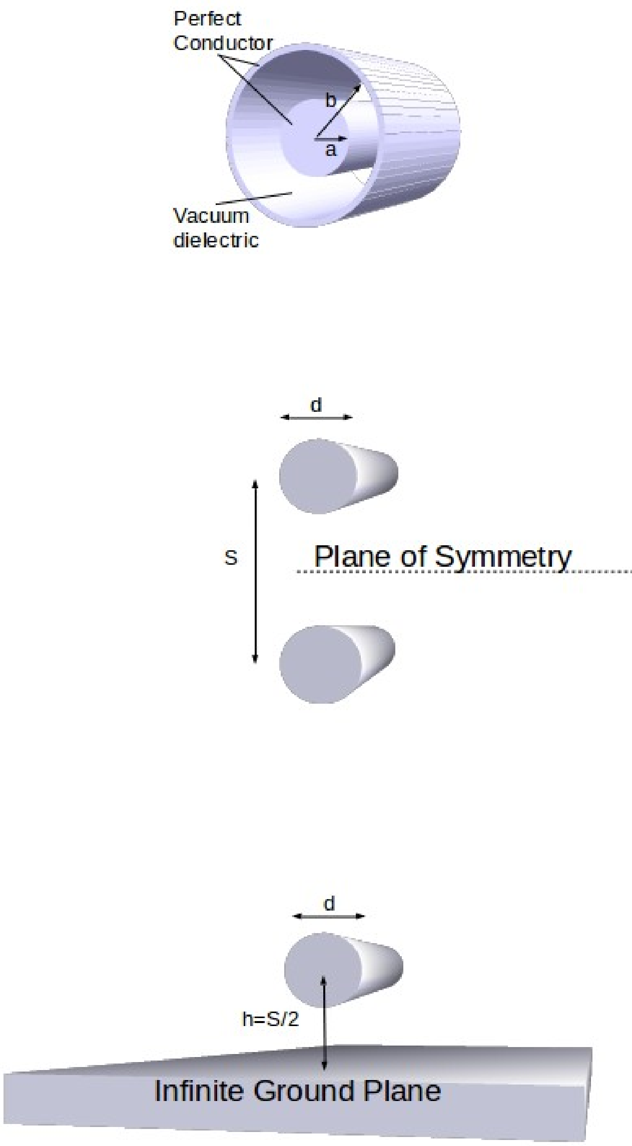


a. Coaxial Cable

b. Balanced Line

c. Wire-over-Ground

Fig. 1

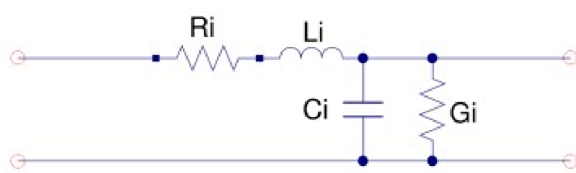
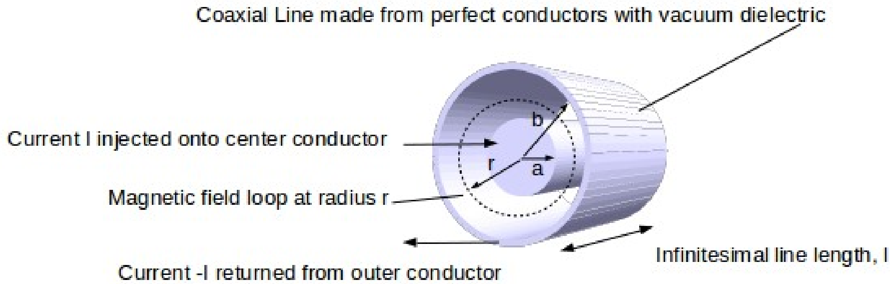


Fig.2



The enclosed current outside the coax is zero so within it at radius r, by Ampère's law

$$\oint \vec{B} \cdot \partial \vec{s} = 2\pi r B = \mu_0 I_{enc} = \mu_0 I$$

where $\vec{B}$ is the magnetic field, I the enclosed current and μ_0 is the permeability of space

so within the coax vacuum $B = \frac{\mu_0 I}{2\pi r}$

The energy density per unit volume $u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

Considering the volume within a short length of line, the energy is

$$U_B = \iiint u_B(\partial Volume) = \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} \cdot 2\pi r\, l\, \partial r = \frac{\mu_0 I^2 l}{4\pi} \int_a^b \frac{1}{r} \partial r = \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right)$$

and energy per unit length is

$$U_{B,per\ length} = U_{B,i} = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

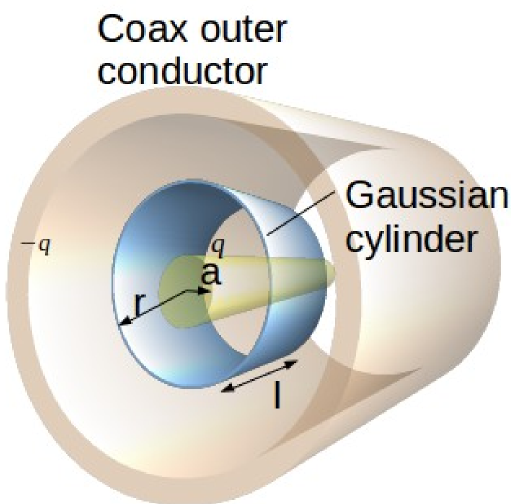
Stored energy is related to inductance and current by

$U_B = \frac{I^2 L}{2}$ which gives, inductance per unit length $L_i = \frac{2 U_{B,i}}{I^2}$

so

$$L_i = \frac{2 U_{Bi}}{I^2} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

Fig. 3



Considering a short section of coaxial transmission line made from perfect conductors,

a vacuum dielectric and a Gaussian surface at radius r with length l and with charge q and -q

on the conductors as shown, the charge/length = $\frac{q}{l}=\alpha$

Gauss' law gives magnitude of electric field, $\vec{E}$ pointed toward the center

$$\int \vec{E}\cdot\partial\vec{A}=\frac{q_{included}}{\epsilon_0}=\frac{\alpha l}{\epsilon_0}$$

$$E\,2\pi r l=\frac{\alpha l}{\epsilon_0}$$

$$E=\frac{\alpha l}{2\pi\epsilon_0 r l}=\frac{q}{2\pi\epsilon_0 r}$$

Referencing the outer conductor as one side of a capacitor with $V_b=0$

the voltage across that capacitor is the potential difference to the inner conductor

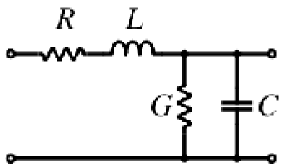
$$V_b-V_a=-V_{capacitor}=\int_a^b \vec{E}\cdot\partial\vec{r}=\int_a^b \frac{q}{2\pi\epsilon_0 r}\partial r=\frac{q}{2\pi\epsilon_0}\int_a^b \frac{1}{r}\partial r$$

$$V_{capacitor}=\frac{q}{2\pi\epsilon_0}(\ln(b)-\ln(a))=\frac{q}{2\pi\epsilon_0}\ln(\frac{b}{a})$$

so the capacitance per unit length is

$$C_i=\frac{C}{l}=\frac{q}{lV_{capacitor}}=\frac{2\pi\epsilon_0}{\ln(\frac{b}{a})}$$

Fig. 4



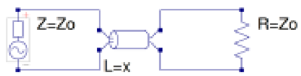
$\frac{dR}{dx}$, series resistance per unit length

$\frac{dL}{dx}$, series inductance per unit length

$\frac{dG}{dx}$, shunt conductance per unit length

$\frac{dC}{dx}$, shunt capacitance per unit length

Inserting Heaviside's model into a circuit with source (transmitter) and load



Using image parameter theory[1], a complex propagation constant

describes the line.

$$\gamma=\alpha+j\beta=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$\alpha=$ attenuation constant, nepers/meter ($\approx$ 8.69 dB/meter) and $\beta=$ phase constant, radians/meter

for low loss line $\alpha\approx\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}$ and $\beta\approx\omega\sqrt{LC}$

For the ideal, lossless case, R and G are zero so that the propagation constant becomes only imaginary.

$$\gamma=\alpha+j\beta=0+j\omega\sqrt{LC}$$

With sinusoidal (CW) drive from the source, the voltage at point l on the line is $\gamma=\alpha+j\beta=0+j\omega\sqrt{LC}$

$$V=V_0\sin(\omega t)e^{-\gamma l}$$

The characteristic impedance is $Z_0=\sqrt{\frac{L}{C}}$

The propagation velocity (phase velocity) of the voltage or current is $U_p=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}$

Using the values derived in Figs 2,3 in the absence of any loss, dielectric or permeable material

The Velocity Factor is $V_r=\frac{U_p}{c}=\frac{1}{c\sqrt{LC}}=\frac{1}{c\sqrt{\mu_0\epsilon_0}}=1$

Whle the Characteristic Impedance is $Z_0=\sqrt{\frac{L}{C}}=\frac{\sqrt{\frac{\mu}{\epsilon}}\ln(\frac{b}{a})}{2\pi}\approx 60\ln(\frac{b}{a})\approx 138\log_{10}(\frac{b}{a})$

Fig. 5

[1]Matthei, Young,Jones, *Microwave Filters; Impedance-matching Networks, and Coupling Structures*,McGraw Hill 164, Chapter 3, p49 ff

with c and μ_0 defined as universal constants ϵ_0 Is derived from them:

$c \equiv 299792458$ meter/second, speed of light in a vacuum

$\mu_0 \equiv 4\pi x 10^{-7}$ Henry/meter, permeability of space

$\epsilon_0 = \frac{1}{c^2 \mu}$ Farad/meter, permittivity of space

within classical physics, it has been accepted that a wave in free space propagates at the speed of light, c, and that space itself sets a maximum impedance of

$Z_{space} = \mu_0 c = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377$ ohms

Referring to Figs. 2-3 and Fig. 5, using Heaviside Telegrapher's Equation

Coax Impedance $Z_0 = \sqrt{\frac{L_i}{C_i}} = \frac{\sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right)}{2\pi}$ exceeds $Z_{space} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ when $\ln\left(\frac{b}{a}\right) > 2\pi$ which occurs at geometries where $\left(\frac{b}{a}\right) > e^{2\pi} \approx 535$

$C_i = \frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}$ becomes less than ϵ_0, the permittivity of space

and

$L_i = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$ becomes greater than μ_0, the permeability of space

so

Z_0 exceeds $\frac{\sqrt{\frac{\mu_0}{\epsilon_0}}(2\pi)}{2\pi} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377$ ohms which is the impedance of free space.

Fig. 6

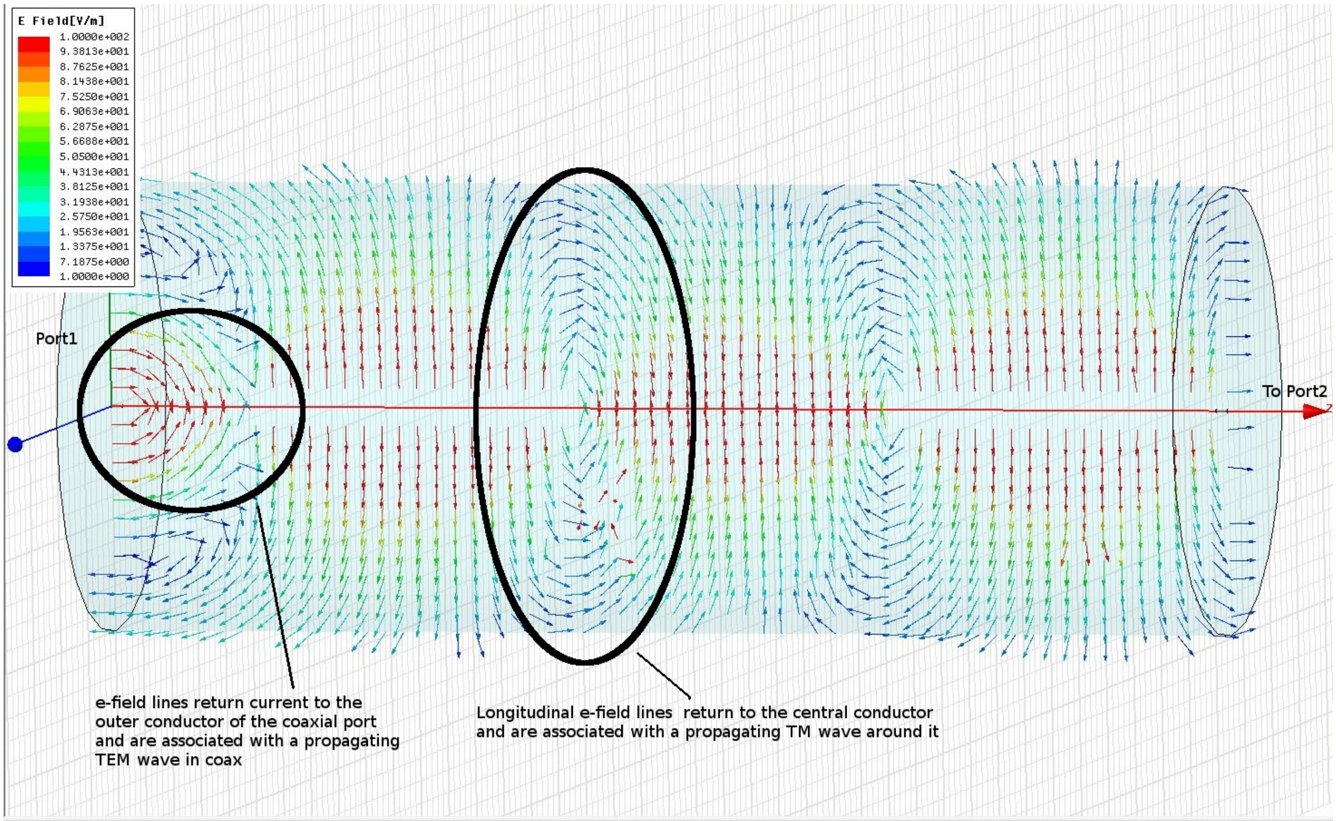

E Field[V/m]
1.0000e+002
9.3813e+001
8.7625e+001
8.1438e+001
7.5250e+001
6.9063e+001
6.2875e+001
5.6688e+001
5.0500e+001
4.4313e+001
3.8125e+001
3.1938e+001
2.5750e+001
1.9563e+001
1.3375e+001
7.1875e+000
1.0000e+000
Port1
To Port2
e-field lines return current to the outer conductor of the coaxial port and are associated with a propagating TEM wave in coax
Longitudinal e-field lines return to the central conductor and are associated with a propagating TM wave around it

Fig. 7

Photo 1