A Versatile Leaky-Wave Antenna Based on Stub-Loaded Rectangular Waveguide: Part I—Theory

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***Abstract*— A new leaky-wave antenna is presented here that possesses many desirable features and is suitable for application to both the millimeter-wave and microwave ranges. These desirable features, some of which are unusual, include a simple configuration, a wide flexibility in the range of available beamwidths, the ability to control the beamwidth and the direction of the beam essentially independently, and negligible cross polarization at all scan angles. The antenna structure consists of a parallel-plate stub guide of small height, less than a half wavelength, located off center on the top of rectangular waveguide. The beamwidth is easily controlled from very wide to very narrow by adjusting the stub width or location. Part I presents the principle of operation and the theory, employing a new transverse equivalent network that is accurate, but also simple, so that it permits rapid and inexpensive numerical calculations. Part II describes and derives the modifications required in the theory to account for flanges and/or finite stub length, and Part III is devoted to measurements at X-band and from 40 to 60 GHz on both the propagation properties and the radiation patterns; excellent agreement with the theory is demonstrated.**

***Index Terms*— Leaky-wave antennas.**

I. Introduction

A leaky-wave line-source antenna is basically an open waveguide possessing a mechanism that permits a slow leakage of power along the length of the waveguide. This length then constitutes the radiating aperture of the line source and the radiation occurs in the form of a conical fan beam that is narrow in the plane of scan. When the aperture is horizontal and faces upward, the radiated beam is customarily scanned in elevation by varying the frequency.

Most of the early leaky-wave antennas were based on closed rectangular waveguide where leakage was produced by introducing a physical cut in the waveguide top or side wall in the form of a long slit or a series of closely spaced holes. Excellent summaries are presented in [1] and [2]. Within the past two decades, interest has shifted to the *millimeter-wave range*, where the smaller wavelengths and higher waveguide losses introduced new problems and, therefore, new challenges. To overcome these new problems, various novel leaky-wave antennas were proposed, analyzed, and measured based on some low-loss open waveguide designed specifically for use at millimeter wavelengths. Since these waveguides are already open, a physical cut to produce leakage is not meaningful and new mechanisms were tried for permitting the initially bound dominant mode to leak, the most common of which was the introduction of *asymmetry*. The most widely used of these open waveguides for antenna purposes are the groove guide, the NRD guide, and the dielectric image guide. Summaries of the most successful of these new antennas together with their principles of operation are presented in [3] and [4].

A desirable leaky-wave line-source antenna should have the following properties. It should involve a simple structure, be easily fed by a standard source, have low metal loss, radiate in a single polarization (with negligible cross polarization at all scan angles), possess flexibility in beamwidth (narrow or wide, as we wish), and permit one to change the beamwidth without affecting the angle of maximum radiation. Most of the previous antennas satisfied most, but not all, of these desirable characteristics, but the antenna described in this paper seems to be the best so far in attempting to satisfy all of these features.

The actual antenna structure is certainly simple in form (as seen in Fig. 1) consisting of a rectangular waveguide with an asymmetrically located stub guide. The antenna can be fed directly from an ordinary rectangular waveguide source, perhaps with a small modification at the feed junction between the source and the antenna. This question is considered further in Part III, when measurements are discussed. It is interesting that this antenna structure was not conceived directly by us from a rectangular guide, but actually evolved as a modification of groove guide and was discussed in much less complete form at two conferences [5], [6]. It was referred to then as the "offset groove-guide" antenna, where the groove-guide top and bottom stubs were shifted over, or offset, to introduce the required *asymmetry* so that the initially bound dominant mode would become leaky. It was only later, when we considered

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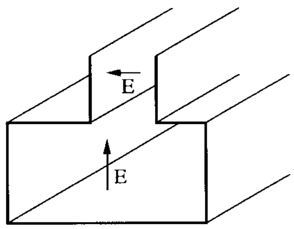

Fig. 1. The stub-loaded rectangular waveguide leaky-wave line-source antenna. The stub guide is located asymmetrically to produce a horizontal electric field component that propagates as a TEM mode at an angle in the stub guide and radiates power from the open end.

how the antenna could be fed, that we realized that it was better and simpler to view the structure as a stub-loaded rectangular waveguide.

The description of this antenna is divided into three parts devoted, respectively, to the principle of operation and the theory in Part I, to the modifications in the theory needed to account for flanges and finite stub length (or height) in Part II, and to the measurements of both the propagation constants and the radiation patterns in Part III.

This paper (Part I) first discusses the *principle of operation* in Section II. That discussion summarizes some of the basic properties of leaky-wave antennas and explains why the structure shown in Fig. 1 possesses the claimed flexibility in beamwidth and why the radiation has negligible cross polarization at all angles.

Two entirely different theoretical approaches were employed in computing the numerical values presented here. One of them is the *mode-matching procedure*, which is well known and is accurate but time-consuming (and costly). The other is an analytical procedure based on a novel *transverse equivalent network* for the cross section of the stub-loaded rectangular waveguide structure. A dispersion relation is then obtained easily from a resonance of the transverse equivalent network. An inspection of the structure in Fig. 1 shows that a major constituent in its cross section and, in fact, central to it is an open E-plane tee junction in parallel-plate waveguide. A recent paper [7], presents a new equivalent network for just such a structure and the transverse equivalent network for this antenna is built upon that new network.

A novel feature of the network for the tee junction is that theoretical expressions have been derived for all of the elements of the network, which are in *simple closed form* and yet are accurate so that numerical values for the antenna performance can be calculated *quickly* and *inexpensively*. An independent verification of the accuracy of these tee junction expressions has been provided in a recent paper [8]. Section III presents the details of the transverse equivalent network and derives the dispersion relation for the propagation characteristics from the transverse equivalent network.

Because the dispersion relation is in simple closed form, one might question its accuracy. For this reason, before the recent comparison in [8] became available, we made computations using the mode-matching procedure (which is a completely different theoretical approach) to compare with the numerical values obtained using the equivalent network approach. Some of these comparisons are also shown in Section III where it is seen that the agreement is excellent. In addition, it is demonstrated in Part III (on measurements) that the measured values agree very well with the numerical calculations made using the equivalent network. We, therefore, have strong evidence that the analytical dispersion relation derived in Section III is *accurate* in addition to being simple.

Numerical results for the propagation behavior are discussed in Section IV. The variations of the normalized phase constant and leakage constant are examined first as a function of several *geometrical parameters*. To assist in the design of the leaky-wave antenna, particularly with respect to sidelobe control, one wishes to know which geometrical parameter can be varied so as to alter the value of the leakage constant while maintaining the phase constant the same. The results in Section IV indicate that changing either the off center location of the stub guide or the width of the stub guide will perform that function, although the former choice is superior with respect to the constancy of the phase constant.

The next set of numerical values in Section IV relates to the performance as a function of *frequency*, which is important since the beam direction is scanned by varying the frequency. The behavior is presented over a very wide range of frequencies, both above and below cutoff. The behavior below cutoff is not generally known and may be of interest in its own right. Because the structure is air-filled, the beamwidth stays constant as the beam is scanned by varying the frequency; this known result is demonstrated numerically.

The dispersion relation derived in Section III and the numerical values presented in Section IV hold for a stub guide of infinite length (or height). With that approximation to reality, we are able to work with simpler analytical expressions and to perform faster calculations and yet obtain good numerical results that provide sufficient insight for design purposes. In practice, however, the stub guide is of *finite* length and we must know the effects on antenna performance of varying the length of the stub guide. These considerations are treated in detail in Part II.

Paper III is concerned with *measurements* made at X band and at millimeter wavelengths (40–60 GHz). In that paper, we describe the specific structures, which employ flanges and explain how the measurements were taken for both the propagation wavenumbers and the radiation patterns. Careful comparisons are presented between the measured and theoretical values, verifying that very good agreement was obtained.

II. Principle of Operation

A. Background

A leaky-wave line-source antenna is based on an open waveguide that is characterized by a complex propagation wavenumber $k_z = \beta - j\alpha$, where β and α are the phase constant and leakage constant of the guided mode, respectively. If there are also metal and/or dielectric losses, the quantity α comprises both the material losses and the leakage (or radiation) losses and is appropriately termed the attenuation constant. We assume that the material losses here may be neglected. Once β and α are known as a function of the waveguide geometry and the frequency, all the antenna properties can be determined readily or, alternatively, the antenna can be designed to conform to specifications. The objective of the theory for a leaky-wave structure is, therefore, to determine β and α as functions of the geometry and frequency; analytical expressions that provide such information are presented in Section III for the antenna discussed in this paper and corresponding numerical results are contained in Section IV.

Before discussing the specific antenna of concern here, we should recall that simple approximate expressions are available [4] that relate the β and α of the leaky guided mode to the angle θ_m of maximum radiation measured from the broadside direction (perpendicular to the leaky waveguide axis) and to the 3-dB beamwidth $\Delta\theta$ of the radiated beam. These relations are

$$\sin\theta_m \cong \beta/k_o \tag{1}$$

$$\Delta\theta \cong \frac{1}{(L/\lambda_o)\cos\theta_m} \tag{2}$$

$$\frac{L}{\lambda_o} \cong \frac{0.183}{\alpha/k_o}. \tag{3}$$

In (1)–(3), k_o is the free-space wavenumber $(= 2\pi/\lambda_o)$ and L is the length of the leaky-wave antenna. Both θ_m and $\Delta\theta$ are in radians in (1) and (2).

The beamwidth $\Delta\theta$ is determined in (2) primarily by the antenna length L, but it is also influenced to a lesser extent by the aperture-field amplitude distribution, being narrowest for a constant aperture field and wider for more peaked distributions. The factor unity in the numerator in (2) is a middle-of-the-range result. Length L is usually chosen corresponding to a given value of α so that about 90% of the power is radiated and the remaining 10% or so is absorbed by a matched load. For 90% of the power radiated and for an untapered geometry, the relation between L and α is given by (3). If both L and α are specified independently, the percentage of power radiated can deviate significantly from the desired 90% and the radiation efficiency can be poor.

We see from (1)–(3) that numerical values in the form of the normalized quantities β/k_o and α/k_o are valuable to have because they relate directly to the antenna radiation angle θ_m and beamwidth $\Delta\theta$. Section IV displays the numerical values in that form.

The aperture amplitude distributions of leaky-wave antennas are customarily tapered in some specified fashion to reduce and otherwise control the sidelobes of the radiation pattern. The procedure to be followed in relating the desired aperture distribution to the variation of the leakage constant α along the aperture length is well known and is the same for all leaky-wave antennas. A problem arises, however, in that for many leaky-wave antennas, a taper in the geometry of the cross section in order to affect the value of α also modifies the value of β, at least to some extent. But β must be maintained constant along the length so that the radiation from all sections of the antenna point in the same direction. Thus, the taper must then be modified slightly to reinstate the constancy of β. This process usually requires only a single iteration, but it points out the strong desirability of being able to identify an appropriate geometrical parameter so that when α is altered β is disturbed negligibly. The antenna described in this paper satisfies this desirable condition. A systematic further discussion of the features considered in this subsection on background may be found in [4].

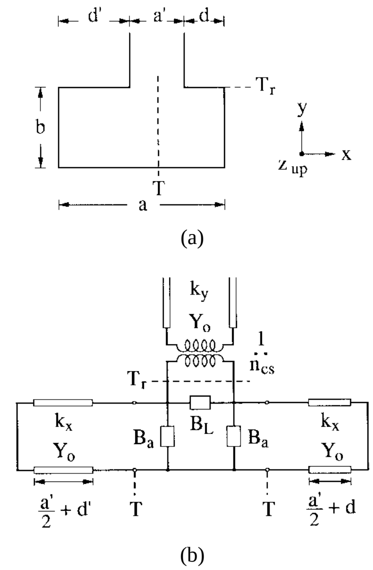

Fig. 2. (a) The cross section of the stub-loaded rectangular waveguide antenna. (b) Its transverse equivalent network, which utilizes the new E-plane tee junction equivalent network presented in [7].

B. The Stub-Loaded Rectangular Waveguide Antenna

The structure of this antenna is shown in Fig. 1, and is seen to consist of a rectangular waveguide with an off-center open-top stub guide. The major electric field directions are also indicated there. The dimensions of the cross section are shown in Fig. 2(a). The leakage mechanism employed here is *asymmetry*, which is provided directly by the off-center stub guide.

Although the stub guide is shown here with an open top in the form of a pair of baffles, it may be more convenient to have the open top connect to a pair of flanges or a ground plane. In our measurements in Part III we employ such flanges and in Part II we present the modifications in the theory required to take into account that change in the geometry.

The principle of operation is very simple and straightforward. The rectangular waveguide is fed in its dominant mode from one end and some field penetrates into the stub guide. Because of the asymmetry, a horizontal component of electric field is excited at the opening to the stub guide in addition to the vertical electric field component due to the rectangular waveguide dominant mode. These horizontal and vertical electric field components give rise, respectively, to a TEM mode and a TM_1 mode with respect to the y direction in the parallel-plate stub guide. Due to the narrower width a' of the stub guide, the TM_1 mode remains below cutoff and decays exponentially in the vertical (y) direction. The TEM mode, however, propagates at an angle upwards in the stub guide, reaches the open top, and radiates power in horizontal polarization. If the stub guide is allowed to be sufficiently long (or high), the field of the TM_1 mode will decay to negligible values as it reaches the open end, and the radiated power will possess *essentially pure horizontal polarization*. The stub guide is, therefore, located asymmetrically to permit the excitation of a horizontal electric field component and made long enough to essentially completely suppress the vertical field component.

To be more precise, we should realize that the two transverse parallel-plate modes actually propagate at an angle with components in both the longitudinal (z) and vertical (y) directions. Furthermore, these transverse modes are part of the overall basic leaky mode, with the radiating "TEM" mode at an angle, in fact, producing the leakage. Nevertheless, the simplified phrasing in the previous paragraph contains the essence of the physical picture and is useful in explaining in simple terms the basic principle of operation.

It is important to understand that the leakage rate (the value of α and, therefore, the beamwidth $\Delta\theta$) can be *controlled* neatly by the amount of offset of the stub guide. If the stub guide is *centered* on the rectangular waveguide top, the only electric field component excited at the stub-guide opening is the vertical one. If the stub guide is sufficiently long (or high), no leakage will occur at all and the situation would resemble that of a slotted-section cut. If the stub guide is moved all the way over to one side $(d = 0)$, the resulting structure becomes the simple L-shaped leaky-wave antenna, which was described and analyzed previously [9], [10]. The L-shaped antenna was extremely simple in shape but was found to have too strong a leakage rate and was, therefore, suitable only when large beamwidths were required. We see, therefore, that by varying the amount of offcenter shift (or, equivalently, by varying the shift d) we can vary the value of α from zero to a large value, so that the structure in Fig. 1 [or 2(a)] is basically *simple* and yet provides a radiation pattern that is *flexible in beamwidth*.

III. Theoretical Description

The theoretical approach used to obtain most of the numerical values presented in this paper involves an analytical procedure based on a novel *transverse equivalent network* for the stub-loaded rectangular waveguide structure. From a resonance of this transverse equivalent network, we then derive a *dispersion relation* from which the numerical values are calculated.

As mentioned in the Introduction, an unusual advantage associated with this transverse equivalent network is that analytical expressions for all the elements in the network are available in *simple closed form*. As a result, the dispersion relation itself is in simple closed form permitting the numerical values obtained from it to be calculated *quickly and inexpensively*. We show later, by comparisons with a completely different method of computation, that these simple relations are *accurate* as well. In Part III (on measurements), we find that the measured values also agree very well with numerical values calculated using this theoretical approach.

The theoretical expressions derived in this section assume that the stub guide is of *infinite length* so that the expressions become simpler and the calculations can be made even more quickly. In practice, the stub guide must be finite in length and the changes introduced by that modification are treated in Part II. The transverse equivalent network is appropriately modified there and the resulting dispersion relation is still in simple analytical form. It was found that the value of β is not affected much by changes in the stub length, but that α varies periodically with that length. The case of infinite length provides a reasonable average within the range of those α values so that numerical values computed under the infinite length assumption are useful for furnishing reasonably accurate and valid information on the effects of changing the frequency or the geometrical parameters. Final designs of practical antennas should take the finite length into account, however, and should use the dispersion relation derived in Part II.

A. The Transverse Equivalent Network

It is evident from even a quick inspection of Fig. 1 or Fig. 2(a) that the central region of the structure resembles that of an open E-plane tee junction. We may, therefore, employ the equivalent network for such a tee junction, discussed in a recent paper [7] as the central element in a transverse equivalent network representation of the stub-loaded rectangular waveguide.

The transverse equivalent network for the antenna structure whose cross section appears in Fig. 2(a) is given in Fig. 2(b). It is built around the network for the E-plane tee junction, which employs the reference plane locations T and T_r shown in Fig. 2. It is seen that the *asymmetry* in the antenna structure due to the stub being offset reflects itself in the two unequal distances d and d' shown on the cross section. In turn, the asymmetry is accounted for in the transverse equivalent network by the unequal line lengths that represent the main guide arms. Only one mode is above cutoff in the main guide arms and in the stub guide and that mode is essentially a TEM mode propagating at an angle between the parallel plates. Therefore, the wavenumbers k_x and k_y in the network transmission lines are equal as are the characteristic admittances Y_o. The different widths b and a' of the parallel-plate sections are absorbed into the turns ratio n_{cs} (see (7) below).

Before we can adapt the expressions from [7] for the elements of the tee junction, we must change the notation appropriately as follows:

$$b' \to a', \quad b \to b, \quad \lambda_g \to 2\pi/k_x.$$

In [7], the tee junction was viewed as a discontinuity in the longitudinal direction; here, the tee junction is part of a cross section. The expressions appropriate to the network in Fig. 2(b) then become

$$\frac{B_a}{Y_o} = -\frac{\pi}{16}\frac{a'}{b}\frac{k_x a'}{2}J_o^2\left(\frac{k_x a'}{2}\right) \tag{4}$$

where J_o is the Bessel function of zero order

$$\frac{B_L}{Y_o} + \frac{1}{2}\frac{B_a}{Y_o} = \frac{1}{n_c^2}\left(\frac{k_x b}{\pi}\right)\left[\ln\left(1.43\frac{b}{a'}\right) + \frac{1}{2}\left(\frac{k_x b}{\pi}\right)^2\right] \quad (5)$$

$$n_c = \frac{\sin(k_x a'/2)}{k_x a'/2} \quad (6)$$

$$n_{\rm cs}^2 = n_c^2(a'/b). \quad (7)$$

As we may readily see, (4)–(7) are in closed form and are surprisingly simple.

B. The Dispersion Relation

The dispersion relation for the propagation behavior is obtained by taking the free resonance of the transverse equivalent network in Fig. 2. We choose a reference plane at T_r just below the turns of the transformer and sum to zero the admittances seen looking up and down from T_r. We then find

$$Y_{\rm up} = \frac{1}{n_{\rm cs}^2}Y_o$$
$$Y_{\rm down} = jB_L + \frac{1}{1/(jB_a + Y_R) + 1/(jB_a + Y_L)} \quad (8)$$

where Y_R and Y_L are the main guide input admittances looking to the right of reference plane T on the right side of the network and to the left of T on the left side, namely

$$Y_R = -jY_o \cot k_x\left(\frac{a'}{2} + d\right)$$
$$Y_L = -jY_o \cot k_x\left(\frac{a'}{2} + d'\right). \quad (9)$$

Setting $Y_{\rm up} + Y_{\rm down} = 0$ and normalizing to Y_o, we obtain

$$\frac{1}{n_{\rm cs}^2} + j\frac{B_L}{Y_o} + j\frac{\left[\frac{B_a}{Y_o} - \cot k_x\left(\frac{a'}{2}+d\right)\right]\left[\frac{B_a}{Y_o} - \cot k_x\left(\frac{a'}{2}+d'\right)\right]}{2\frac{B_a}{Y_o} - \left[\cot k_x\left(\frac{a'}{2}+d\right) + \cot k_x\left(\frac{a'}{2}+d'\right)\right]} = 0 \quad (10)$$

where the elements B_a/Y_o, $n_{\rm cs}$, and B_L/Y_o are given by (4)–(7). Wavenumber k_x is related to k_z, the result we seek by

$$k_z = \beta - j\alpha = \sqrt{k_o^2 - k_x^2}. \quad (11)$$

We may observe that dispersion relation (10) is in *closed form*. From this dispersion relation, we see that k_x will be complex so that β and α will yield the phase constant and leakage constant of the leaky-wave antenna. By employing (1)–(3), these values of β and α yield information on the properties of the radiation pattern.

C. Comparison with the Mode-Matching Procedure

In order to check the *accuracy* of the theoretical approach described above, numerical calculations were made of some of the performance properties by using a *totally different* theoretical method—the accurate *mode-matching procedure*.

The mode matching is performed in the *vertical* direction in the stub-loaded rectangular guide structure shown in Fig. 2(a). In the upper (stub) waveguide of narrower width a', only the TEM mode is above cutoff; in the lower guide of width a, three modes are vertically above cutoff—the TEM mode and the first TE and TM higher order modes. The mode matching is set up rigorously and, for most points, 100 TE and TM modes are included in the numerical computations. Convergence in the mean-square sense is employed.

We should note that this method is actually entirely different from the one based on the tee-junction equivalent network, not simply that the mode-matching procedure is basically numerical and the network one is basically analytical. In the network approach, the discontinuity involved is viewed as an E-plane tee; in the mode-matching procedure it is treated as a transverse discontinuity—a change in guide width. Another major difference is that the network has closed-form expressions for its elements and is approximate; the mode-matching procedure is asymptotically rigorous.

Since the mode-matching procedure (to be accurate) must include many modes, it is computationally relatively costly. As a result, only two cases were selected with which comparisons were made with the numerical results obtained by means of the dispersion relation (10) together with (11). One of these cases involves the dependence on geometry (specifically, on d/a) and the other case is concerned with variations with frequency.

The first comparison is shown in Fig. 3, for the behavior of β/k_o and α/k_o as a function of d/a, which is a measure of the offset of the stub guide. The dashed lines represent the numerical values that were calculated using the equivalent network approach. The solid lines correspond to numerical values obtained from the accurate mode-matching procedure. It is seen that the agreement is really very good over the complete range of d/a.

The second comparison (in Fig. 4) concerns the variations of β/k_o and α/k_o over the complete frequency range. Let us note the level of agreement between the two methods of calculation; as with Fig. 3, the dashed and solid lines represent the network and mode-matching approaches, respectively. Again, the agreement is very good over the very wide range of frequencies. The behavior with frequency in this figure may seem unfamiliar to some readers; it is discussed in Section IV-B.

The principal conclusion from these comparisons is that in both cases the agreement is very good so that we may have confidence in the validity of our numerical results and in the reliability of both of the theoretical approaches.

IV. Numerical Results for the Propagation Behavior

Dispersion relation (10) together with (11) permits one to compute the variations of the phase constant β and the leakage constant α as a function of frequency and various geometrical

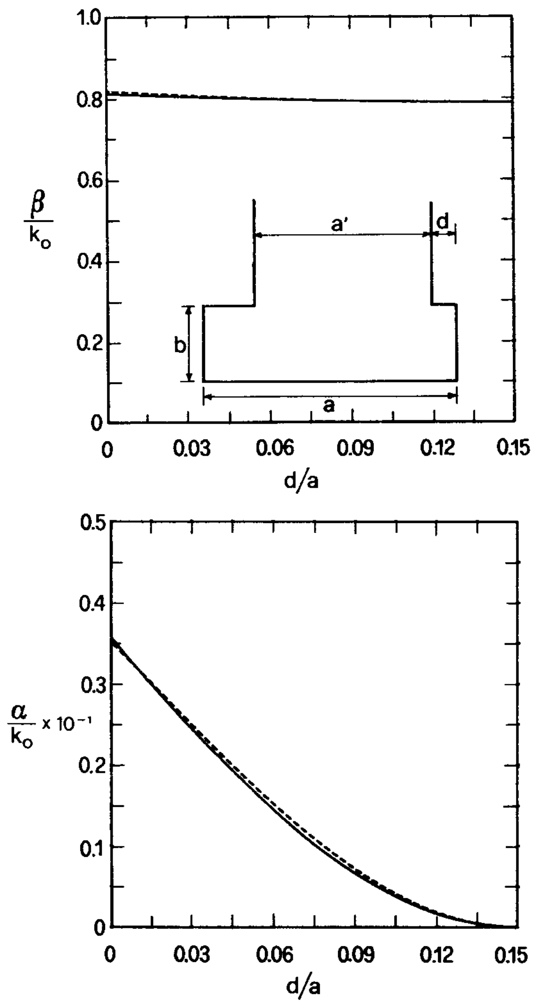

Fig. 3. Comparison of numerical results for β/k_o and α/k_o versus d/a obtained via two completely different theoretical methods. The dashed and solid lines correspond, respectively, to the tee-junction network approach and the mode-matching procedure: $f = 28$ GHz; $a = 10.0$ mm; $b = 3.0$ mm; $a' = 7.0$ mm.

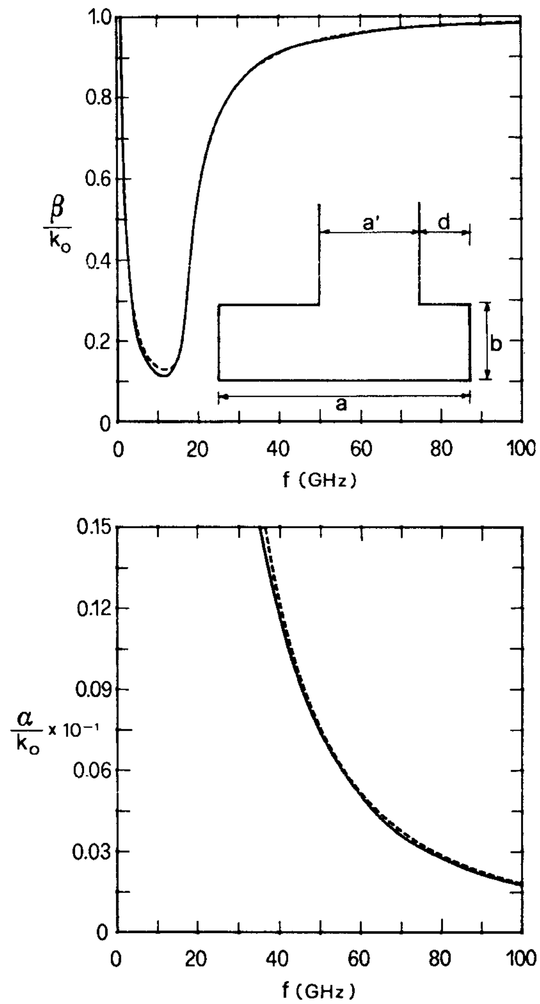

Fig. 4. Comparison of numerical results for β/k_o and α/k_o versus frequency, over a very wide frequency range obtained via two completely different theoretical methods. The dashed and solid lines correspond, respectively, to the tee-junction network approach and the mode-matching procedure: $a = 10.0$ mm; $b = 2.0$ mm; $a' = 4.0$ mm; $d = 2.0$ mm.

parameters. The numerical values will be displayed in normalized form as β/k_o and α/k_o, because those quantities are related directly to the angle θ_m of maximum radiation and the beamwidth $\Delta\theta$, as seen in (1)–(3).

A. Dependence on Geometrical Parameters

It was pointed out earlier that it is useful for design purposes to know which parameters to vary in order to adjust the radiation angle θ_m and the beamwidth $\Delta\theta$ as independently as possible. Furthermore, in order to achieve low sidelobes in the radiation pattern, it is necessary to *taper* the amplitude of the aperture distribution of the leaky-wave antenna, which means that the value of α/k_o must be slowly varied in a specified way along the antenna length. At the same time, the value of β/k_o must remain constant along the length so that all sections of the antenna radiate at the same angle. We have, therefore, two reasons for needing to know what parameter or parameters can vary α/k_o while changing β/k_o negligibly.

The first step in the design is to specify the angle θ_m of maximum radiation, which, in view of (1), means the value of β/k_o. It is clear that the dimension that controls the value of β/k_o to the largest extent is the rectangular guide width a. Anything relating to the stub guide is basically a perturbation on the rectangular guide dominant mode.

The first sets of numerical values apply to a rectangular guide aspect ratio $a/b = 2$, since that ratio is present in most rectangular waveguides at millimeter wavelengths. The antenna structure can then be fed directly from the rectangular waveguide.

The behavior of β/k_o and α/k_o as a function of *stub offset* measured as d/a, is shown in Fig. 5. The precise cross-section shape is indicated in the inset. As expected, the values of α/k_o range from zero when the stub is centered to a maximum value when the stub is at one end. This maximum value for α/k_o is about 0.077, which, for the value of $\beta/k_o = 0.76$, corresponds to a radiation pattern beamwidth of about 33° when the antenna length is taken to radiate 90% of the incident power (as is customary) and the structure is not tapered. As before, it is clear that beamwidths can be readily achieved that range from this very

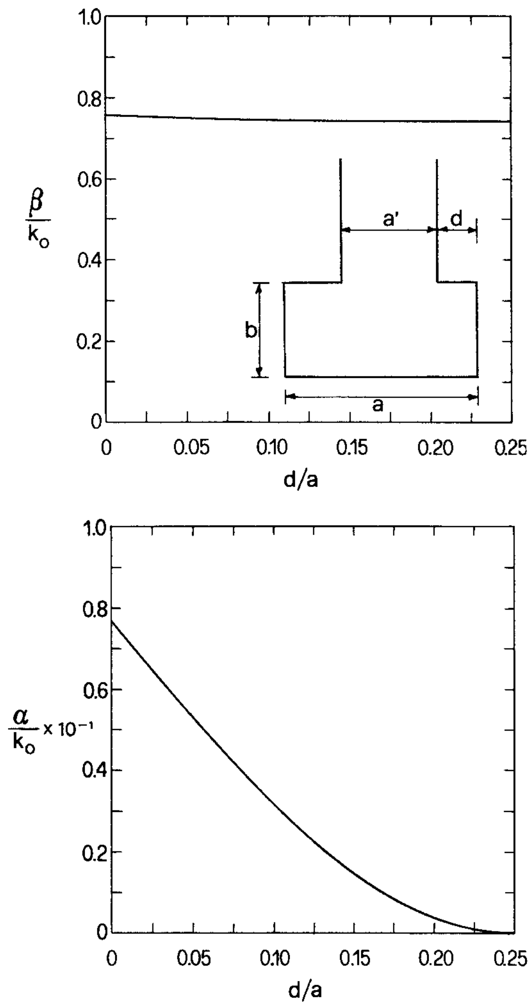

Fig. 5. Behavior of the normalized phase and leakage constants as a function of off-center shift of the stub guide when the aspect ratio is that of millimeter-wave rectangular waveguide. Note that very desirable characteristics are present since β/k_o remains very flat over most of the range while α/k_o varies greatly: $f = 50$ GHz; $a = 4.8$ mm; $b = 2.4$ mm; $a' = 2.4$ mm.

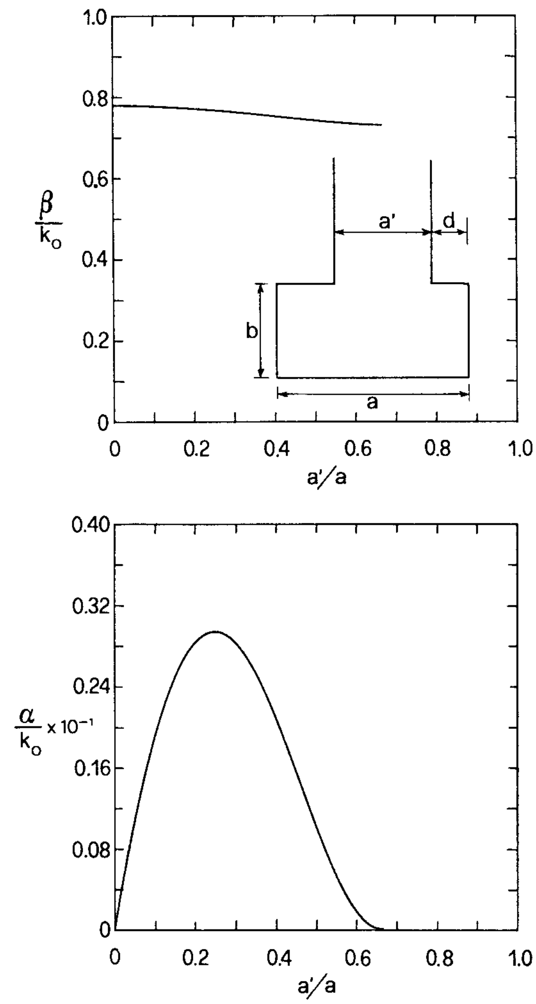

Fig. 6. Behavior of the normalized phase and leakage constants as a function of stub-guide relative width a'/a when the aspect ratio is that of millimeter-wave rectangular waveguide: $f = 50$ GHz; $a = 4.8$ mm; $b = 2.4$ mm; $d = 0.8$ mm.

large value (33°) down to widths that are as narrow as one pleases.

What is particularly interesting now is the behavior of β/k_o. This quantity is seen to be remarkably flat as d/a varies. In fact, for the range of d/a from 0.15 to 0.25, the value of β/k_o is so flat that the corresponding beam angle, which is about 48° from broadside, is constant to within about 0.1°. At $d/a = 0.15$ the value of α/k_o is 0.015, which corresponds to a beamwidth of 6.4°, so that the beamwidth can be varied over quite a large range while the beam angle stays very constant. With almost the same flatness, the range can be extended to $d/a = 0.10$, at which location the beamwidth becomes as large as 13.4°. For the set of dimensions and the frequency specified in Fig. 5, therefore, we see that a variation in off-center shift will indeed permit a large choice of beamwidths while hardly disturbing the beam angle at all.

The variations with stub guide *relative width* a'/a are illustrated next in Fig. 6. For these curves, the stub-guide offset position is held constant. We observe that there is an optimum stub-guide width that yields the largest value of α/k_o. When the stub guide is too narrow, the leakage rate and, therefore, the beamwidth drops quickly. For this frequency and set of dimensions, the maximum value of beamwidth (for 90% radiation and a uniform nontapered structure) is about 13.1°, which is fairly large. The curve of β/k_o versus a'/a is not as flat as that for variations in d/a, but nevertheless the variation is small. Suppose, for example, that we examine the range of a'/a from zero to 0.25 at which the leakage rate is maximum. We find that the beam angle is about 50.0° when $a'/a = 0.25$ and about 50.9° when $a'/a = 0$. The change is only slightly less than 1° over the whole range of α/k_o from zero to maximum, which is not bad. The curves in Fig. 6 have been calculated only up to $a'/a \cong 0.67$ at which value the stub guide is centered; for higher values of a'/a, the curve for α/k_o will increase again.

By examining the curves in Figs. 5 and 6 simultaneously, we note that the maximum value of α/k_o in Fig. 6 could have easily been increased by choosing a smaller value of d/a and that the whole α/k_o curve in Fig. 5 would have been lifted (to

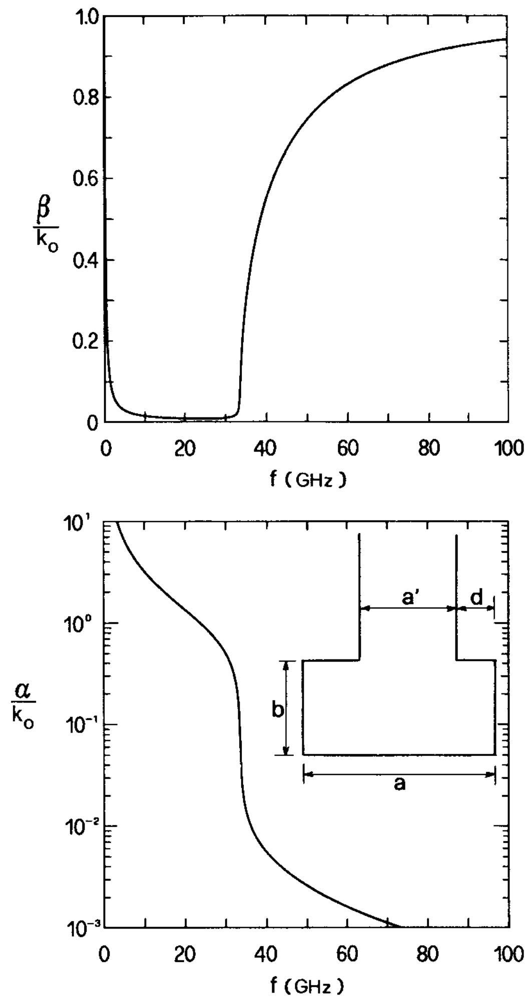

Fig. 7. The behavior of the normalized phase and leakage constants as a function of frequency both above and below cutoff when the leakage rate is small: $a = 4.8$ mm; $b = 2.4$ mm; $a' = 2.4$ mm; $d = 1.0$ mm.

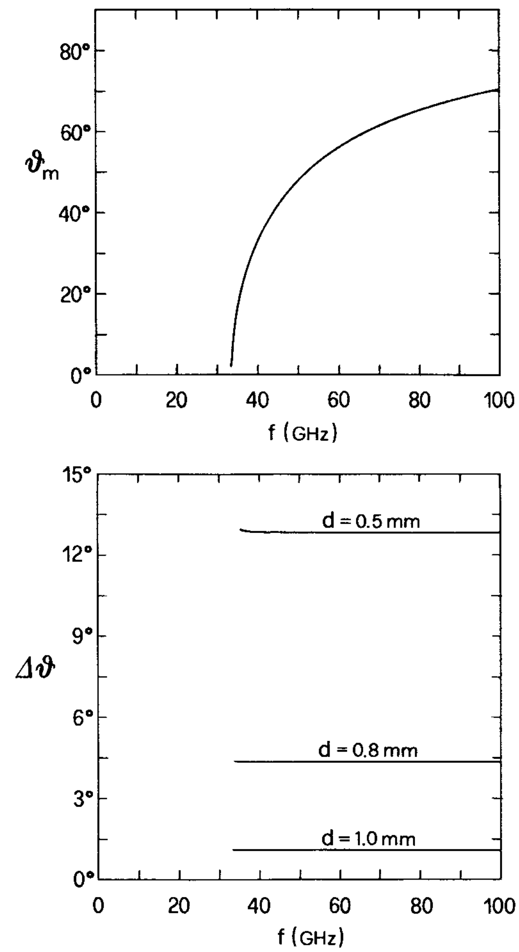

Fig. 8. The variations in beam angle θ_m and beamwidth $\Delta\theta$ of the radiation pattern as a function of frequency. Note the constancy of beamwidth with frequency scan: $a = 4.8$ mm; $b = 2.4$ mm; $a' = 2.4$ mm.

higher values of leakage constant) if the value of a'/a selected for Fig. 5 were made somewhat smaller than 0.5. Thus, we see that the dimensions were not optimized with respect to leakage rate in either case, but that nevertheless the range available in α/k_o values was very large.

We therefore find that the desirable property we seek, namely that β/k_o be maintained the same while α/k_o changes strongly, can be furnished in two ways: by varying a'/a or by changing d/a. One method will be superior to the other depending on the parameter values. It is also clear that by adjusting a combination of the values of d/a and a'/a almost any desired value of beamwidth can be obtained.

B. Variations with Frequency

Fig. 4 shows how β/k_o and α/k_o vary over a very wide frequency range when the antenna has a high leakage rate. In contrast, when the leakage rate is much lower the behavior changes to that seen in Fig. 7.

These curves can be divided into two basic parts—for the region above cutoff and for the one below cutoff. *Above cutoff*, which corresponds roughly to $f = 17$ and 33 GHz, respectively, the behavior is as expected, with β/k_o following the standard curve shape and with α/k_o decreasing monotonically with increasing frequency.

The transverse wavenumber k_t is equal to k_x in the main guide region, which, in turn, is equal to k_y in the stub-guide region (see Fig. 2). If the stub guide were closed and the leakage were absent k_x would be real and equal to π/a. In the actual antenna, k_t is *complex* with a predominantly real part. For the parameter values given in Fig. 7, we calculate that

$$k_t = \mathrm{Re}\, k_t + j\, \mathrm{Im}\, k_t = 0.702 \text{ mm}^{-1} + j\, 3.00 \times 10^{-3} \text{ mm}^{-1} \tag{12}$$

and is *independent of frequency* since only one medium (air) is present. For comparison, $\pi/a = 0.654 \text{ mm}^{-1}$.

The behavior *below cutoff* is not generally known. First of all, β/k_o does not approach zero or a constant as the frequency is reduced far below cutoff, as is generally believed. It turns up again and, in fact, exceeds unity as the frequency becomes very low. When the leakage is stronger (as in Fig. 4), the minimum value of β/k_o is larger, the turn-up faster, and the value unity is reached at a higher frequency.

The explanation for the turn up in the value of β/k_o at low frequencies is that β itself approaches a constant as the frequency approaches zero, so that β/k_o must then increase without limit as the frequency goes to zero. Therefore, the value of β/k_o will certainly exceed unity, but the solution has no meaning in that range. It may be shown by employing the steepest-descent plane that when $\beta/k_o > 1$ the pole representing the leaky wave solution is no longer captured by the deformation of the original path into the steepest-descent path [12]. Physically, this means that the leaky wave is no longer a part of the total field for any angle of observation.

The behavior of α/k_o is also different above cutoff and below it. Above cutoff, the value of α represents the leakage of power. Below cutoff, α represents primarily the *reactive decay* of the field, although some leakage of power also contributes a little to its value. Therefore, the value of α/k_o increases substantially once the leaky mode goes below cutoff, as one may see in Fig. 7.

In addition, α approaches a constant as the frequency approaches zero, as may be shown readily when (11) is expanded into real and imaginary parts. Recognizing that $\alpha \gg \beta$ and $\mathrm{Re}\, k_t \gg \mathrm{Im}\, k_t$ as $f \rightarrow 0$, we obtain from the real part that $\alpha \rightarrow \mathrm{Re}\, k_t$ as $f \rightarrow 0$. Furthermore, from the imaginary part of (11) and the above limit value of α, we immediately see that $\beta \rightarrow \mathrm{Im}\, k_t$, as $f \rightarrow 0$.

From the portions of the curves in Fig. 7 *above cutoff*, we may determine the behavior shown in Fig. 8. The curve of θ_m, the angle of the maximum of the radiated beam measured in degrees from the broadside direction follows directly from (1). The beamwidth $\Delta\theta$ is given approximately by (2).

When only one medium is present in the antenna cross section, the transverse wavenumber k_t is a constant independent of frequency, as in (12). It then follows that when the beam angle θ_m is scanned by varying the frequency, the value of beamwidth $\Delta\theta$ *remains constant*. The proof of this statement appears in many references (e.g., [4]). The curves for $\Delta\theta$ in Fig. 8, for three different values of off-center shift, are seen to be very constant with frequency. This feature is an added advantage of this antenna structure.

REFERENCES

[1] F. J. Zucker, "Surface- and leaky-wave antennas," in *Antenna Engineering Handbook*, 1st ed., H. Jasik, Ed. New York: McGraw-Hill, 1969, ch. 16.

[2] C. H. Walter, *Traveling Wave Antennas*. New York: McGraw-Hill, 1965 (reprint Los Altos, CA: Peninsula, 1990).

[3] F. Schwering and A. A. Oliner, "Millimeter-wave antennas," in *Antenna Handbook*, Y. T. Lo and S. W. Lee, Eds. New York: Van Nostrand Reinhold, 1988, ch. 17.

[4] A. A. Oliner, "Leaky-wave antennas," in *Antenna Engineering Handbook*, 3rd ed., R. C. Johnson, Ed. New York: McGraw-Hill, 1993, ch. 10.

[5] A. A. Oliner and P. Lampariello, "A simple leaky-wave antenna that permits flexibility in beam width," in *Dig. Nat. Radio Sci. Meet. (URSI)*, Philadelphia, PA, June 1986, p. 26.

[6] P. Lampariello, F. Frezza, H. Shigesawa, M. Tsuji, and A. A. Oliner, "Guidance and leakage properties of offset groove guide," in *Dig. IEEE Int. Microwave Symp.*, Las Vegas, NV, June 1987, pp. 731–734.

[7] P. Lampariello and A. A. Oliner, "New equivalent networks with simple closed-form expressions for open and slit-coupled E-plane tee junctions," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 839–847, May 1993.

[8] A. Abdelmonem, H.-W. Yao, and K. A. Zaki, "Slit-coupled E-plane rectangular tee junctions using single-port mode-matching technique," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 903–907, May 1994.

[9] A. A. Oliner and P. Lampariello, "A new simple leaky-wave antenna for millimeter waves," in *Dig. North Amer. Radio Sci. Meet. (URSI)*, Vancouver, Canada, June 1985, p. 57.

[10] A. A. Oliner, "Scannable millimeter wave arrays," Final Rep. RADC Contract F19 628-84-K-0025, Polytechnic Univ., Brooklyn, NY, Sept. 1988, ch. VI.

[11] ——, "Scannable millimeter wave arrays," Final Rep. RADC Contract F19 628-84-K-0025, Polytechnic Univ., Brooklyn, NY, Sept. 1988, ch. VII.

[12] ——, "Scannable millimeter wave arrays," Final Rep. RADC Contract F19 628-84-K-0025, Polytechnic Univ., Brooklyn, NY, Sept. 1988, ch. IX.

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