A New Way of Bandpass Filter Design Based on Zeroth-Order and Negative-Order Resonance Modes

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***Abstract* — A new method of bandpass filter design is proposed by using the CRLH transmission line theory. The method starts from the dispersion diagram of CRLH transmission line and is based on the negative first order and zero order resonance modes of an unbalanced CRLH transmission line. The relationship between the new method and conventional coupling method of filter design is discussed. By using the new method, a two-order mushroom filter is designed, fabricated and tested in this paper. The filter achieves a small size and a wide-stopband.**

***Index Terms*—Bandpass filter, Mushroom, Negative order resonator, Zeroth-order resonator, Wide stop-band.**

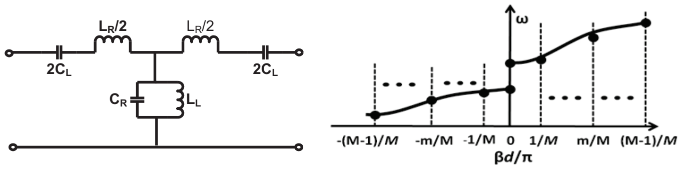
I. Introduction

Recently, there has been significant increase of interest in research and development of the devices using the metamaterial concept. Especially, a composite Right/Left-Handed transmission Line (CRLH-TL)[1] has been studied for microwave applications [2-3]. Among these applications, negative and zeroth-order resonators have been proposed based on CRLH-TL. These resonators have much smaller size compared to the conventional (Right-Handed) transmission line (RH-TL). Thus, these resonators have significant advantages in building filters with small size. Some Filters based on CRLH-TL negative or zeroth-order resonances have been built in [4][5]. During the design of these filters, a negative or zeroth-order resonator is usually chosen first, and then the classic filter synthesis process in [8][9] is applied to obtain the final filter performance. In [6], a relationship between the Chebyshev filter and balanced CRLH-TL is discussed.

In this paper, a new method of filter design based on unbalanced CRLH-TL is discussed. The method starts from the dispersion diagram of CRLH-TL and is based on the negative first order and zeroth-order resonance modes of an unbalanced CRLH transmission line. By changing the slope of the dispersion diagram of the CRLH-TL, the frequency difference between the negative first order resonant frequency and zeroth-order resonant frequency of the CRLH-TL is varied. When this difference becomes small enough, there will be coupling between these two resonant modes, which forms a filter passband. A two-pole mushroom filter is proposed to verify the design process. In addition, a filter with wide stopband can be easily obtained by applying the CRLH-TL theory. A comparison between this new method and the standard filter design method is also discussed in the paper.

II. Filter design in CRLH Transmission theory

A purely left-handed transmission line is physically unrealized, so a Composite Right/Left Handed Transmission Line (CRLH-TL) is used. Fig. 1(a) shows the equivalent circuit of a symmetric CRLH TL unit cell. By applying periodic boundary conditions (PBCs) related to the Bloch-Floquet theorem, the CRLH-TL's unit cell's dispersion relation is determined as given in (1)



(a) (b)

Fig. 1. a) Equivalent circuit of a symmetric unit cell of CRTH-TL. b) Dispersion diagram of a CRLH-TL with M unit cells.

$$\beta(\omega) = \frac{1}{d}\cos^{-1}\left(1-\frac{1}{2}\left(\frac{\omega_L{}^2}{\omega^2}+\frac{\omega^2}{\omega_R{}^2}-\frac{\omega_L{}^2}{\omega_{se}{}^2}-\frac{\omega_L{}^2}{\omega_{sh}{}^2}\right) \right) \tag{1}$$

where

$$\omega_L = \frac{1}{\sqrt{C_L L_L}},\ \omega_R = \frac{1}{\sqrt{C_R L_R}},\ \omega_{se} = \frac{1}{\sqrt{C_L L_R}},\ \omega_{sh} = \frac{1}{\sqrt{C_R L_L}}$$

The specific case where the series resonance ω_{se} and the shunt resonance ω_{se} are equal is called a balanced structure while, for more general case where $\omega_{se} \neq \omega_{sh}$, it is called an unbalanced structure. Dispersion diagrams are often used to determine the resonant properties of a CRLH structure. Fig. 1(b) shows the dispersion diagram for M CRLH-TL unit cells. Generally, in a CRLH TL structure with M unit cells, there are (M-1) positive resonances that occur on the right-handed dispersion curve (1,…m, … (M-1)). These resonances are termed positive because according to dispersion diagram, the phase constant β is positive. There are also (M-1) negative resonances that occur on the left-handed dispersion curve (-1,…-m, … -(M-1)). Additionally, there is a zeroth order resonance, corresponding to either ω_{se} or ω_{sh} depending on the applied boundary conditions. If open circuit boundary conditions are assumed, the zeroth order resonance will coincide with ω_{sh}. When the short circuit boundary conditions

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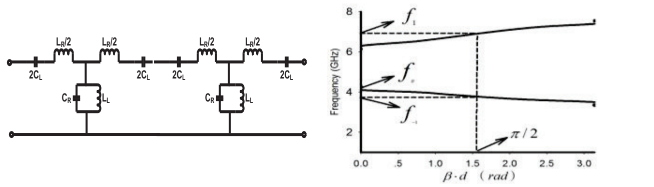
are assumed, the zeroth order resonance will coincide with ω_{se}.

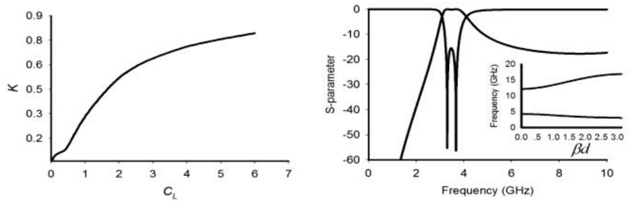
When the number of unit cells in CRLH TL is M=2, there will be three resonant modes, which are m=-1, 0, 1, and the resonant frequencies of these modes will be termed as negative first order resonance f_{-1}, zeroth order resonance f_0 and positive first order resonance f_1 in this paper. Fig. 2 shows the equivalent circuit of 2 CRLH-TL unit cells and the corresponding dispersion diagram. The equivalent circuit is an open ended circuit, so f_0 is determined by the shunt resonance ω_{sh}. f_1 and f_{-1} will take place at frequencies where

$$\beta d = \frac{\pi}{2} \ \ and \ \beta d = -\frac{\pi}{2}$$



(a) (b)

Fig. 2. (a) Equivalent circuit mode of 2 CRLH-TL unit cells. (b) Dispersion diagram of the circuit in (a).

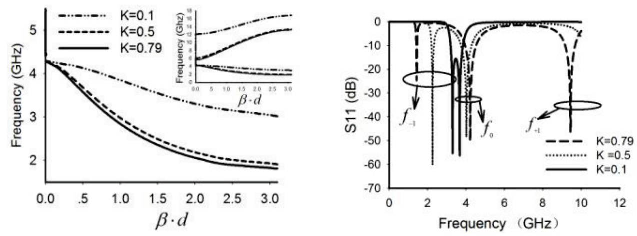


(a) (b)

Fig. 3. a) The relationship between the C_L and K. b) The observed filter performance when K =0.1 and the corresponding dispersion diagram.

Here, the goal is to build filters based on f_{-1} and f_0. If f_{-1} and f_0 is close enough, the two resonant modes will be coupled and then form a filter passband. According to the dispersion diagram in Fig. 2(b), it can be easily found that the frequency difference between f_{-1} and f_0 depends on the slope of the dispersion curve in the left-handed region. The flatter the dispersion curve is, the closer the f_{-1} and f_0 are, and the more likely a filter can be formed. In an extreme case where the dispersion curve is nearly horizontal, the f_{-1} will be almost equal to f_0. To verify this, the series capacitance C_L in Fig. 2(a) is tuned while other parameters are kept constant, and then the position of f_{-1} and f_0 is observed. An index, K, defined in (2), is introduced to indicate the frequency difference between f_0 and f_{-1}. Fig. 3(a) shows the relationship between C_L and K. It can be found that K decreases as C_L decreases. It also can be easily found from Fig. 4(a) that as the K becomes smaller, the dispersion curve becomes more flat, and the frequency difference between f_0 and f_{-1} becomes smaller. When the K is small enough, a filter passband is formed, which can be easily found from Fig. 4(b). When K is around 0.1, a filter passband is observed as is shown Fig. 3(b). This indicates that the negative first resonance mode is coupled with the zeroth order resonance mode so that a bandpass filter is built based on these two modes. Note that the filter is based on an unbalanced CRLH-TL.

$$K = \frac{{f_0}^2 - {f_{-1}}^2}{{f_0}^2 + {f_{-1}}^2} \tag{2}$$

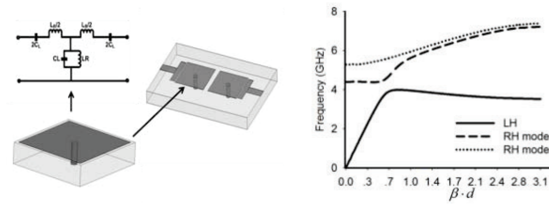


(a) (b)

Fig. 4. a) The relation between the dispersion diagram and index K. b) The relationship between the reflection S_{11} and index K.

III. Mushroom filter design

To apply the theory discussed above, a mushroom structure filter based on CRLH-TL is designed, fabricated and tested in this paper. The proposed mushroom structure is shown in Fig. 5(a).

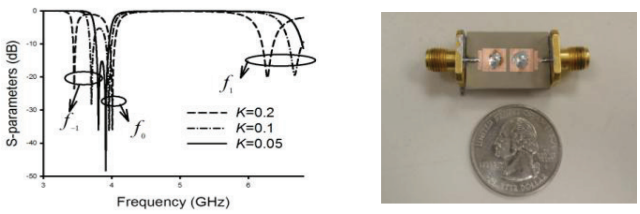


(a) (b)

Fig. 5. a) The proposed mushroom structure. b) The dispersion diagram of the mushroom by using the periodic boundary condition.

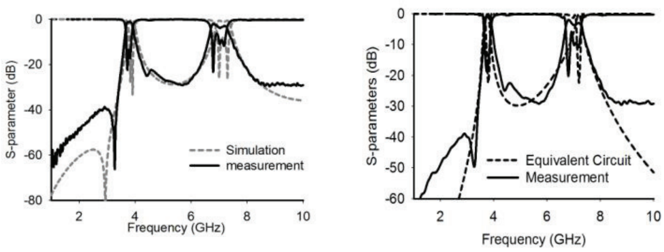
The mushroom's left-handed (LH) capacitance (C_L) is attributed to the edge coupling between the unit-cells while the mushroom's LH inductance (L_L) is due to the shorting post to ground. The right-handed (RH) effects are due to the capacitive coupling (C_R) between the patch and ground plane and the current flow along the patch (L_R). The equivalent circuit of unit cell mushroom is the same as that in Fig. 1(a), and the equivalent circuit of the two mushrooms cascaded is the same as the one in Fig. 2(a). Thus, this mushroom structure can be described by the theory in Section Ⅱ. By changing the physical properties of mushroom unit-cell(e.g., patch size, shorting post radius, dielectric constant, etc.), the equivalent capacitances and inductances can be controlled. Two input/output ports are added to the ends of the mushroom structure to feed the filter.

To build the filter based on the negative first order resonance mode and the zeroth order resonance mode, the physical size of the mushroom must be carefully chosen to make the LH dispersion curve flat, so the negative first resonance mode and the zeroth order resonance mode can be close enough to couple with each other. The dispersion diagram of the unit cell mushroom structure can be calculated by applying periodic conditions, as shown in Fig. 5(b). The f_{-1} and f_0 can be indicated from the dispersion diagram. The gap between the unit cells, which determinates C_L, is tuned to change the frequency difference between f_{-1} and f_0 with other physical parameters kept constant. As discussed in section Ⅱ, the smaller C_L is, the smaller K is and more likely a filter can be formed. A careful tuning process is applied to filter design. Fig. 6(a) shows the reflection parameter of the filter which changes with K. A filter passband is formed when the K becomes small enough. A filter is fabricated when K is 0.05, as shown in Fig. 6(b). Fig. 7(a) shows the simulation and measured results. A parameter extraction in [7] is used to extract the equivalent circuit parameters of the mushroom structure. Fig. 7(b) shows the equivalent circuit results and measurement results. It can be seen that the three results agree well with each other.



(a) (b)

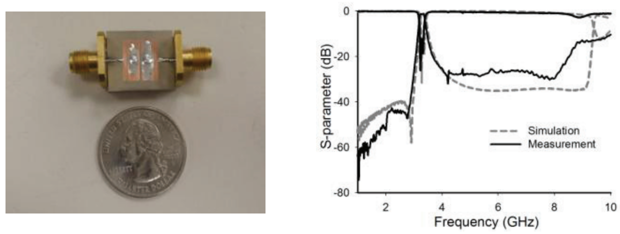
Fig. 6. a) The different filter performance when K varies. b) Photograph of the fabricated mushroom filter.



(a) (b)

Fig. 7. a) Simulation results and measurement results. b) Equivalent circuit and measurement results.

There is a spurious passband around the 7 GHz, this is due to the positive first order resonance f_1 and higher RH mode, which can be seen in dispersion diagram shown in Fig. 5(b). It can be easily known from (1) that the ω_R and ω_{se} will increase by decreasing L_R. When ω_R and ω_{se} increase, the f_1 will increase consequently. However, when L_R changes, ω_{sh} and ω_L will keep unchanged. In an open boundary condition, f_0 and f_{-1} are only determined by ω_{sh} and ω_L, so when L_R increases, f_0 and f_{-1} will keep unchanged. It means that the spurious passband can be moved far away from main passband by decreasing L_R and increasing the f_1, while the main passband keeps unchanged. Thus, filter with wide stop-band can be achieved. To move the spurious passband of the mushroom filter shown in Fig. 6(b), L_R is decreased by decreasing the length and increasing the width of the metal patch. To keep the main passband unchanged, the C_R and C_L must be unchanged. To keep the C_R unchanged, the total area of the metal patch must keep unchanged. To keep the C_L unchanged, the gap between mushrooms needs to be increased to compensate the longer fringe coupling. The new mushroom filter is shown in Fig. 8(a). The simulation and measurement are shown in Fig. 8(b). It can be seen that the spurious passband which located at 7 GHz in Fig. 7(a) is now moved further away to 9 GHz, and the main pass-band is nearly unchanged at 3.8 GHz compared to Fig. 7(a). Thus, a wide-stop band mushroom filter is achieved by using the CRLH-TL theory.



(a) (b)

Fig. 8. a) Photograph of the new mushroom filter. b) Simulation and measurement result of the filter with wide stop-band.

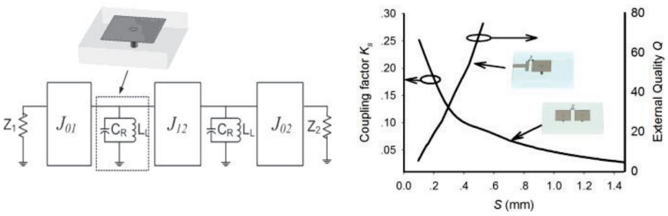
IV. RELATION WITH THE CONVENTIONAL FILTER THEORY

The single open mushroom structure shown in Fig. 5(a) can be considered as a resonator in classic coupling filter theory [8], [9]. This resonator has infinite number of resonant frequencies, and the lowest resonant frequency is main resonant frequency, f_c, which can be used as the center frequency of a filter. At the main resonant frequency, the single open mushroom is equivalent to a shunt resonator and has an equivalent circuit shown Fig. 9(a). Because the single mushroom is an open structure, the series inductor (L_R) due to the current flow along the patch does not contribute to the main resonance mode. Meanwhile, there is no left-handed capacitance (CL) for a single mushroom. Thus, the resonance frequency is only dominated by the capacitive coupling (C_R) between the patch and ground plane and the inductance (L_L) of the shorting post to ground. When two mushroom resonators are cascaded just as shown in Fig. 5(a), a filter prototype can be formed by adding an input and an output coupling port. This filter is equivalent to the circuit shown in Fig. 9(a). By using the classic filter theory in [9], a filter can be well designed. The coupling factor, K_s, and external quality, Q, which are applied to indicate the coupling between mushroom resonators and the coupling at the input/output ports, are defined as.

$$K_s = \frac{f_{p1}{}^2 - f_{p2}{}^2}{f_{p1}{}^2 + f_{p2}{}^2}\ , \quad Q = \frac{\omega_0 \bullet \tau(\omega_0)}{4} \tag{3}$$

where the f_{p1} and f_{p2} represent the characteristic frequency of the two resonant peaks when two resonators cascaded. ω_0 is resonant angular frequency, $\tau(\omega_0)$ is the group delay at ω_0. Fig. 9(b) shows the relationship between the coupling factor K_s and gap size, S, between the resonators, and the relationship between the external quality and the input/output coupling.

According to the physical parameters of filter obtained in section Ⅲ, the corresponding coupling factor, K_s, and external quality, Q, can be read from Fig. 9(b), which shows to be Q=21, K_s=0.05. A two order filter synthesis process is applied with Q=21and K_s=0.05 by using the theory in [9]. Fig. 10 shows the synthesized filter performance and the measured filter results given in section Ⅲ. The two results agree well with each other. It means that the filter given in section Ⅲ can also be designed in standard filter theory.



(a) (b)

Fig. 9. a) The equivalent circuit of the mushroom filter in conventional filter theory. b) The coupling factor K_s and the external quality Q with different gap size.

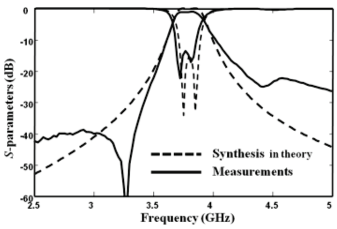
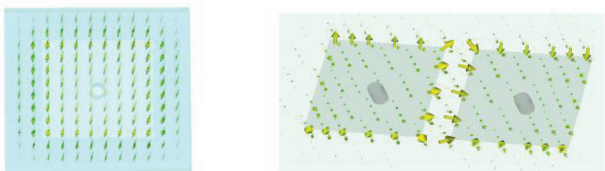


Fig. 10. The synthesized filter performance and the measurement results given in Fig. 7(a).



(a) (b)

Fig. 11. a) The electric field distribution of single mushroom. b) The electric field distribution of the cascaded mushrooms.

Using the classic filter theory [8], [9], the field in the single resonator at the main resonant frequency is shown in Fig. 11(a). The field is the same as the field of the zeroth order resonance in a CRLH transmission line. Thus, the main resonant frequency can be viewed as zeroth order resonance. When two resonators are cascaded, the field of each resonator is coupled. The coupled field excites a new resonant mode, and the field of this mode is shown in Fig. 11(b). This new resonate mode is same as the negative first order resonance in a CRLH transmission line. So this mode can be viewed as negative first order resonance. It means that the coupling factor calculated in (3) is actually the coupling between negative first order mode and zeroth order mode. Thus, two methods discussed above converge together. It can be considered that there is no conflict between two methods. They are just two methods to the same case from two different points of view.

IV. CONCLUSION

From discussions above, we can find that there is a new way to design filter by using the CRLH handed transmission theory. The designed filter can be based on negative first order mode and zeroth order mode. This method is different from the standard filter theory. However, this method does not conflict with the standard filter theory. It is a method from the view point of CRLH transmission. Using this method, filter based on negative first order mode and zeroth order mode can easily be built with small size and can achieve a wide stop-band by moving positive order resonance modes far from the center frequency.

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