

100:1 Bandwidth Balun Transformer*

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Summary—The theory and design of a Tchebycheff tapered balun transformer which will function over frequency bandwidths as great as 100:1 is presented. The balun is an impedance matching transition from coaxial line to a balanced, two-conductor line. The transition is accomplished by cutting open the outer wall of the coax so that a cross-sectional view shows a sector of the outer conductor removed. As one progresses along the balun from the coaxial end, the open sector varies from zero to almost 2π , yielding the transition to a two-conductor line.

The balun impedance is tapered so that the input reflection coefficient follows a Tchebycheff response in the pass band. To synthesize the impedance taper, the impedance of a slotted coaxial line was obtained by means of a variational solution which yielded upper and lower bounds to the exact impedance. Slotted line impedance was determined experimentally by painting the line cross section on resistance card using silver paint and measuring the dc resistance of the section.

The measured VSWR of a test balun did not exceed 1.25:1 over a 50:1 bandwidth. Dissipative loss was less than 0.1 db over most of the range. Measurements show that the unbalanced current at the output terminals is negligible.

INTRODUCTION

IN utilizing some of the recently developed broad-band antennas such as the logarithmically periodic antenna, it is sometimes advantageous to excite the antenna from balanced, two-conductor terminals.¹ In order to match the balanced antenna impedance to the unbalanced impedance of a coaxial line, a balun transformer is required. Moreover, the balun transformer must be capable of operating over a very large frequency range if it is to be compatible with the antenna performance. This paper presents the theory and design of a Tchebycheff tapered balun transformer which will function over bandwidths as great as 100:1.

The balun transformer is illustrated in Fig. 1. The balun is an impedance matching transition from coaxial line to a balanced two-conductor open line. The transition is accomplished by cutting open the outer wall of the coax so that a cross-section view shows a sector of the outer conductor removed. The angle subtended by the open sector is denoted by 2α . As one progresses along the balun from the coaxial end, the angle 2α varies from zero to almost 2π , yielding the transition from coax to an open two-conductor line. The cross section of the conductors is then varied as required. One is not limited to conductors having a circular cross section; a transition from coaxial cable to a balanced strip line is one of the possible configurations.

The broad-band impedance matching properties of

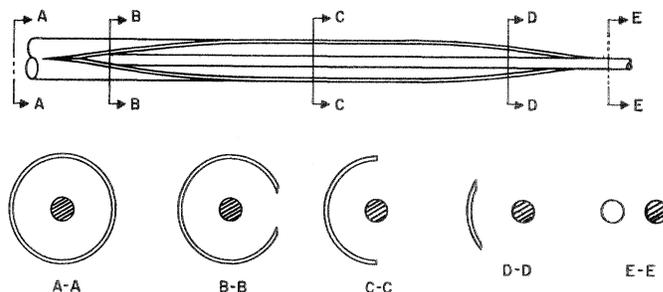


Fig. 1—Tapered balun transformer.

the balun are obtained by utilizing a continuous transmission line taper described by Klopfenstein.² The characteristic impedance of the balun transformer is tapered along its length so that the input reflection coefficient follows a Tchebycheff response in the pass band. The length of the balun is determined by the lowest operating frequency and the maximum reflection coefficient which is to occur in the pass band. The balun has no upper frequency limit other than the frequency where higher order coaxial modes are supported or where radiation from the open wire line becomes appreciable.

Before discussing the "balun" property of the device, a brief review of balance conditions on an open transmission line is in order. A balanced two-conductor transmission line has equal currents of opposite phase in the line conductors at any cross section. System unbalance is evidenced by the addition of codirectional currents of arbitrary phase to the balanced transmission line currents. The order of unbalance is measured by the ratio of the codirectional current to the balanced current. Now in a coaxial line, the total current on the inside surface of the outer conductor is equal and opposite to the total current on the center conductor. The ideal balun functions by isolating the outside surface of the coax from the transmission line junction so that all of the current on the inside surface of the coax outer conductor is delivered in the proper phase to one of the two balanced conductors. Unbalance of the transmission line currents results if current returns to the generator on the outside surface of the coaxial line.

Consider the Tchebycheff tapered balun transformer which is formed by increasing the slot aperture in the outer wall of the coax until an open two-conductor line is obtained. Over the length of the transition the electromagnetic field changes from a totally confined field in the coax to the "open" field of a two-wire transmission line. It is evident that the total current on the out-

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¹ R. H. DuHamel and F. R. Ore, "Log periodic feeds for lens and reflectors," 1959 IRE NATIONAL CONVENTION RECORD, pt. 1, pp. 128-137.

² R. W. Klopfenstein, "A transmission line taper of improved design," PROC. IRE, vol. 44, pp. 31-35; January, 1956.

side surface of the coax at the balun input must result from the summation of wave reflections which originate over the entire length of the open transition. But the slot transition is purposely tapered so that the net reflection at the balun input is arbitrarily small. Consequently, negligible current appears on the outside of the coaxial line at the balun input and electrical balance at the output terminals is very good. In other words, the physical geometry of the transition which produces negligible wave reflections and leads to a broad-band impedance transformer also results in the operation of the device as a balun.

Assuming that the characteristic impedance of the balun at any cross section is equal to the characteristic impedance of a uniform, slotted coaxial line of that particular cross section, it is possible to synthesize the required impedance taper by providing the appropriate cross section at each position along the balun transformer. In order to carry out this procedure, one must know the characteristic impedance of a uniform, slotted coaxial line as the angle 2α varies from zero to 2π . This information was obtained by theoretical analysis and verified experimentally. The characteristic impedance of the slotted line was determined from a variational solution of the two-dimensional boundary value problem. The variational expressions yield upper and lower bounds to the exact characteristic impedance. The upper bound is obtained from a variational expression involving the charge distribution on the outer conductor of the slotted coaxial line, while the lower bound is obtained from a variational expression involving the potential distribution in the slot aperture. Characteristic impedance was determined experimentally by painting the slotted line cross section on resistance card, using silver paint and measuring the dc resistance of the cross section. These data are presented as curves which show characteristic impedance of the slotted coaxial line as a function of the angular opening. The curves allow one to design a balun for matching a large range of impedances with an arbitrarily small standing wave ratio. We proceed to derive variational expressions for the characteristic impedance of the slotted line. The method of analysis is similar to that used by Collin to solve the problem of a symmetrically slotted coaxial line.³

UPPER BOUND TO THE CHARACTERISTIC IMPEDANCE

Consider the cross-sectional view of the uniform, slotted coaxial line shown in Fig. 2. We choose the cylindrical coordinate system r, θ, z , where r, θ are in the transverse plane and z is the direction of wave propagation along the line. The radius of the inner conductor is $r=a$, while the outer conductor occurs at $r=b$. The slot opening in the outer conductor is defined by the angle

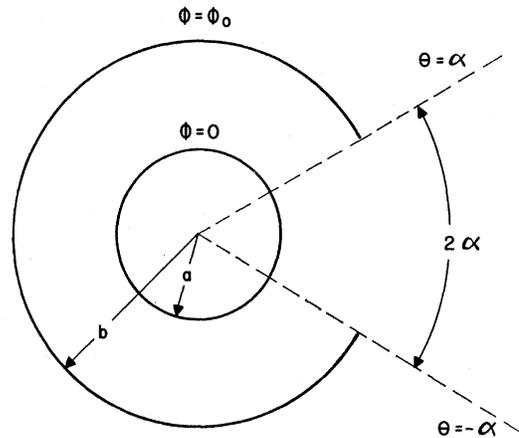


Fig. 2—Cross section of uniform slotted coaxial line.

2α . We assume that there is a homogeneous, isotropic medium about the conductors with permeability μ and permittivity ϵ .

It may be verified that the solution of Maxwell's equations for the TEM mode of propagation on the line reduces to solving Laplace's equation for the static potential distribution $\phi(r, \theta)$ in the transverse plane. The electric field $\bar{E}(r, \theta)$ is defined by the relation

$$\bar{E}(r, \theta) = -\text{grad } \phi(r, \theta). \quad (1)$$

It follows from Maxwell's equations that the transverse field components are given by

$$E_r = -\frac{\partial \phi}{\partial r} = \frac{1}{\epsilon v} H_\theta$$

and

$$E_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{1}{\epsilon v} H_r \quad (2)$$

where $v=1/\sqrt{\mu\epsilon}$ is the velocity of light in the surrounding medium. Thus, all field components may be derived from the scalar potential function $\phi(r, \theta)$ which is the solution of Laplace's equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0, \quad (3)$$

subject to the boundary conditions of the problem.

We define the potential on the inner conductor $r=a$ as $\phi=0$, while the outer conductor $r=b$, $\alpha \leq \theta \leq 2\pi - \alpha$ is maintained at the constant potential ϕ_0 . The potential $\phi(r, \theta)$ at any point in the plane may be expressed in terms of the Green's function $G(r, \theta | r', \theta')$ for the problem. The Green's function is the solution of the inhomogeneous equation

$$\nabla^2 G(r, \theta | r', \theta') = -\frac{1}{\epsilon} \frac{\delta(r-r')\delta(\theta-\theta')}{r} \quad (4)$$

where the polar coordinate form of the delta function

$$\frac{\delta(r-r')\delta(\theta-\theta')}{r}$$

³ R. E. Collin, "The characteristic impedance of a slotted coaxial line," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 4-8; January, 1956.

represents a unit line source at $r=r', \theta=\theta'$. The Green's function satisfies Laplace's equation throughout the r, θ plane except at the source point r', θ' where $G(r, \theta | r', \theta')$ and all its derivatives are singular. Denoting R as the scalar separation between observation point r, θ and source point r', θ' , the singularity of G is such that

$$G(r, \theta | r', \theta') \rightarrow -\frac{1}{2\pi\epsilon} \ln R \text{ as } R \rightarrow 0.^4$$

The Green's function is subject to the boundary condition $G=0$ on the inner cylinder $r=a$. $G(r, \theta | r', \theta')$ may be viewed as the potential at the point r, θ because of a unit line charge located at r', θ' .

Because of the symmetry of the problem, it is convenient to write the Green's function in the form which derives from unit line sources located as shown in Fig. 3. The positive unit charges are located at $r=b, \theta = \pm\theta'$. The images of these line charges in the grounded cylinder $r=a$ occur at $r=a^2/b, \theta = \pm\theta'$. The harmonic expansion of the potential caused by this system of sources with the condition that $G=0$ at $r=a$, yields the appropriate Green's function which is

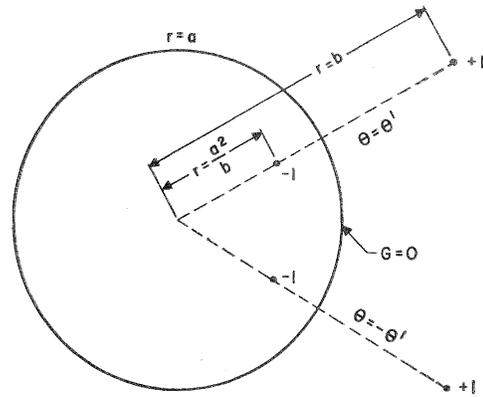


Fig. 3—Unit line charges and images.

(7) by $\sigma(\theta)$ and integrate with respect to θ over $\alpha \leq \theta \leq \pi$; thus

$$\phi_0 = \frac{b \int_{\alpha}^{\pi} \int_{\alpha}^{\pi} G(b, \theta | b, \theta') \sigma(\theta) \sigma(\theta') d\theta d\theta'}{\int_{\alpha}^{\pi} \sigma(\theta) d\theta} \quad (8)$$

$$G(r, \theta | b, \theta') = \frac{1}{\epsilon\pi} \begin{cases} \ln\left(\frac{r}{a}\right) + \sum_{n=1}^{\infty} \frac{2 \sinh\left(n \ln \frac{r}{a}\right) \cos(n\theta) \cos(n\theta')}{n \left[\sinh\left(n \ln \frac{b}{a}\right) + \cosh\left(n \ln \frac{b}{a}\right) \right]} & \text{where } a \leq r \leq b \\ \ln\left(\frac{b}{a}\right) + \sum_{n=1}^{\infty} \frac{2 \sinh\left(n \ln \frac{b}{a}\right) e^{-n \ln(r/b)} \cos(n\theta) \cos(n\theta')}{n \left[\sinh\left(n \ln \frac{b}{a}\right) + \cosh\left(n \ln \frac{b}{a}\right) \right]} & \text{where } r \geq b. \end{cases} \quad (5)$$

It now follows that the potential $\phi(r, \theta)$ caused by an arbitrary (but necessarily symmetrical) charge distribution $\sigma(\theta')$ at $r=b$ is given by

$$\phi(r, \theta) = \int_{\alpha}^{\pi} G(r, \theta | b, \theta') \sigma(\theta') b d\theta'. \quad (6)$$

The charge distribution $\sigma(\theta')$ is still unknown, however, imposing the boundary condition that $\phi(r, \theta) = \phi_0$ when $r=b, \alpha \leq \theta \leq \pi$ leads to the following integral equation for $\sigma(\theta')$:

$$\phi_0 = b \int_{\alpha}^{\pi} G(b, \theta | b, \theta') \sigma(\theta') d\theta'. \quad (7)$$

To obtain a variational expression for Z_0 , we multiply

The total charge Q on the outer conductor resulting from the charge distribution $\sigma(\theta')$ is given by

$$Q = \int_{\alpha}^{2\pi-\alpha} \sigma(\theta') b d\theta' = 2b \int_{\alpha}^{\pi} \sigma(\theta') d\theta'. \quad (9)$$

The characteristic impedance of a uniform, lossless transmission line is given by $1/vC$, where C is the capacitance of the line per unit length and v is the wave velocity. It is sufficient, therefore, to determine C in order to evaluate Z_0 . Since C is equal to the ratio of charge on the outer conductor to the potential difference ϕ_0 between the conductors, we obtain

$$Z_0 = \frac{\frac{1}{v} \phi_0}{Q} \quad (10)$$

⁴ P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill Book Co., Inc., New York, N. Y., pt. 1, pp. 808-810; 1953.

Substituting (8) and (9) into (10) yields the variational form

$$Z_0 = \frac{\frac{1}{2v} \int_{\alpha}^{\pi} \int_{\alpha}^{\pi} G(b, \theta | b, \theta') \sigma(\theta) \sigma(\theta') d\theta d\theta'}{\left[\int_{\alpha}^{\pi} \sigma(\theta) d\theta \right]^2} \quad (11)$$

It may be shown that Z_0 as given by (11) is stationary with respect to arbitrary first order variations in the form of $\sigma(\theta)$ about the correct distribution. (See the Appendix.) The stationary value is an absolute minimum for the "best" approximation to the actual distribution so that (11) yields an upper bound to Z_0 . We approximate the true charge distribution by an N term function containing N arbitrary parameters c_1, c_2, \dots, c_N . This function is substituted into (11) for $\sigma(\theta)$ and the expression for Z_0 is minimized with respect to the parameter constants c_v . To do this, Z_0 is differentiated with respect to the N parameters and the results equated to zero which leads to N homogeneous linear equations in the N unknowns c_v . Solving for the c_v and substituting back into (11) yields the stationary value of Z_0 .

A suitable expansion for $\sigma(\theta)$ is the cosine series

$$\sigma(\theta) = \sum_{\nu=0}^N c_{\nu} \cos \frac{\nu\pi}{\pi - \alpha} (\theta - \alpha).$$

As one uses a larger number of terms to represent $\sigma(\theta)$, the variational solution converges to the exact value of Z_0 ; however, the labor of computations increases enormously with N . It will be seen that sufficiently accurate results are obtained by using the simple two term series

$$\sigma(\theta) = c_0 + c_1 \cos k(\theta - \alpha), \quad (12)$$

where

$$k = \frac{\pi}{\pi - \alpha}.$$

Without loss of generality we may define $c_0 = 1$. Proceeding as outlined above, one obtains

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \left(\frac{b}{a} \right) + \frac{\sqrt{\frac{\mu}{\epsilon}}}{\pi(\pi - \alpha)^2} \sum_{n=1}^{\infty} \frac{\sin^2(n\alpha) \left[1 + \frac{c_1 n^2}{n^2 - k^2} \right]^2}{n^3 \left[1 + \coth \left(n \ln \frac{b}{a} \right) \right]} \text{ ohms,} \quad (13)$$

where

$$-c_1 = \frac{\sum_{n=1}^{\infty} \frac{\sin^2(n\alpha)}{n(n^2 - k^2) \left[1 + \coth \left(n \ln \frac{b}{a} \right) \right]}}{\sum_{n=1}^{\infty} \frac{n \sin^2(n\alpha)}{(n^2 - k^2)^2 \left[1 + \coth \left(n \ln \frac{b}{a} \right) \right]}}.$$

Selecting $\sqrt{\mu/\epsilon}$, (b/a) , and α , one may compute c_1 and evaluate (13) which is an upper bound to the exact characteristic impedance. Before presenting the numerical results obtained with (13) we shall derive a lower bound to Z_0 .

LOWER BOUND TO THE CHARACTERISTIC IMPEDANCE

The fundamental principle that a system in equilibrium is characterized by a minimum of potential energy consistent with the constraints imposed on the system applies to an electrostatic field.⁵ A lower bound to the characteristic impedance may be derived from the integral which yields the total potential energy W of the electrostatic field. For the two dimensional problem under consideration, the total field energy per unit length is given by

$$W = \frac{1}{2} \epsilon \int_0^{2\pi} \int_a^{\infty} \left[\left(\frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \phi}{\partial \theta} \right)^2 \right] r dr d\theta \quad (14)$$

which may be recognized as the Dirichlet integral in polar coordinates.

By definition, $W = (1/2) C \phi_0^2$, where C is the capacitance per unit length. Recalling the relation between characteristic impedance and C , we may express $1/Z_0$ in terms of the integral for the total field energy.

$$\frac{1}{Z_0} = vC = \frac{2v}{\phi_0^2} W. \quad (15)$$

It follows from (15) that if W is minimized with respect to the constants of a parameter-laden function, we obtain a *lower bound* to Z_0 . The function used to minimize W is an N term approximation to the potential distribution $\phi(b, \theta)$ in the slot aperture.

We pause to discuss briefly the variational expression $1/Z_0$. A *necessary* condition for the integral (14) to be stationary is that its first variation vanish. This condition implies that $\phi(r, \theta)$ must satisfy Laplace's equation.⁶ In other words, if a function $\phi(r, \theta)$ exists which minimizes (14), it must necessarily satisfy $\nabla^2 \phi = 0$ and the boundary conditions of the problem. The reader is referred to Kellogg⁷ for proofs that unique solutions of

⁵ J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 114-116; 1941.

⁶ F. B. Hildebrand, "Methods of Applied Mathematics," Prentice-Hall, Inc., Englewood Cliffs, N. J., pp. 138-139; 1952.

⁷ O. D. Kellogg, "Foundations of Potential Theory," Dover Publications, Inc., New York, N. Y., pp. 311-315; 1953.

the Dirichlet problem exist under proper conditions on the region, boundary values, and the functions ϕ eligible for the minimization of the integral.

Based on the Green's function analysis we write the following expansion for the potential function $\phi(r, \theta)$ which satisfies the boundary conditions $\phi=0$ at $r=a$ and ϕ continuous at $r=b$.

$$\phi(r, \theta) = \begin{cases} a_0 \ln\left(\frac{r}{a}\right) + \sum_{n=1}^{\infty} a_n \sinh\left(n \ln \frac{r}{a}\right) \cos(n\theta) \\ \quad \text{where } a \leq r \leq b \\ a_0 \ln\left(\frac{b}{a}\right) + \sum_{n=1}^{\infty} a_n \sinh\left(n \ln \frac{b}{a}\right) \\ \quad \cdot e^{-n \ln(r/b)} \cos(n\theta). \\ \quad \text{where } r \geq b \end{cases} \quad (16)$$

It follows that the potential at $r=b$ is given by

$$\phi(b, \theta) = a_0 \ln\left(\frac{b}{a}\right) + \sum_{n=1}^{\infty} a_n \sinh\left(n \ln \frac{b}{a}\right) \cos(n\theta). \quad (17)$$

Multiplying (17) by $\cos(n\theta)d\theta$ and integrating with respect to θ over $-\pi \leq \theta \leq \pi$ yields

$$a_0 = \frac{1}{\pi \ln\left(\frac{b}{a}\right)} \int_0^{\pi} \phi(b, \theta) d\theta, \\ a_n = \frac{2}{\pi \sinh\left(n \ln \frac{b}{a}\right)} \int_0^{\pi} \phi(b, \theta) \cos(n\theta) d\theta, \quad (18)$$

since $\phi(b, \theta)$ is an even function of θ . If the true potential distribution over the slot aperture were known, the constants a_0, a_n would be determined uniquely by (18), and (16) would yield the exact solution $\phi(r, \theta)$. Instead, we approximate $\phi(b, \theta)$ over the slot by using an appropriate function and then minimize the integral for W with respect to the arbitrary constants.

Substituting the series (16) into (14) and then performing the integration leads to

$$\frac{1}{Z_0} = 2\pi\epsilon v \frac{a_0^2}{\phi_0^2} \ln\left(\frac{b}{a}\right) + \frac{\pi\epsilon v}{\phi_0^2} \sum_{n=1}^{\infty} n a_n^2 \sinh\left(n \ln \frac{b}{a}\right) \cdot \left[\cosh\left(n \ln \frac{b}{a}\right) + \sinh\left(n \ln \frac{b}{a}\right) \right]. \quad (19)$$

Substituting (18) into (19), we obtain the variational expression

$$\frac{1}{Z_0} = \frac{2\epsilon v}{\pi\phi_0^2 \ln\left(\frac{b}{a}\right)} \left[\int_0^{\pi} \phi(b, \theta) d\theta \right]^2 + \frac{4\epsilon v}{\pi\phi_0^2} \sum_{n=1}^{\infty} n \left[1 + \coth\left(n \ln \frac{b}{a}\right) \right] \cdot \left[\int_0^{\pi} \phi(b, \theta) \cos(n\theta) d\theta \right]^2 \quad (20)$$

which is stationary with respect to arbitrary first-order variations in the form of $\phi(b, \theta)$ over the slot aperture.

A suitable representation for the potential $\phi(b, \theta)$ is

$$\phi(b, \theta) = \phi_0 \begin{cases} 1 & \text{where } \alpha \leq \theta \leq 2\pi - \alpha \\ 1 + \sum_{\nu=1,3,5,\dots}^N c_{\nu} \cos \frac{\nu\pi}{2\alpha} \theta & \text{where } -\alpha \leq \theta \leq \alpha \end{cases}$$

Proceeding as outlined for the upper bound, one may substitute this series into (20) and minimize the expression with respect to the c_{ν} . However, a prohibitive number of terms is needed to describe properly $\phi(b, \theta)$ over the slot for α approaching π . We know that for large α , the potential over the slot remains very small until one approaches the outer conductor at $\theta = \pm\alpha$; consequently one would expect an even-powered polynomial in (θ/α) to provide a good approximation to the true distribution. Excellent results were obtained by using the following simple function containing the single arbitrary constant c_1 .

$$\phi(b, \theta) = \phi_0 \begin{cases} 1 & \text{where } \alpha \leq \theta \leq 2\pi - \alpha \\ 1 - c_1 + c_1 \left(\frac{\theta}{\alpha}\right)^4 & \text{where } -\alpha \leq \theta \leq \alpha. \end{cases} \quad (21)$$

Note that when $\theta=0$, $\phi(b, \theta) = \phi_0(1 - c_1)$. Substituting (21) into (20) and minimizing (20) with respect to c_1 , yields

$$Z_0 = \frac{\frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right)}{1 - \frac{4}{5} \left(\frac{\alpha}{\pi}\right) c_1} \text{ ohms} \quad (22)$$

where

$$\frac{1}{c_1} = \frac{4}{5} \left(\frac{\alpha}{\pi}\right) + \frac{40}{\pi} \ln\left(\frac{b}{a}\right) \sum_{n=1}^{\infty} \frac{\left[1 + \coth\left(n \ln \frac{b}{a}\right) \right]}{(n\alpha)} \left[\frac{A_n \cos(n\alpha) - B_n \sin(n\alpha)}{(n\alpha)^4} \right]^2,$$

$$A_n = (n\alpha)^3 - 6(n\alpha),$$

$$B_n = 3(n\alpha)^2 - 6.$$

Eq. (22) provides a lower bound to the exact characteristic impedance of the slotted line. The numerator of

(22) may be recognized as the characteristic impedance of a closed coaxial cable with conductor radii b and a . The denominator of (22) is always less than unity for non-zero α . Since (22) is a lower bound to the exact impedance, we see that the slotted coax impedance is always greater than the impedance of closed coaxial line.

Selecting free space values for μ and ϵ so that $\sqrt{\mu_0/\epsilon_0} = 120\pi$, (13) and (22) were evaluated for $\log_e (b/a) = 0.833, 1.00,$ and 1.25 which corresponds to closed coaxial lines of 50-, 60-, and 75-ohm impedance, respectively. Numerical computations were carried out on an IBM 650 computer. These data are presented in the dashed curves of Figs. 4-6 which show the upper and lower bound to Z_0 as a function of the angle 2α for a particular $\log_e (b/a)$. It is evident that the functional approximations to $\sigma(\theta)$ and $\phi(b, \theta)$ were sufficiently accurate since the difference between the bounds is very small over most of the range 2α . The greatest difference occurs as α approaches π . Since the exact characteristic impedance of the slotted line must lie between the upper and lower bounds, the curves allow one to determine quite accurately the angle 2α required to give a certain impedance Z_0 .

The impedance of the slotted line was also determined by using the well-known method where the line cross section is painted on a two dimensional resistive surface and the dc resistance of the cross section is measured.⁸ Measurements were performed for $\log_e (b/a) = 0.833, 1.00,$ and 1.25 . These experimental data appear as the plotted points in Figs. 4-6. The solid curve is the arithmetic mean of the upper and lower bound to Z_0 for each $\log_e (b/a)$. The experimental data agree quite closely with theory except for the $\log_e (b/a) = 1$ data which diverge slightly for large α . Apparently the cross section was not drawn with sufficient accuracy in this case.

BALUN DESIGN AND PERFORMANCE

Having established the characteristic impedance of the uniform, slotted coaxial line, a specific balun design was undertaken. A transition from 50-ohm coaxial line to 150-ohm two-conductor line was selected for the balun. As mentioned previously, the characteristic impedance of the balun transformer is tapered along its length so that the input reflection coefficient follows a Tchebycheff response in the pass band. The maximum allowable reflection coefficient in the pass band was chosen as 0.055. This corresponds to a maximum standing wave ratio of 1.11 to 1. It follows that the length of the balun is $l = 0.478 \lambda$, where λ is the largest operating wavelength.⁹ The lowest frequency was selected as 50 mc which fixed the length l as approximately 2.86 meters.

⁸ J. D. Kraus, "Electromagnetics," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 426-427; 1953.
⁹ Klopfenstein, *op. cit.*, p. 32.

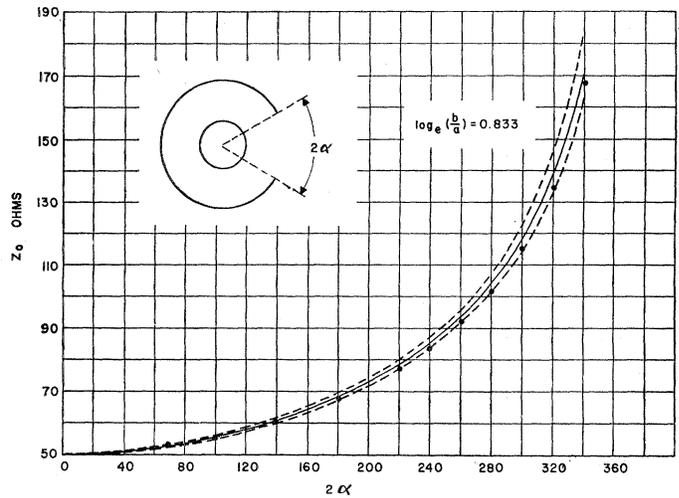


Fig. 4—Characteristic impedance of uniform, slotted coaxial line.

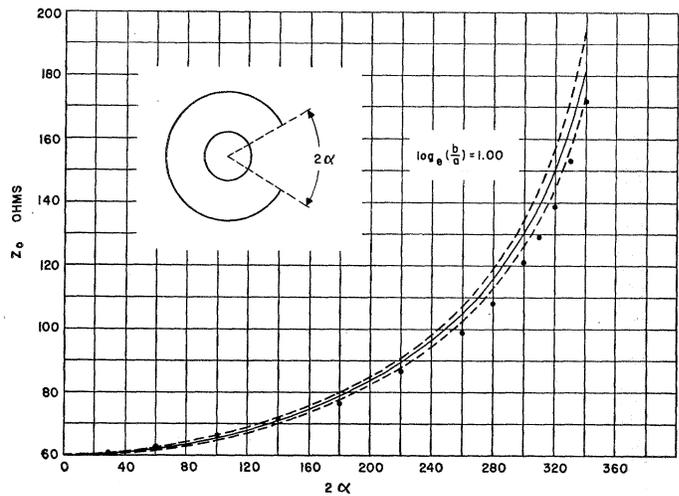


Fig. 5—Characteristic impedance of uniform, slotted coaxial line.

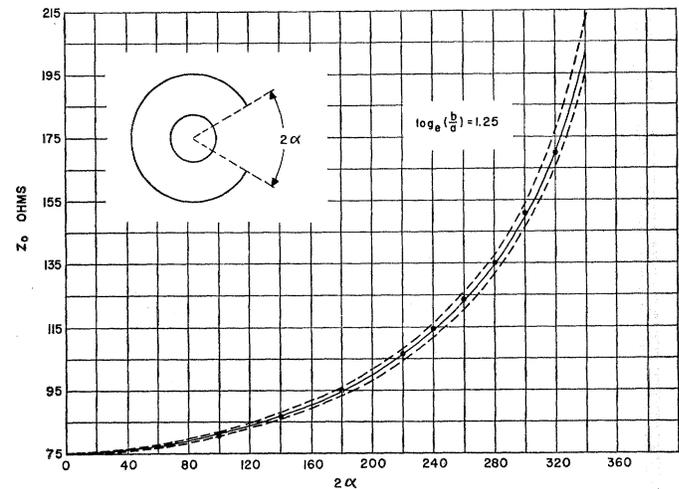


Fig. 6—Characteristic impedance of uniform, slotted coaxial line.

Let the total length l of the balun be defined from $z = -l/2$ to $z = l/2$. Fig. 7 shows the impedance contour required for Tchebycheff response under the prescribed design criteria. The angle 2α which yields the proper impedance at each position along the balun may be extracted from Fig. 4. The outer conductor of the coaxial line had an inside diameter of 1.527 inches. The balun was fabricated by milling through the coax outer conductor to the depth which yielded the angle 2α . The milling cut was performed in discrete 6-inch increments along the balun until the outer conductor was reduced to a thin concave strip having a width equal to the center conductor diameter. This occurred at the position $z/l = 0.373$ where $2\alpha = 312^\circ$ and $Z_0 = 131$ ohms. The strip outer conductor was transformed to a circular cylinder identical to the center conductor over a 6-inch length from $z/l = 0.373$ to $z/l = 0.426$. The spacing between cylindrical conductors at $z/l = 0.426$ was such that the impedance was the required 136 ohms as shown in Fig. 7. From $z/l = 0.426$ to $z/l = 0.5$ the spacing of the cylindrical conductors was gradually increased so that the impedance followed the contour of Fig. 7.

Since the balun may be viewed as a two-port waveguide junction, it was convenient to measure its performance by means of Deschamps' method.¹⁰ The two-conductor output of the balun was terminated in a large, reflecting metal sheet mounted perpendicular to the line. The dissipative loss and scattering matrix coefficients of the balun are readily obtained by locating the reflecting sheet at four equally spaced positions and measuring the corresponding reflection coefficient at the coaxial input.¹¹ Since the scattering coefficient S_{11} corresponds to the input reflection coefficient for a reflectionless termination of the output line, one thereby obtains the input VSWR for a matched termination of the two-conductor line. This procedure also avoids the considerable difficulties encountered in providing a matched termination for an open wire line. Over the 40- to 500-mc frequency range, measurements were performed by using a General Radio admittance bridge.

The voltage standing wave ratio as a function of frequency is presented in Fig. 8. It may be seen that the VSWR never exceeded 1.25:1 over the 43- to 2200-mc spectrum which represents a 50:1 bandwidth. The rapid increase in VSWR below the 50-mc cutoff frequency is quite apparent. The balun dissipative loss was not measurable below 500 mc. At 1000 mc, the loss was approximately 0.1 db and increased to 0.3 db at 2000 mc. The spacing between cylindrical conductors at 2000 mc was 0.21λ . It is evident that the tapered balun can be designed to operate over frequency bandwidths as large as 100:1.

It should be noted that the characteristic impedance

¹⁰ G. A. Deschamps, "Determination of reflection coefficients and insertion loss of a waveguide junction," *J. Appl. Phys.*, vol. 24, pp. 1046-1050; August, 1953.

¹¹ F. L. Wentworth and D. R. Barthel, "A simplified calibration of two-port transmission line devices," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-4, pp. 173-175; July, 1956.

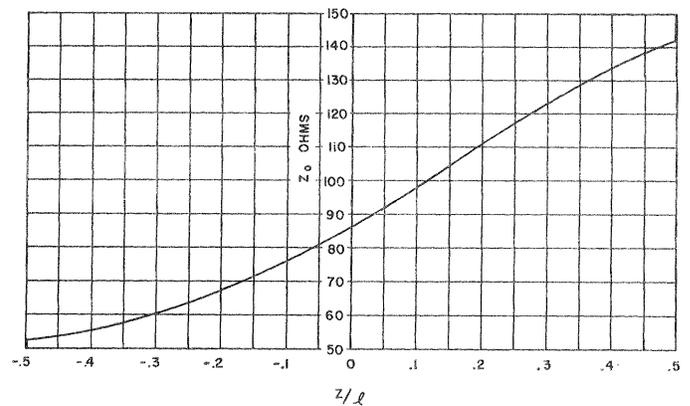


Fig. 7—50- to 150-ohm Tchebycheff impedance taper.

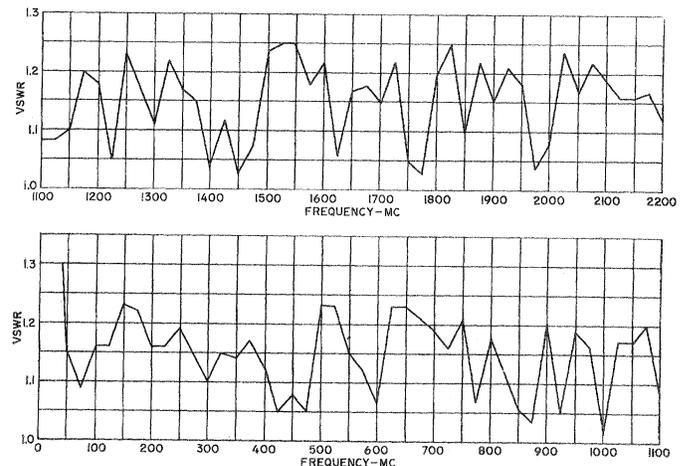


Fig. 8—Experimental performance of tapered balun transformer.

at any cross section of the balun is slightly different than the Z_0 assumed from theory since the slotted line analysis applied to a coax with infinitely thin outer conductor. The effect of finite wall thickness on impedance is greatest for large apertures 2α . Consequently, the synthesis of the required Tchebycheff impedance contour was not accomplished precisely. It appears that the measured VSWR exceeded the design maximum of 1.11 because of reflections from teflon spacers which were used for mechanical support of the line and because the synthesis of the impedance contour was not exact.

Concerning the electrical balance of the balun, it would be fine to prescribe the exact complex ratio of unbalanced to balanced current which results at the two-conductor output of the balun, but, unfortunately, serious questions arise as to the validity or meaning of such a measurement on the open, two-conductor system. We know that the TEM field of the coaxial line is gradually transformed to the TEM field of an open, two-wire transmission line as one traverses the length of the tapered balun transformer. Obviously, not all of the incident power is converted to the transmission line mode. A fraction of the incident power is lost as stray radiation from the slot aperture which forms the tapered transition. That is, the efficiency of excitation of the transmission line mode is necessarily less than 100 per cent.

In addition to the usual TEM transmission line mode, the so-called parallel wave or mode will also be excited.¹² The parallel wave is a transverse magnetic surface wave akin to Sommerfeld's single-wire wave. The parallel wave is evidenced by the superposition of an unbalanced current component (parallel excitation or codirectional currents) with the push-pull currents of the TEM mode. In fact, the common engineering description of this wave phenomenon is to note that the transmission line currents are not balanced, which implies the existence of the parallel wave component of current. The amplitude of the parallel wave field decreases much more slowly with radial distance than does the TEM mode. Because of this fact, the surface wave is quite sensitive to its surroundings and we say that the wave is very loosely bound to the transmission line. At any bends, changes in line cross section, or discontinuities such as line spacers, a significant portion of the mode power will be converted to a radiation field. This is a well-known property of surface wave fields; in fact, some types of surface wave antennas specifically depend upon radiation from obstacles as the mechanism for operation. Wherever radiation occurs, the magnitude of the parallel wave (unbalanced) current will be attenuated. Obviously, then, the measured unbalance on the open two-conductor line will depend upon the line position where the measurement is performed. One question, therefore, the utility or meaning of an "exact" balance measurement on such an open system.

In order to excite any surface wave mode efficiently, the launching source must produce a field which is quite similar to the mode distribution. If the physical parameters of the problem are such that the surface wave field is of large transversal extent, then the launching source must necessarily have a large physical aperture. It so happens that the parallel wave does have a very large transverse distribution so that the tapered balun transformer, which accomplishes a very gradual transition between two TEM field distributions, is a very poor source of the parallel wave mode. Thus, the initial magnitude of the unbalanced current is quite small compared to the balanced current. Furthermore, it is possible to attenuate the unbalanced current in a short distance from the balun terminals by placing several radiating discontinuities such as spacers on the line. Since only the TEM mode exists at a sufficient distance from the balun output, a reflecting plate may be placed there and a network measurement of dissipative attenuation (Deschamps' method) is valid. In view of the foregoing circumstances it would seem more realistic to evaluate electrical balance by the measurement of balun radiation loss since the "net" result of the unbalanced current is, precisely, radiation which may be included in the total dissipative attenuation of the balun. If the total balun attenuation is small, we can be sure that the

unbalanced current is insignificant compared to the balanced current. As a result of the extremely low dissipative attenuation which was measured with the test balun, we conclude that the magnitude of the unbalanced current is negligible and that the tapered balun transformer is inherently a balanced device.

As a final demonstration of the electrical balance resulting from the tapered balun, a scaled model of the previous design was constructed for operation in the kilomegacycle frequency region. The balun was fabricated from $\frac{1}{4}$ -inch-diameter brass tubing and the total length was approximately 12 inches to permit operation down to 500 mc. The impedance taper of the microwave model was identical to the taper of the low-frequency balun. The balun was used to excite dipole radiators at various frequencies from 500 to 5000 mc. *No asymmetry caused by unbalanced excitation currents was evident in the dipole radiation patterns.*

CONCLUSION

The performance of the Tchebycheff tapered balun transformer is unique; it provides near perfect impedance matching over frequency bandwidths as great as 100:1. The balun geometry is not limited to a transition from coax to two-wire transmission line; other output configurations such as a balanced strip line are possible. The basic design allows one to match a large range of impedances with an arbitrarily small standing wave ratio. The balun length is determined by the lowest frequency of operation and the maximum reflection coefficient which is to occur in the pass band. It is evident from the very small dissipative attenuation that negligible radiation results from the balun and that the balun is inherently balanced. From the satisfactory performance of the test model baluns, we know that, by simple scaling according to wavelength and with careful regard to construction, tapered baluns may be operated in the kilomegacycle frequency region. It should also be noted that the balun is well suited to high power applications.

APPENDIX

The formation of a variational principle for the eigenvalue equation

$$\mathcal{L}(\psi) = \lambda \mathcal{M}(\psi) \quad (23)$$

is discussed by Feshbach and Morse.¹³ Here \mathcal{L} and \mathcal{M} are differential or integral operators, ψ is the function upon which \mathcal{L} and \mathcal{M} operate, and λ is the quantity (eigenvalue) whose value is desired. Morse and Feshbach show that if \mathcal{L} and \mathcal{M} are self-adjoint operators, a variational principle for λ is the form

$$\delta[\lambda] = \delta \left[\frac{\int \psi \mathcal{L}(\psi) dv}{\int \psi \mathcal{M}(\psi) dv} \right] = 0, \quad (24)$$

¹² A. Sommerfeld, "Electrodynamics," Academic Press, Inc., New York, N. Y., pp. 198-211; 1952.

¹³ Morse and Feshbach, *op. cit.*, pt. 2, pp. 1108-1109.

which means that the eigenvalue λ is stationary with respect to arbitrary first order variations in the functional form of ψ .

Let the integral operators \mathfrak{L} and \mathfrak{M} , and the function ψ be defined as follows:

$$\mathfrak{L} = \frac{b}{v} \int_{\alpha}^{\pi} G(b, \theta | b, \theta') d\theta',$$

$$\mathfrak{M} = 2b \int_{\alpha}^{\pi} d\theta$$

$$\psi = \sigma(\theta').$$

Then

$$\mathfrak{L}(\psi) = \frac{b}{v} \int_{\alpha}^{\pi} G(b, \theta | b, \theta') \sigma(\theta') d\theta' = \frac{1}{v} \phi_0,$$

$$\mathfrak{M}(\psi) = 2b \int_{\alpha}^{\pi} \sigma(\theta') d\theta' = Q, \quad (25)$$

and (23) takes the form

$$\frac{1}{v} \phi_0 = \lambda Q; \quad (26)$$

i.e., the eigenvalue λ is the characteristic impedance Z_0 . Substituting (25) and ψ into (24), we obtain the variational principle

$$\delta[Z_0] = \delta \left[\frac{\frac{1}{2v} \int_{\alpha}^{\pi} \int_{\alpha}^{\pi} G(b, \theta | b, \theta') \sigma(\theta) \sigma(\theta') d\theta d\theta'}{\left\{ \int_{\alpha}^{\pi} \sigma(\theta') d\theta' \right\}^2} \right] = 0, \quad (27)$$

which shows that Z_0 as given by (11) is stationary with respect to arbitrary first order variations in the functional form of $\sigma(\theta)$.

ACKNOWLEDGMENT

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CORRECTIONS

W. K. Weihe, author of "Classification and Analysis of Image-Forming Systems," which appeared on pages 1593-1604 of the September, 1959, issue of PROCEEDINGS has requested that the following corrections be made to his paper.

In the second paragraph of Section I, on page 1593, the description following the colon on the third line is incomplete. It should read: ". . . the radiation which is being emitted by each individual element and the radiation which is being reflected by the same element and which has its origin inside or outside the scene."

On page 1599, second column, the dimensions in the fourth line after (7) should read $\text{cm}^{-1} \text{deg}^{-1}$.

On page 1602, $4\Gamma_0$ in (23) should be replaced by $\Gamma/4L_0$.

In the equation in the middle of the first column on page 1603, r^2 should be replaced by Γ^2 .

R. Parthasarathy, R. P. Basler, and R. N. DeWitt, authors of the correspondence entitled "A New Method for Studying the Auroral Ionosphere Using Earth Satellites," which appeared on page 1660 of the September, 1959, issue of PROCEEDINGS, have requested that the following corrections be made to their letter.

In the first paragraph of the second column, the time difference mentioned on the tenth line should be 33 ± 1 seconds and the corresponding height given in the next sentence should be $104 \text{ km} \pm 3 \text{ km}$.