Quantum coherence and entanglement in the avian compass

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Tremendous efforts are underway to build technologies that harness the deep quantum phenomena of superposition and entanglement. These properties have proven fragile, often decaying rapidly unless cryogenic temperatures are used. Could life have evolved to exploit such phenomena [1]?

Certain migratory birds have the ability to sense very subtle variations in the Earth’s magnetic field [2]. Here we use recent experimental observations [3] together with the well developed ‘radical pair’ model of the avian compass [4], and employ a master equation with general decoherence operators in order to examine the system’s vulnerability to environmental noise. Remarkably, the room temperature noise tolerance in this natural system appears greater than that of the best man-made molecular radical, N@C60 [5]. We find that entanglement, though probably not an essential feature of this process, appears to persist to tens of microseconds, or more.

The intriguing possibility that living systems already use non-trivial quantum effects to optimise some tasks has led to a number of recent suggestions, ranging from the role of quantum physics in natural selection itself [3], through to the observation that ‘warm and wet’ living systems can embody entanglement given a suitable cyclic driving [4-6]. In particular, the physics of photosynthesis has received considerable attention; a remarkable idea here is that environmental noise is not only tolerated, but even beneficial [9, 10, 11, 12]. In this letter, we examine a particularly important form of natural information processing known as magnetoreception. Unlike the other senses, the organ of magnetoreception is difficult to find, because organic tissue is mostly transparent to magnetic fields.

There are three main mechanisms explaining biological compasses: electromagnetic induction, ferromagnetism, and radical pairs (RP) [2]. Although the first two are essentially classical effects, there is growing body of evidence in certain species (including birds [4], fruit flies [13] and even plants [14]) for the RP mechanism — which relies on the quantum evolution of a spatially-separated interacting pair of electron spins. This is also supported by the growing field of spin chemistry [15-19] and recent experiments which were able to demonstrate a chemical compass [20].

We examine the quantum dynamics of the simplest RP model, using an open system master equation approach. Using our model together with recent experimental data, we infer a timescale for the compass mechanism and then use it to estimate a lower bound for the noise tolerance of the constituent quantum spins. Our first conclusion is that the system tolerates noise to a remarkable extent, greater than that of the best performing man-made molecular system. We speculate that this tolerance is a combination of the inherent robustness of the RP mechanism to specific noise types, together with a high level of protection from other noise sources. We then go on to examine the extent and evolution of entanglement throughout the process, and its likely significance in magnetoreception by the RP mechanism.

There have been a series of experiments in which migratory birds are captured and exposed to artificial magnetic fields [8, 21, 22, 23]. By manipulating a bird’s environment and recording its response, one can make inferences about the mechanism of the magnetic sensor. For European Robins, it is found that the birds are only sensitive to the inclination and not the polarization of the magnetic field [21]. Secondly, if the ambient photons are of low energy (i.e. yellow, red), then the birds’ preferred direction becomes random. If the ambient light is of higher energy (i.e. blue, green, white), then birds show a strong preference for a specific direction [22]. Finally, a very small oscillating field can disrupt the magnetic orientation behaviour [5, 23]. All these experiments can be explained with the Radical Pair Model.

The intriguing idea behind the Radical Pair (RP) Model is that birds ‘see’ the geomagnetic field. Orientated molecules, embedded in the eye’s retina, form a signal pattern dependent on the inclination of Earth’s magnetic field (see Fig.1). The simplest RP model describes the spin of two electrons [4, 24] and one nucleus of the molecule. Absorption of a photon and subsequent transfer of one electron to an acceptor part of the molecule, gives rise to the radical pair. Due to the spatial separation it now becomes meaningful to talk about electron spin entanglement. Without the hyperfine interaction both electrons would precess around the same magnetic field, leaving the singlet state invariant. With the nuclear
interaction present, the singlet state is no longer an eigenstate of this Hamiltonian leading to an angle dependent singlet-triplet oscillation. In other words, both electrons are subject to different local operations. Recombination occurs either from the singlet or triplet state, leading to different chemical endproducts. The concentration of those products constitutes a macroscopic chemical signal, which, due to the HF interaction, is sensitive to the orientation of the molecule with the magnetic field.

We employ the Hamiltonian corresponding to the system once the two electrons have become separated. The anisotropic hyperfine tensor, coupling the nucleus and electron 1, is conveniently written in its diagonal basis

\[
A = \begin{pmatrix}
A_x & 0 & 0 \\
0 & A_y & 0 \\
0 & 0 & A_z
\end{pmatrix}
\]

and we assume an axially symmetric (or cigar-shaped) molecule with \(A_x = 10^{-3}\) meV and \(A_z = A_y = A_z/2\). This is the simplest assumption that can provide us with directionality. It is also possible to choose a more general tensor with lower symmetry, though such a choice complicates the analysis (necessitating the use of two angles to describe the orientation). Nevertheless, we have compared the two and found that the results are qualitatively similar. The Hamiltonian is

\[
H = \hat{I} A \hat{S}_1 + \frac{\gamma}{\hbar} \mathbf{B} \cdot (\hat{S}_1 + \hat{S}_2),
\]

where \(\hat{I}\) is the nuclear spin operator, \(\hat{S}_i = (\sigma_x, \sigma_y, \sigma_z)_i\) are the electron spin operators \((i = 1, 2)\), \(\mathbf{B}\) is the magnetic field vector and \(\gamma = \frac{1}{2}\mu_0 g = \) the gyromagnetic ratio with \(\mu_0\) being Bohr’s magneton and \(g = 2\) the g-factor. The factor \(1/2\) in the gyromagnetic ratio accounts for the fact that we have a spin one-half system, but we will use Pauli matrices such as \(\sigma_z = \text{diag}\{1,-1\}\) etc. Generally

the magnetic field which we employ is

\[
\mathbf{B} = B_0 (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta) + B_{\text{rf}} \cos \omega t (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta),
\]

where \(B_0 = 47\ \mu\text{T}\) is the Earth’s magnetic field in Frankfurt [3], and the angles describe the orientation of magnetic field to the basis of the HF tensor. \(B_{\text{rf}} = 150\ \text{nT}\) is an additional oscillatory field only applied in our simulations where explicitly mentioned. For resonant excitation with the uncoupled electron spin, \(\hbar \omega = 2 \gamma B_0\), so that \(\nu = \omega/(2 \pi) = 1.316\text{MHz}\).

Because of the axial symmetry of the HF tensor we can set \(\varphi = 0\) and focus on the \(\theta\) in the range \([0, \pi/2]\) without loss of generality. Furthermore, for the oscillatory field, we set \(\phi = 0\) and introduce \(\kappa = \theta - \varphi\) as the angle enclosed by oscillatory and static field.

We model the dynamics of the system with a quantum master equation (ME) approach. We employ operators representing the relaxation processes; specifically, we include two ‘shelving states’ which represent the system having decayed either from an electron singlet state, or from one of the triplet states. Ultimately one of these two forms of relaxation will occur. The three spins span an 8 dimensional Hilbert space to which we therefore add two further levels \(|S\rangle\) and \(|T\rangle\) for the singlet and triplet decay outcomes, respectively. The populations of these levels will then correspond the singlet and triplet yield.

With the usual definiton of singlet \(|s\rangle\) and triplet states \(|t_i\rangle\) in the electronic subspace, while \(|\uparrow\rangle\) and \(|\downarrow\rangle\) describing the states of the nuclear spin, we define the following decay operators:

\[
\begin{align*}
P_{S,\uparrow} &= |S\rangle \langle s, \uparrow| \\
P_{S,\downarrow} &= |S\rangle \langle s, \downarrow| \\
P_{T_{\uparrow,\uparrow}} &= |T\rangle \langle t_{\uparrow}, \uparrow| \\
P_{T_{\uparrow,\downarrow}} &= |T\rangle \langle t_{\uparrow}, \downarrow| \\
P_{T_{\downarrow,\uparrow}} &= |T\rangle \langle t_{\downarrow}, \uparrow| \\
P_{T_{\downarrow,\downarrow}} &= |T\rangle \langle t_{\downarrow}, \downarrow|
\end{align*}
\]

and similarly for the ‘down’ nuclear states. This gives us a total of two singlet projectors and six triplet projectors. For simplicity and because this choice corresponds exactly to the expression for singlet yield used in previous literature, all eight projectors have the same decay rate \(\Gamma_P = k\).

For our model we start from an initial density matrix \(\rho(0)\) corresponding to the electrons in a pure singlet state, and a completely mixed nuclear state, i.e.,

\[
\rho(0) = id_s \otimes |s\rangle \langle s| = |s, \downarrow\rangle \langle s, \downarrow| + |s, \uparrow\rangle \langle s, \uparrow|.
\]

The decay to the two shelving levels is then described using a standard quantum optical ME with above decay operators which effectively discriminate singlet and triplet decay events

\[
\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - k \left( \sum_{i=1}^{8} P_i \rho P_i^\dagger - \frac{1}{2} \left( P_i^\dagger \rho P_i + \rho P_i^\dagger P_i \right) \right)
\]

FIG. 1: Schematic of the bird’s eye. The back of the eye contains numerous molecules, fixed with specific orientations. In the simplest RP model, each such molecule involves three crucial components (see inset): there are two electrons, initially photo-excited to a singlet state, and a nuclear spin that couples to one of the electrons. This coupling is anisotropic, so that the molecule has a directionality to it.
For comparison, we performed a more general approach with coherent decay from each basis state into its own corresponding bin level, with subsequent projection onto singlet and triplet components. That approach gives rise to identical system dynamics and singlet yield and so is unnecessarily complicated for the problem at hand.

In the absence of noise operators (discussed below) it is possible to compare the predictions of our full ME approach with the singlet yield integral that is commonly used in the prior literature. Specifically, we compare the ultimate population of our singlet ‘shelf’ $|S\rangle$ with the quantity $\Phi = \int_0^\infty \langle \psi^- | Tr_n(\rho(t)) | \psi^- \rangle k e^{-kt} dt$. As expected, we find that in the limit where $k$ is small, i.e. the process takes a long time compared to the rate at which the coherent dynamics take place, the two quantities agree.

We now wish to determine an appropriate choice for our parameter $k$ in Eqn. 2. In Ref. 3 the authors report that a perturbing magnetic field of frequency of 1.316 MHz (i.e. the resonance frequency of the ‘remote’ electron) can disrupt the avian compass. They note that this immediately implies a bound on the decay rate (since the field would appear static for sufficiently rapid decay). Here we aim to refine this bound on $k$ by considering the oscillating magnetic field strength which suffices to completely disorient the bird’s compass, i.e. 150 nT. (Indeed, even a 15 nT field was reported as being disruptive, but to be conservative in our conclusions we take the larger value here.) To model this effect, we activate the oscillatory field component defined in Eqn. 1 and examine the singlet yield (i.e. the eventual population of shelving state $|S\rangle$) as a function of the angle between the Earth’s field and the molecular axis. Consistent with the experimental work, we find that there is no effect at such weak fields when the oscillatory field is parallel to the Earth’s field. Therefore for our analysis we set the oscillatory field to be perpendicular. The results are shown in Figure 2. We conclude that if the oscillating field is to disorient the bird, as experiments showed, then the decay rate $k$ should be approximately $10^4$ s$^{-1}$ or less. For higher values of $k$ (shorter timescales for the overall process) there is no time for the weak oscillatory field to significantly perturb the system; it relaxes before it has suffered any effect. Such a value for the decay rate is consistent with the long RP lifetimes in certain candidate cryptochrome molecules found in migratory birds.

Taking the value $k = 10^4$ s$^{-1}$, we are able to move to the primary question of interest: how robust this mechanism is against environmental noise. There are several reasons for decoherence. For example dipole interaction, electron-electron distance fluctuations and other particles’ spin interaction with the electrons will cause decoherence. We describe generic environmental noise with a standard Lindblad ME technique [26], where the following dissipator is additional to the terms in Eqn. 2 above:
where the angle between the Earth’s field and the molecular axis in \( \pi/4 \), although the behavior at other angles is similar. The entanglement metric is negativity as defined in the text.

\[
\dot{\rho} = \text{above terms} + \sum_i \Gamma_i \left( L_i \rho L_i^\dagger - \frac{1}{2} \left( L_i^\dagger L_i \rho + \rho L_i^\dagger L_i \right) \right) \tag{3}
\]

Noise operators \( L_i \) are \( \sigma_x, \sigma_y, \sigma_z \) for each electron spin individually (i.e. tensored with identity matrices for the nuclear spin and the other electron spin). This gives a total of six different noise operators \( L_i \) and we use the same decoherence rate \( \Gamma \) for all of them. We are now in a position to determine the approximate level of noise which the compass suffers, by finding the magnitude of \( \Gamma \) for which the angular sensitivity fails. This is shown in Fig. [3] Conservatively, we can say that when \( \Gamma \geq k \), the angular sensitivity is highly degraded. This is remarkable, since it implies the decoherence time of the two-electron compass system is of order 100 \( \mu s \) or more! For comparison, the best laboratory experiment involving preservation of a molecular quantum state has accomplished a decoherence time of 80\( \mu s \) [5].

It is interesting to ask, what is the significance of entanglement between the spins in the avian compass? Having inferred approximate values for the key parameters, we can plot an appropriate entanglement measure over the course of the process, from the initial singlet state to the eventual decay. The metric we use is negativity:

\[
N(\rho) = \frac{\|\rho^{TA}\|}{2}
\]

where \( \|\rho^{TA}\| \) is the trace norm of the partial transpose of the system’s density matrix. The transpose is applied to the uncoupled electron, thus performing the natural partitioning between the electron, on one side, and the coupled electron plus its nucleus, on the other. Fig. [4] shows how this negativity evolves under our noise model. Clearly, the initial singlet state is maximally entangled. Under noise, entanglement falls off at a faster rate than the decay of population from the excited state.

Although a physical mechanism for pure dephasing is somewhat questionable, we can apply such noise in our model to investigate the role of entanglement in this process. We have studied the effect of pure dephasing noise on the system and observed far less degradation on angular visibility for a given strength \( \Gamma \), than the generic \( x, y, z \) noise model described above. Interestingly, if we begin the simulation with a completely dephased state: \( \langle s\rangle\langle s| + |f_0\rangle\langle f_0|)/2 \), then the classical correlations are still sufficient for achieving adequate angular visibility. Thus, entanglement does not appear to play an essential role in the efficiency of the avian compass.

In summary, we used an open system master equation approach to examine the implications of recent experimental data on European Robins upon spin coherence in biological magnetoreception. We have been conservative in our line of inference (indeed, we could reasonably have started with an oscillatory field of 15 nT rather than 150 nT) and yet found remarkably long spin coherence times under a general noise model. We further conclude that while entanglement does not appear necessary to the process of avian magnetoreception under the RP mechanism, it is most likely not only present, but persists out to long times.

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We note that in the final stages of preparation of this manuscript, a related work has appeared which considers a chemical magnetometer in the context of quantum control [27].

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